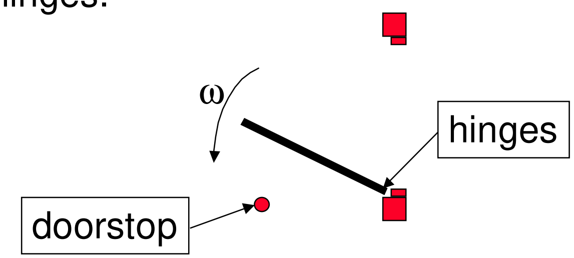
Lecture 11

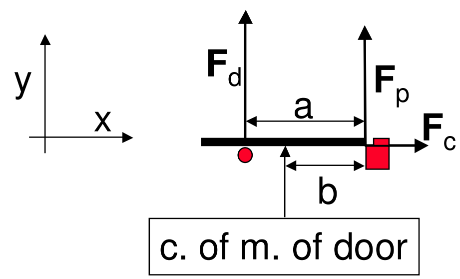
- Centre of Percussion
- Precession of the Equinox
- Stability of Spinning Objects
- Rolling Coin
- Gyrocompass

Centre of Percussion

- Example, placing door stop in position which minimises forces on hinges.



- Consider forces at moment of impact



Centre of percussion cont.

- F_d and F_p are large impact forces, F_c is smaller centripetal force. Minimise damage to hinge by making F_p as small as possible.
- Consider change of ang. mom. of door about hinges (along z axis)

$$L_f - L_i = 0 - (-I\omega)$$
$$= \int \tau \, dt = \int F_d a \, dt$$
$$I\omega = a \int F_d \, dt \qquad (1)$$

- Consider change of linear momentum of door's c. of m.

$$p_f - p_i = 0 - (-Mb\omega)$$
$$= \int F_d + F_p \, dt$$
$$Mb\omega = \int F_d \, dt + \int F_p \, dt \qquad (2)$$

Centre of percussion cont.

- Now substitute expression for F_d from (1) in (2) to get

$$Mb\omega = \frac{I\omega}{a} + \int F_p \, dt$$
$$\int F_p \, dt = \left(Mb - \frac{I}{a} \right) \omega$$

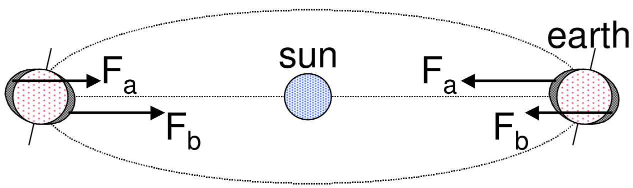
- Minimise by choosing $a = \dfrac{I}{Mb}$
- Using $I = \frac{1}{3} M w^2$
where w=2b is width of door we see

$$a = \frac{4Mb^2}{3Mb} = \frac{4b}{3} \text{ or } \frac{2w}{3}$$

- This point is centre of percussion. Another e.g. “sweet spot” on squash or tennis racket. (Ball on sweet spot, least reaction on player’s hands.)

Precession of the Equinox

- Direction of earth's axis changes (slowly) w.r.t fixed stars, why?



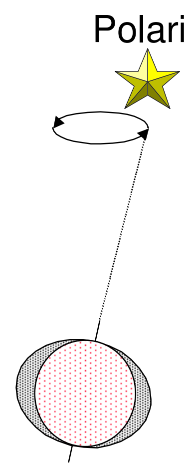
winter $F_a < F_b$ (further from sun) torque out of plane	summer $F_a > F_b$ (nearer to sun) torque out of plane

- Bulge due to earth's rotation, radius at equator 21 km larger than at poles.
- At spring and autumn equinoxes no torque, averaged over year have torque in one direction.

Precession of the equinoxes cont.

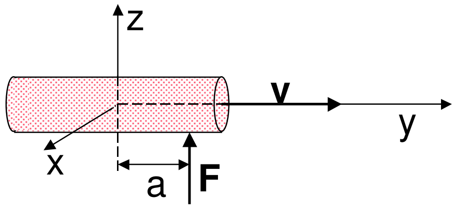
- Similar effect due to moon, torque about two times larger.
- Average torque perpendicular to spin ang. mom. and in plane of ecliptic, c.f. "Precession of Gyroscope".

Period of precession 26000 years. 13000 years from now polar axis will point 47° away from present pole star Polaris.



Stability of Spinning Objects

- Look at stability of moving object subject to small disturbing force



- Apply force F for time Δt. Torque about x is Fa so angular impulse is $Fa\Delta t$.

$$\Delta L_x = I_x \Delta\omega = Fa\Delta t$$

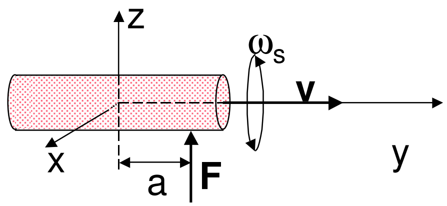
$$\Delta\omega = (\omega_x - \omega_0) = \frac{Fa\Delta t}{I_x}$$

$$\omega_x = \frac{Fa\Delta t}{I_x}$$

- Causes body to “tumble”.

Stability of spinning objects cont.

- Stabilise object by spinning about direction of motion, frequency ω_s, angular momentum L_s.



- Torque along x axis now causes precession about y axis with frequency

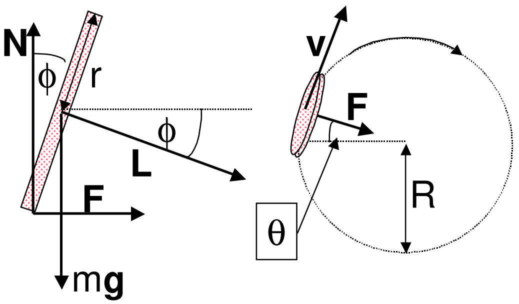
$$\Omega = \frac{Fa}{L_s}$$

Stability of spinning objects cont.

- When force removed precession stops, angular displacement is
  $$\phi = \Omega \Delta t = \frac{F a \Delta t}{L_s}$$
- Spinning object does not tumble, slightly changes orientation while force applied then stops precessing.
- Note, spin has no effect on c. of m. which in both cases acquires velocity
  $$\Delta v_y = \frac{F \Delta t}{M}$$

Rolling Coin

- Why does coin roll in "circle"?



- We have: $N = mg$
  $$F = \frac{Mv^2}{R}$$
- Torque about c. of m.
  $$\tau = Nr\sin\phi - Fr\cos\phi$$
  $$= mgr\sin\phi - \frac{Mv^2}{R} r\cos\phi \qquad (1)$$

Rolling coin cont.

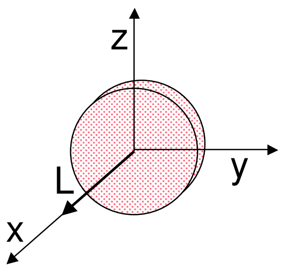
- Horizontal component of ang. mom.
  $L_h = L\cos\phi$
- Precesses due to torque, tends to prevent coin falling over (cf. bicycle problem)
  $$\tau = \frac{dL_h}{dt} = L_h \frac{d\theta}{dt} = L_h \Omega \qquad (2)$$
- What is condition for rolling in circle? Need precession freq. same as freq. of rotation around circle!
  $$\Omega = \frac{v}{R}$$
- Relate ang. mom. to speed of coin
  $$L = I\omega = \frac{Mr^2}{2}\frac{v}{r} = \frac{Mrv}{2}$$

Rolling coin cont.

- Substitute for L_h and Ω in (2)
  $$\tau = \frac{Mv^2 r\cos\phi}{2R}$$
- Now substitute for τ using (1)
  $$Mgr\sin\phi - \frac{Mv^2 r\cos\phi}{R} = \frac{Mv^2 r\cos\phi}{2R}$$
  $$Mgr\sin\phi = \frac{3Mv^2 r\cos\phi}{2R}$$
  $$\tan\phi = \frac{3v^2}{2gR}$$

Gyrocompass

- Consider gyroscope spinning about x axis in suspension free to rotate about y axis. Apply torque about z axis.



- Torque causes ang. mom. along z axis to increase. Happens by causing precession so component of **L** along z axis.
- No ang. mom. along y axis.

Gyrocompass cont.

- Precession stops when **L** along z axis.
- Gyrocompass is gyroscope constrained so axis can only move in horizontal plane.
- Rotation of earth causes horizontal plane to shift w.r.t. inertial plane.
- Gyroscope axis “would like” to remain stationary, but rotation of earth exerts torque through bearings causing precession.
- Torque causes gyroscope axis to line up with axis of rotation of earth.