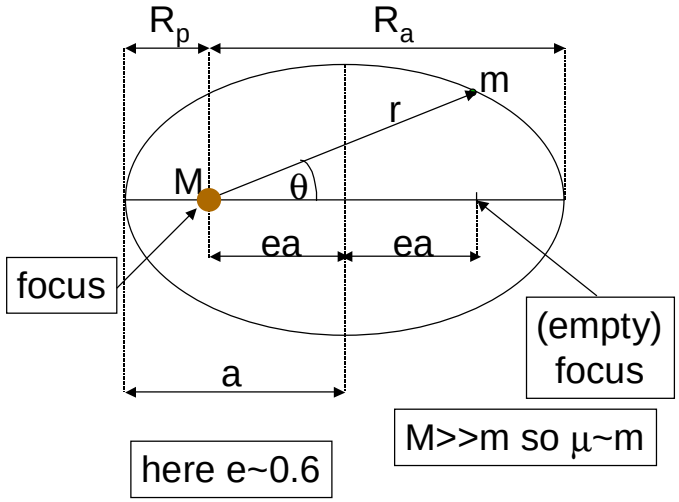
Lecture 17

- Kepler's Laws
- Orbits and Energy
- Escape Speed
- Principle of Equivalence
- Deflection of Starlight

Kepler's Laws

- Kepler's empirical laws are consequence of gravitation, we show this with some restrictions.
- 1) All planets move in elliptical orbits with the sun at one focus.

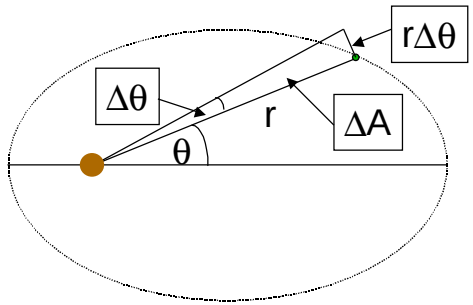


Kepler's Laws cont.

- Quantities describing elliptical orbit:
  - semi-major axis a (r for circle)
  - eccentricity e (0 for circle)
  - perihelion (position at closest approach to sun, dist. R_p)
    - » called perigee if orbit round Earth
  - aphelion (position at greatest distance from sun, dist. R_a)
    - » called apogee if orbit round Earth
- Eccentricity of Earth's orbit 0.0167 (both foci within sun)
- See "Maths Appl. to Mech.s" by Thomas for more detail.

Kepler's Laws cont.

- 2) A line that connects a planet to the sun sweeps out equal areas in equal times.
- Prove this is equivalent to conservation of angular momentum.

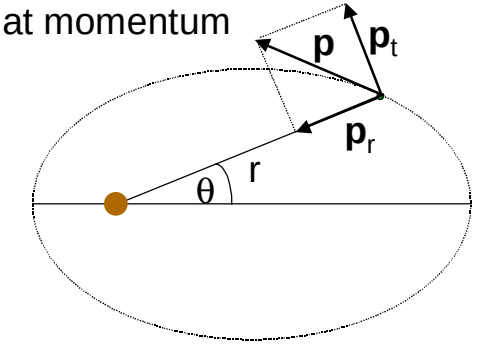


$$\Delta A \approx \frac{r^2 \Delta\theta}{2} \Rightarrow \frac{\Delta A}{\Delta t} \approx \frac{r^2}{2}\frac{\Delta\theta}{\Delta t}$$

$$\frac{dA}{dt} = \frac{r^2}{2}\frac{d\theta}{dt} = \frac{r^2\omega}{2}$$

Kepler's Laws cont.

- Look at momentum



- Angular momentum given by

$$L = rp_t = (r)(mv_t) = (r)(m\omega r) = mr^2\omega$$

$$\text{cf. } \frac{dA}{dt} = \frac{r^2\omega}{2}$$

$$\Rightarrow \frac{dA}{dt} = \frac{L}{2m}$$

- Equal areas in equal times same as const. L

Kepler's Laws cont.

- 3) The square of the orbital period of any planet is proportional to the cube of the semi-major axis of its orbit.
- Prove for circular orbit

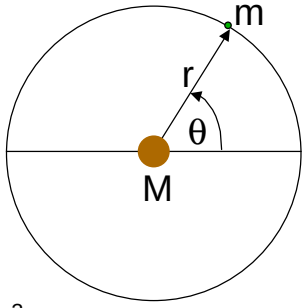
$$F = ma$$

$$\Rightarrow \frac{GMm}{r^2} = mr\omega^2$$

$$\text{Now } \omega = \frac{2\pi}{T}$$

$$\therefore \frac{GM}{r^2} = r\left(\frac{2\pi}{T}\right)^2$$

$$\text{Hence } T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$



Orbits and Energy

- Potential energy of satellite-earth system (zero at infinity, negative at earth's surface!)

$$U = -\frac{GMm}{r}$$

- Kinetic energy from

$$\frac{GMm}{r^2} = m\frac{v^2}{r} \Rightarrow v^2 = \frac{GM}{r}$$

$$\therefore K = \frac{mv^2}{2} = \frac{GMm}{2r}$$

- For circular orbit see

$$K = -\frac{U}{2} \text{ and}$$

$$E = K + U = -\frac{GMm}{2r}$$

Orbits and energy cont.

- Latter holds also for elliptical orbits if replace radius by semi-major axis a, ie.

$$E = -\frac{GMm}{2a}$$

- Note, for elliptical orbits it is <u>not</u> true that

$$K = -\frac{U}{2}$$

- Escape speed is that necessary for object to just escape from earth's gravitational field, need

$$\Delta K = -\Delta U \Rightarrow \frac{mv^2}{2} = \frac{GMm}{R}$$

$$\Rightarrow v = \sqrt{\frac{2GM}{R}}.$$

Principle of Equivalence

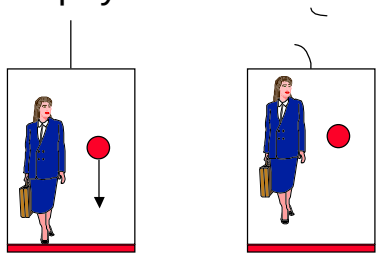
- There are two types of mass
  - The mass that causes inertia, ie. the "m" in $F = ma$
  - The mass which is the source of the gravitational field, ie. the "m's" in

$$F = -\frac{Gm_1m_2}{r^2}$$

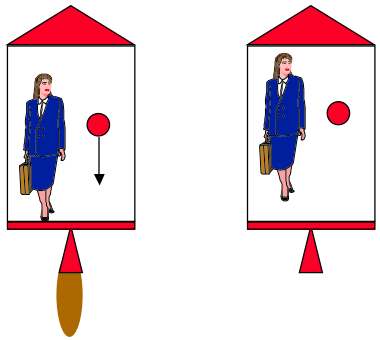
- We have assumed these are the same and experiment shows this is the case to 1 part in 10^{12}!
- Einstein realised this means we can't tell the difference between gravitation and acceleration, they are equivalent. This is at the heart of General Relativity.

Principle of equivalence cont.

- Compare physicist in lift



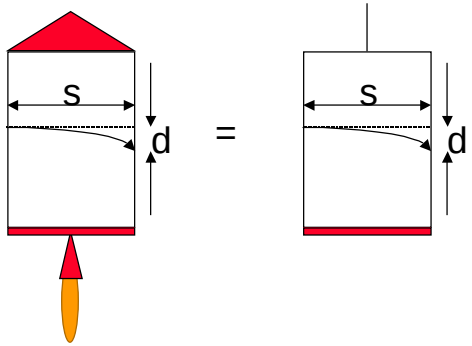
- with physicist in rocket



- Can't distinguish lift from rocket unless you look out of window!

Principle of equivalence cont.

- Implies light is bent by “gravity”



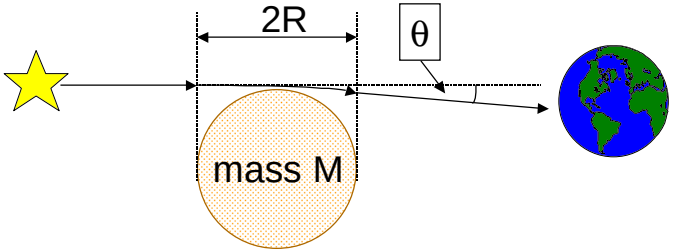
- Rocket accelerates in time light takes to travel across lift/rocket, so light hits right wall lower than height at left wall by distance

$$d = \frac{at^2}{2} = \frac{a}{2}\left(\frac{s}{c}\right)^2$$

- Angle of deflection $\theta \approx \frac{d}{s} = \frac{as}{2c^2}$

Principle of equivalence cont.

- For starlight grazing sun



$$\theta \approx \frac{gs}{2c^2} = \frac{GM}{R^2}\frac{2R}{2c^2}$$

$$\approx \frac{GM}{Rc^2} = \frac{6\cdot67\times10^{-11}\times2\cdot0\times10^{30}}{7\cdot0\times10^{8}\times\left(3\cdot0\times10^{8}\right)^2}$$

$$\approx 2\times10^{-6}\ \text{rad} = 0\cdot4''$$

- General relativity gives $\theta = 1\cdot75''$
- Eddington (1919) measured $\theta = 1\cdot93'' \pm 0\cdot15''$