Lecture 18

- Fluids at rest
  - Density
  - Pressure
  - Hydrostatic pressure
  - Pascal's principle
  - Archimedes principle

Fluids at Rest

- A fluid cannot support a shearing stress (cannot sustain force tangential to its surface), so fluids:
  - take on shape of container;
  - flow.
- Density at point in fluid. Consider vol. ΔV about point, mass Δm, density is

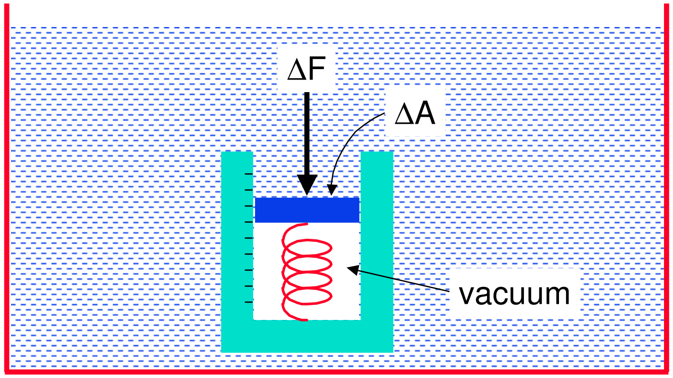
$$\rho = \frac{\Delta m}{\Delta V} \text{ (units kg m}^{-3}\text{)}$$

Typical densities	kg/m^3
Interstellar space	10^{-20}
Best lab vacuum	10^{-17}
Air (20 C, 1 atm)	1.21
Water (20 C, 1 atm)	998
Iron	7.9×10^3
Black hole (solar mass)	10^{19}

Fluids at rest cont.

- Pressure at point

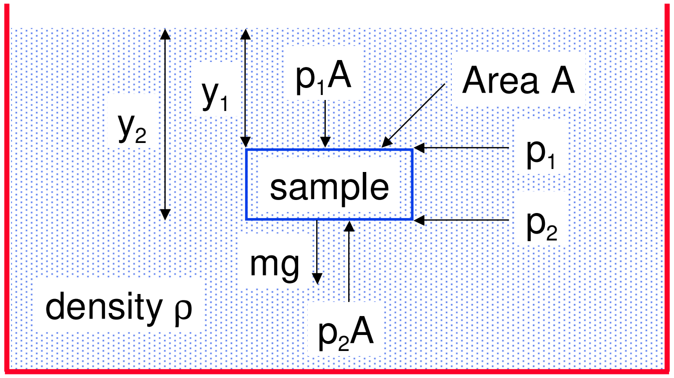
$$p = \frac{\Delta F}{\Delta A} \text{ (units } Nm^{-2} \text{ or Pa)}$$



Typical pressures	Pa
Centre of sun	2×10^{16}
Highest lab pressure	1.5×10^{10}
Stilletto heels	1×10^{6}
Atm at sea level	1.0×10^{5}
Best lab. vacuum	10^{-12}

Hydrostatic Pressure

- Pressure due to fluid at rest.



- Fluid at rest, so for test sample:

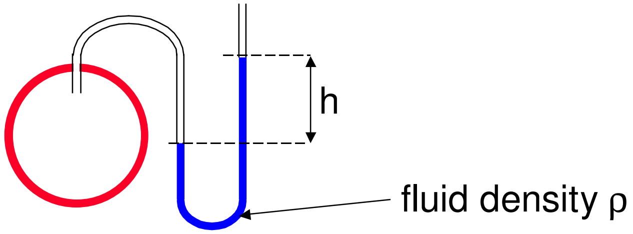
$$p_2A = p_1A + mg$$

$$= p_1A + \rho gA(y_1 - y_2)$$

$$\Rightarrow p_2 = p_1 + \rho g(y_1 - y_2)$$

Gauge Pressure

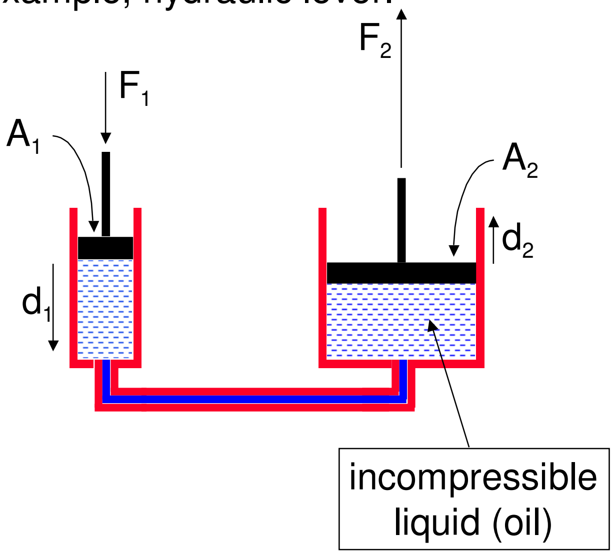
- Hence, if pressure at surface is p_0, pressure at depth h in fluid is: $p = p_0 + \rho g h$
- In a manometer, this is used to measure pressure:



- Pressure in vessel $p = p_0 + \rho g h$ where p_0 is atmospheric pressure.
- The difference between absolute pressure p and atmospheric pressure is called gauge pressure. In above e.g. gauge pressure $p_g = \rho g h$

Pascal's Principle

- Any change in pressure of fluid in container is communicated to every portion of fluid and to walls of container.
- Example, hydraulic lever.

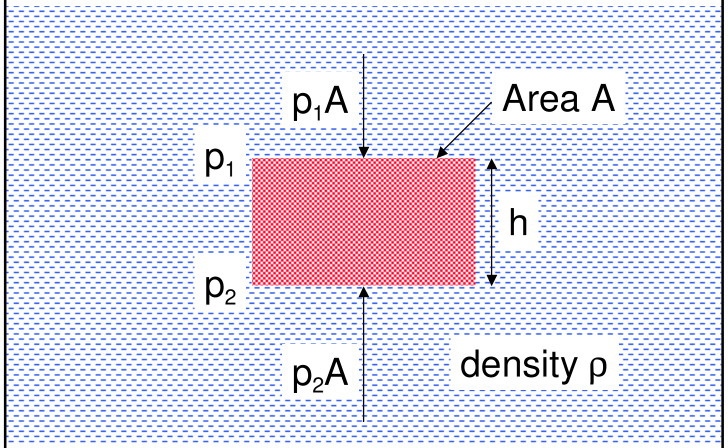


Pascal's principle cont.

- Force F_1 applied on piston one causes pressure change: $\Delta p = F_1 / A_1$
- From Pascal's principle, same pressure change occurs at piston two, hence: $\Delta p = F_2 / A_2$
- Hence force can be magnified:
$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_2 = \frac{A_2}{A_1} F_1$$
- Same volume, V, of liquid displaced at both pistons:
$$V = A_1 d_1 = A_2 d_2 \Rightarrow d_2 = \frac{A_1}{A_2} d_1$$
- Work done by piston two same as work done on piston one:
$$W = F_2 d_2 = \frac{A_2}{A_1} F_1 \frac{A_1}{A_2} d_1 = F_1 d_1$$

Archimedes' Principle

- Buoyancy force exerted on object immersed in fluid is equal to weight of fluid displaced.
- Proof for rectangular prism:



Archimedes' principle cont.

- Buoyancy force B given by:

$$\begin{aligned} B &= p_2 A - p_1 A \\ &= (p_1 + \rho g h)A - p_1 A \\ &= \rho g h A \end{aligned}$$

  which is the weight of the fluid displaced.
- Objects float if buoyancy force equal to weight of object.
- Example, how much of iceberg is underwater?
  Volume of iceberg V_i,
  density of ice ρ_i=917 kg/m^3,
  of sea water ρ_w=1024 kg/m^3.

Archimedes' principle cont.

- Weight of iceberg $W_i = V_i \rho_i g$
- Archimedes' principle says weight of displaced water is same, so volume of displaced water can be found from:

$$W_i = V_w \rho_w g = V_i \rho_i g$$

$$\Rightarrow V_w = \frac{\rho_i}{\rho_w} V_i$$

- Proportion of iceberg under water (by volume) is then

$$\frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = \frac{917}{1024} = 0.896$$