Lecture 19

- Fluids in motion
  - Ideal fluids
  - Streamlines
  - Equation of continuity
  - Bernoulli’s equation
  - Venturi Meter

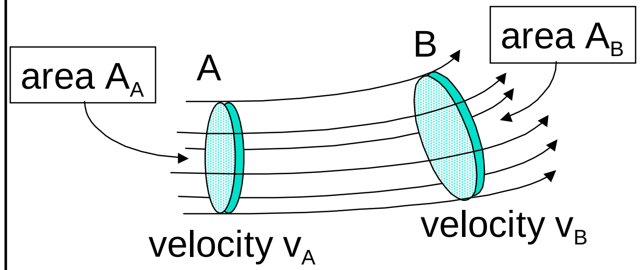
Ideal Fluids

- The motion of real fluids is very complicated.
- Using some simplifying assumptions to define an ideal fluid allows a reasonable description of much of the behaviour of fluids.
- An ideal fluid is <u>incompressible</u> and its flow is:
  - <u>laminar</u>
  - <u>non-viscous</u>
  - <u>irrotational.</u>

Streamlines

- A streamline is the path traced out by a tiny element of the fluid (a fluid "particle").
- The velocity of the fluid particle is tangential to the streamline.
- Streamlines never cross.
- A set of streamlines can define a tube of flow, the borders of which the fluid does not cross.

Equation of Continuity



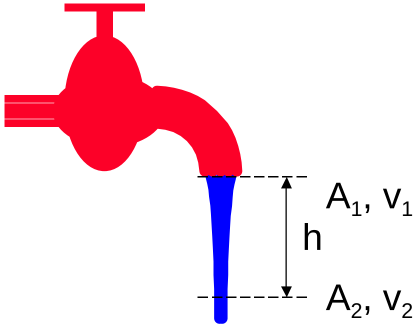
- Fluid incompressible, so volume ΔV through A in time Δt same as volume through B, i.e.

  $$\Delta V = A_A v_A \Delta t = A_B v_B \Delta t$$

  $$\Rightarrow A_A v_A = A_B v_B$$
- The volume flow rate $R = Av$ is constant (units $m^3 s^{-1}$).
- As fluid has constant density (it is incompressible) the mass flow rate $R\rho = Av\rho$ is also constant (units $kg\ s^{-1}$).

Equation of continuity cont.

- Example, decrease in area of stream of water coming from tap.



- Continuity equation $A_1 v_1 = A_2 v_2$
- Water is falling under influence of gravity so
  $v_2^2 = v_1^2 + 2gh$

Equation of continuity cont.

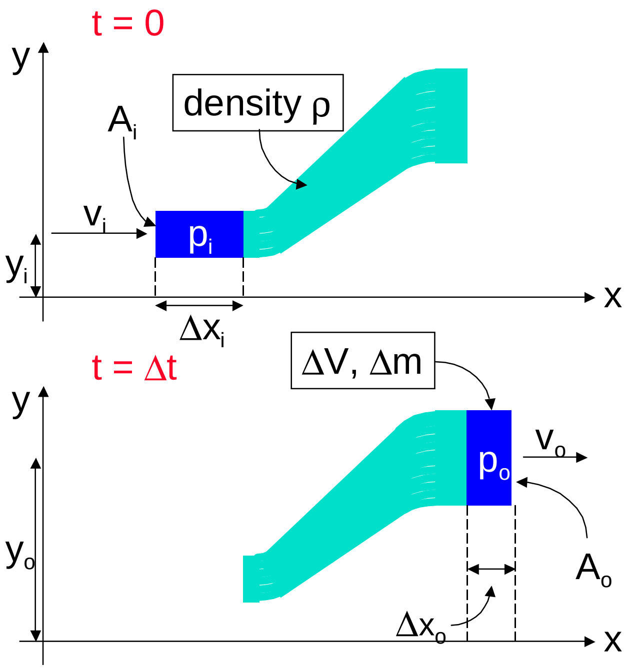
- Measurement of rate of flow
  R = Av, plus known x-sect of tap (A_1)
  allows calc. of x-sect throughout fall.

$$A_2^2 = \frac{A_1^2 v_1^2}{v_2^2} = \frac{(A_1 v_1)^2}{v_1^2 + 2gh} = \frac{(A_1 v_1)^2}{\left(A_1 v_1 / A_1\right)^2 + 2gh}$$

$$A_2 = \frac{R_1}{\sqrt{\left(R_1 / A_1\right)^2 + 2gh}}$$

Bernoulli's Equation

- Consider fluid flow through pipe, or tube of flow delimited by streamlines.
- Take two snapshots separated by Δt.



Bernoulli's equation cont.

- In time Δt, volume ΔV (mass Δm) flows through pipe (dark blue in diagram).
- KE of this vol. of fluid as it enters pipe $K_i = \frac{1}{2}\Delta m v_i^2 = \frac{1}{2}\rho \Delta V v_i^2$
- KE as it leaves pipe $K_o = \frac{1}{2}\rho \Delta V v_o^2$
- Change in KE $\Delta K = \frac{1}{2}\rho \Delta V\left(v_o^2 - v_i^2\right)$
- Work done by fluid against gravity $W_g = -\Delta m g(y_o - y_i)$
- Work done on fluid to push it into pipe $W_i = F_i \Delta x_i = p_i A_i \Delta x_i = p_i \Delta V$
- Work done by fluid in leaving pipe $W_o = -p_o \Delta V$
- Total work done

$$W = W_g + W_i + W_o$$

$$= -\rho g \Delta V(y_o - y_i) + \Delta V(p_i - p_o)$$

Bernoulli's equation cont.

- Equate KE and work

$$\Delta K = W$$

$$\tfrac{1}{2}\rho\Delta V\left(v_o^{\,2} - v_i^{\,2}\right) = -\rho g\Delta V(y_o - y_i) + \Delta V(p_i - p_o)$$

$$\Rightarrow \tfrac{1}{2}\rho\left(v_o^{\,2} - v_i^{\,2}\right) = -\rho g(y_o - y_i) + (p_i - p_o)$$

- Rearranging gives the standard form of Bernoulli's equation

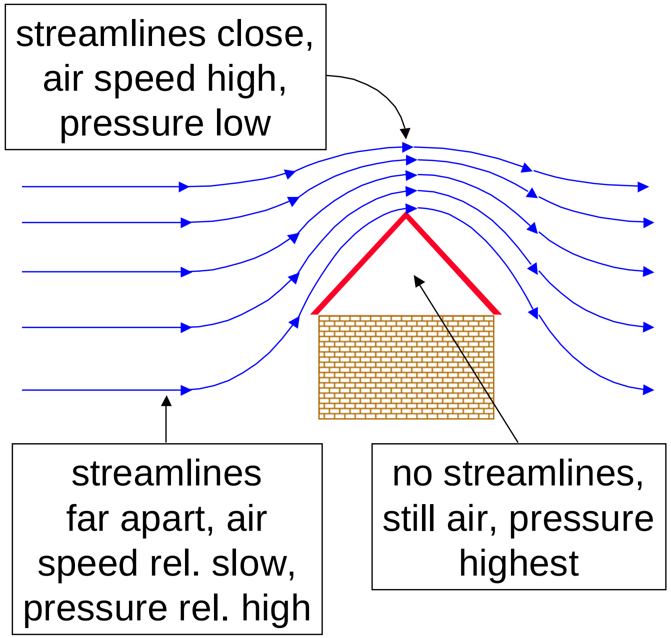
$$p_i + \tfrac{1}{2}\rho v_i^{\,2} + \rho g y_i = p_o + \tfrac{1}{2}\rho v_o^{\,2} + \rho g y_o$$

- Alternatively

$$p + \tfrac{1}{2}\rho v^2 + \rho g y = \text{const.}$$

Bernoulli's equation cont.

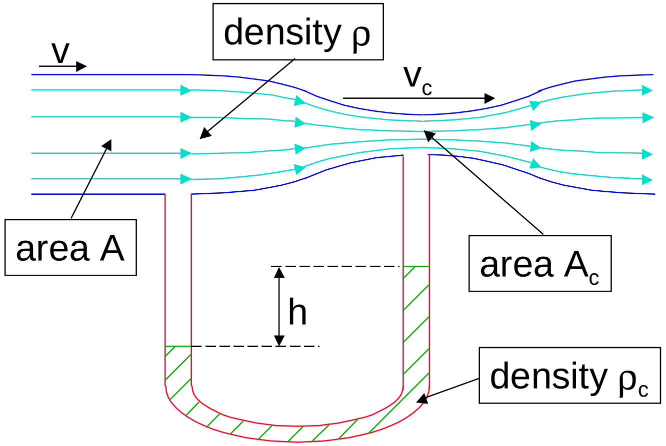
- Example, tiles lifting off roof in a gale.



- Higher pressure inside roof cavity causes upward force on tiles.

Venturi Meter

- Used to measure speed, or rate, of fluid flow in a pipe.



- Pressure difference $\Delta p = h\rho_c g$
- Continuity equation

$$Av = A_c v_c$$

$$\Rightarrow v_c = \frac{A}{A_c} v$$

Venturi meter cont.

- Bernoulli's equation for level flow

$$p + \tfrac{1}{2}\rho v^2 = p - \Delta p + \tfrac{1}{2}\rho v_c{}^2$$

$$\Rightarrow v^2 = \frac{2}{\rho}\left(\tfrac{1}{2}\rho v_c{}^2 - \Delta p\right)$$

$$= \frac{A^2}{A_c{}^2} v^2 - \frac{2\rho_c gh}{\rho}$$

$$\Rightarrow v^2\left(\frac{A^2}{A_c{}^2} - 1\right) = 2\frac{\rho_c}{\rho} gh$$

$$\Rightarrow v^2 = 2\frac{A_c{}^2}{A^2 - A_c{}^2}\frac{\rho_c}{\rho} gh$$

$$\text{and } v = \sqrt{2\frac{A_c{}^2}{A^2 - A_c{}^2}\frac{\rho_c}{\rho} gh}$$