Kinematics: 1

Important concepts

- **Lab frame:**
  - Where measurements are made
- **CM / rest frame:**
  - Where particle/system is at rest
- **Invariant mass:**
  - Particle mass in every frame
- **CM energy:**
  - Invariant mass of system of particles

Frames

We can view collisions from many standpoints (“frames”).

Observer in laboratory

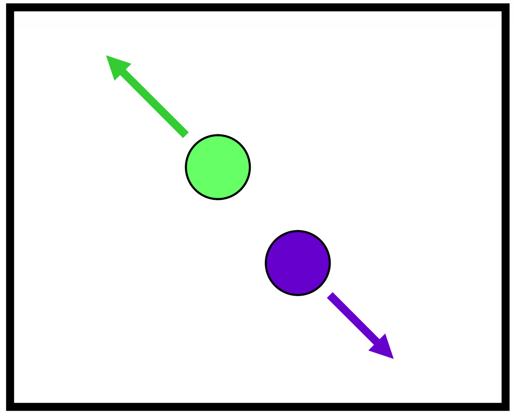
Observer in centre of collision

In HEP

We detect particles in lab frame

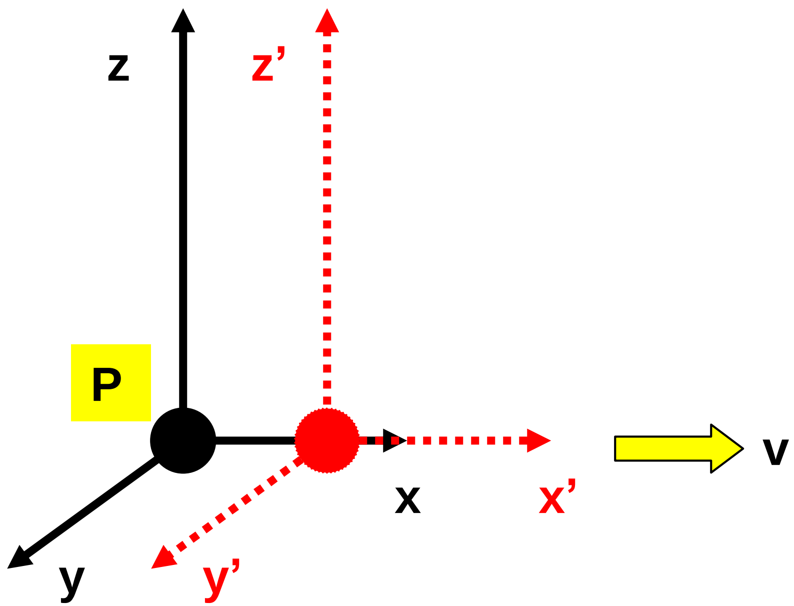
We deduce their properties in CM frame

LAB FRAME



CM FRAME

Relating frames



Consider particle p in lab and CM frame:
CM frame at rest (black)
Lab frame moves with velocity v (red)
Frames coincide at t=0.

Classically:
(x,y,z)   in cm frame
(x',y',z') in lab frame

$x' = x - vt$

$y' = y,\ z' = z$

$t' = t$

In HEP particles move near speed of light; use special relativity instead.

Four vectors

Compact notation for defining position (+ time), energy + momentum.

- Particle position; (ct,x,y,z)
- Particle energy/momentum; (E, cp_x, cp_y, cp_z)

Magnitude of vector (**a,b,c,d**): $\mathbf{L^2 = a^2 - b^2 - c^2 - d^2}$

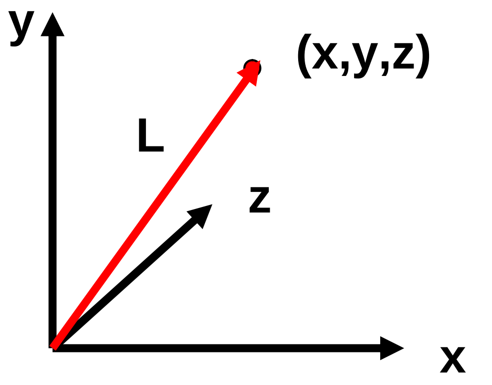
- **invariant** between frames

eg. Position: $c^2t^2 - x^2 - y^2 - z^2$

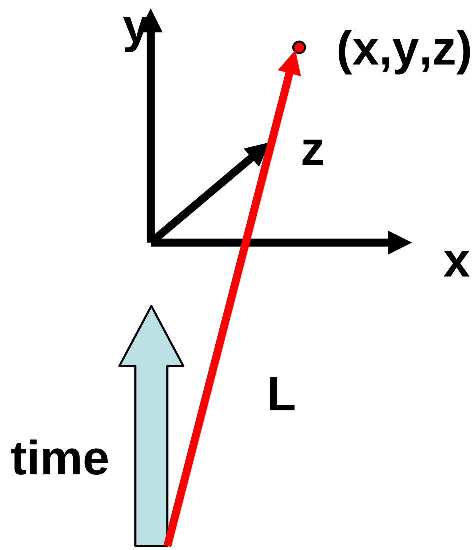
or energy-momentum: $E^2 - c^2px^2 - c^2py^2 - c^2pz^2$

Four vectors

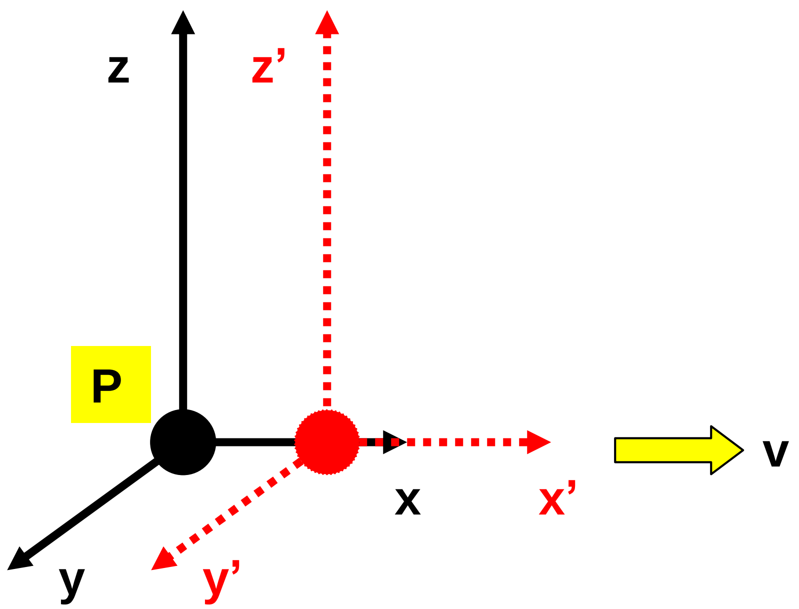
In 3D; $\mathbf{L^2 = x^2+y^2+z^2}$
invariant under rotation
(ie. x,y,z change, but L
doesn't)



In special relativity; must
include fourth dimension



Rules for relating frames



Position:

(ct',x',y',z')   lab frame

(ct,x,y,z)   cm frame

$x' = \gamma(x - vt)$

$y' = y$

$z' = z$

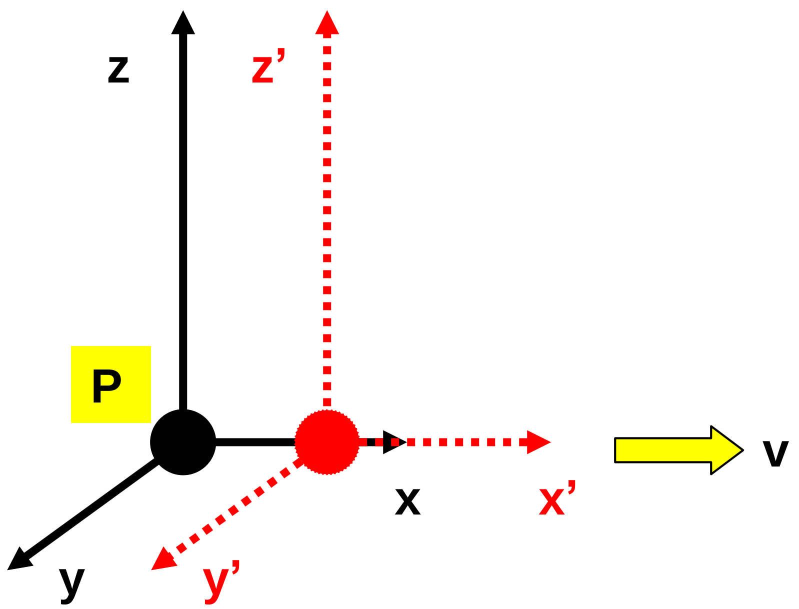
$t' = \gamma(t - vx/c^2)$

Exercise: prove $c^2t^2-x^2-y^2-z^2$ invariant

$\beta = v/c$

$\gamma = 1/\sqrt{(1 - \beta^2)}$

Rules for relating frames



Energy/momentum:

$(E', cp_x', cp_y', cp_z')$ lab frame

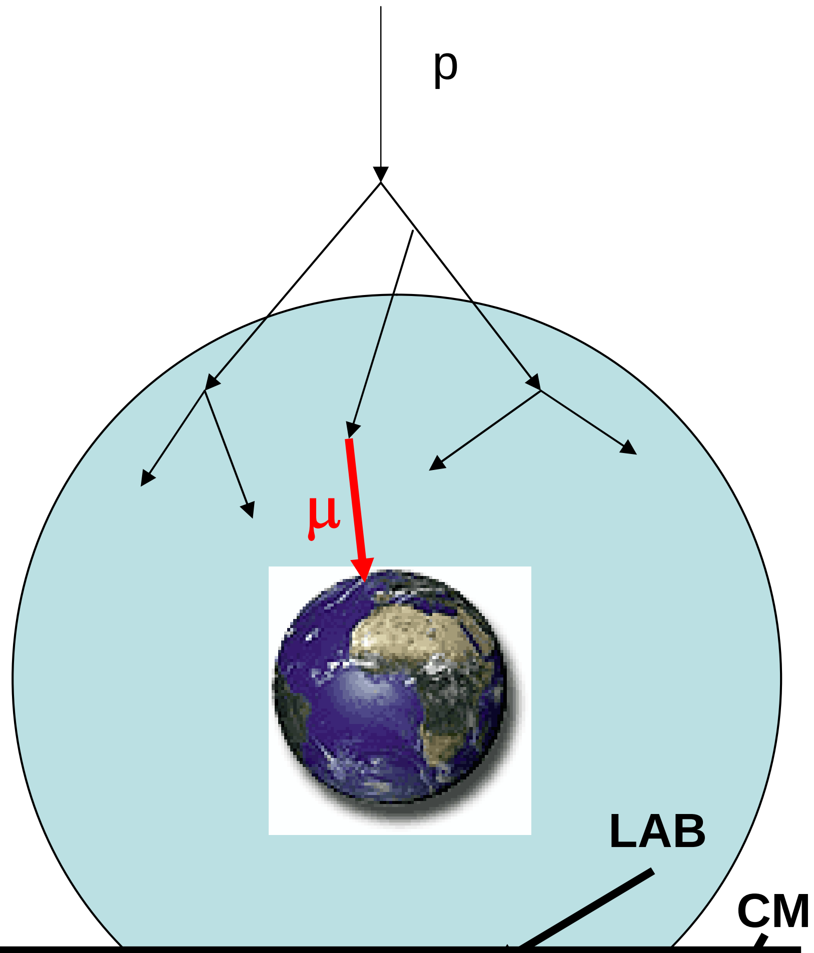
(E, cp_x, cp_y, cp_z) cm frame

$$p_x' = \gamma(p_x - vE/c^2)$$
$$p_y' = p_y$$
$$p_z' = p_z$$
$$E' = \gamma(E - vp_x)$$

Exercise: prove $E^2 - c^2p_x^2 - c^2p_y^2 - c^2p_z^2$ invariant

Example: detecting μ in cosmic rays

Muons are typically produced in cosmic rays as protons hit the upper atmosphere. Would we detect them on the Earth's surface?



Energy of muon $E_\mu \sim 4$ GeV
Lifetime of muon $T_\mu \sim 2.10^{-6}$ s
Mass of muon $M_\mu \sim 0.106$ GeV/c^2

Invariant mass

Follows from energy-momentum 4 vector.

Eg. Single particle.

invariant $= E^2 - c^2p^2$ (general case: all frames)

$= m^2c^4$ (rest frame; $p^2=0$)

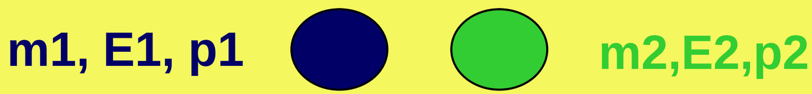
For single particle invariant mass = rest mass

- characteristic of particle type

Invariant mass/ CM energy

Eg.(2) System of > 1 particle.

invariant $=(\Sigma E)^2 - c^2(\Sigma p)^2$ (combined 4 vector)



General solution: $(E1 + E2)^2 - c^2(p1 + p2)^2$

$= E1^2 + E2^2 + 2.E1E2 - c^2p1^2 - c^2p2^2 - 2c^2p1.p2$

$= m1^2c^4 + m2^2c^4 + 2E1E2 - 2c^2p1.p2$

…. Valid for any frame … but horrible maths!

Often easier to consider rest frame: $= \Sigma E^2$ (rest frame; $\Sigma p^2=0$)

Special case if both particles at rest $= (m1 + m2)^2c^4$

If > 1 particle, invariant mass called CENTRE-OF-MASS ENERGY

Summary

- Important HEP concepts:
  - Centre of mass frame, lab frame
  - Invariant mass
  - CM energy
- Special relativity reminder:
  - How to relate position between frames
  - How to relate energy and momentum between frames