Lecture 4

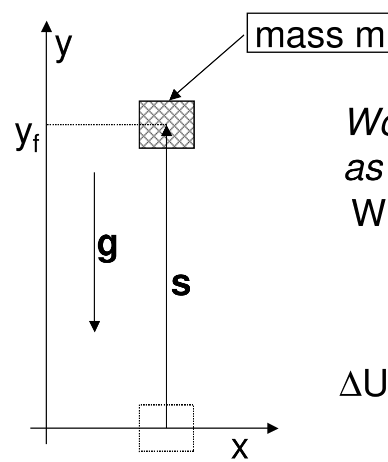
- Potential Energy
  - Gravitational potential energy
- Energy Conservation
  - Motion under gravity
- Using Conservation of Energy
  - Back to spring problem
- Conservative and Non-conservative Forces
- Energy Conservation Summary

Potential Energy

- P.E depends on configuration of system. Change in P.E. equal to negative of work done to change configuration.

  $\Delta U = -W$

- Gravitational potential energy.

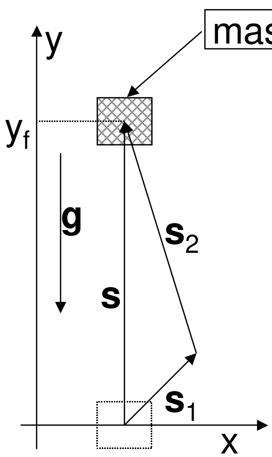


Work done by gravity as weight raised:

$$W = \mathbf{F} \cdot \mathbf{s} = m\mathbf{g} \cdot \mathbf{s}$$
$$= m(0,-g) \cdot (0, y_f)$$
$$= -mgy_f$$
$$\Delta U = mgy_f$$

Potential energy cont.

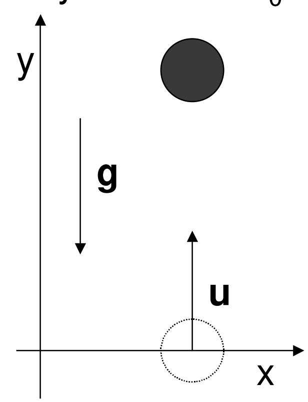
- Work done by gravity depends on end-points, not path taken, it is a conservative force.



$$
\begin{aligned}
W &= m\mathbf{g}\cdot\mathbf{s}_1 + m\mathbf{g}\cdot\mathbf{s}_2 \\
&= m\mathbf{g}\cdot(\mathbf{s}_1+\mathbf{s}_2) \\
&= m\mathbf{g}\cdot\mathbf{s} \\
&= m(0,-g)\cdot(0,y_f) \\
&= -mgy_f \\
\Delta U &= mgy_f \quad \textit{as before.}
\end{aligned}
$$

Energy Conservation

- Look at change of sum of kinetic and potential energy:
- Example, motion under gravity. At y=0 define U_0=0 then:



$$
\begin{aligned}
E_0 &= K_0 + U_0 \\
&= \tfrac{1}{2}mu^2
\end{aligned}
$$

The velocity at time t is given by

$$
\begin{aligned}
v &= \int -g\,dt \\
&= -gt + u
\end{aligned}
$$

Energy conservation cont.

The height at time t *is*

$$y = \int -gt + u\,dt$$

$$= -\frac{gt^2}{2} + ut$$

Hence the total energy at time t *is*

$$E = \tfrac{1}{2}m(-gt+u)^2 + mg(\frac{-gt^2}{2} + ut)$$

$$= \frac{m}{2}(g^2t^2 - 2gtu + u^2) +$$

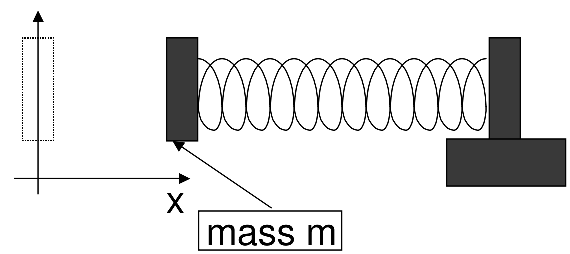
$$\frac{m}{2}(-g^2t^2 + 2gtu)$$

$$= \tfrac{1}{2}mu^2$$

$$= E_0$$

Using conservation of energy

- Back to spring problem



Spring compressed distance d, *released at* t=0, *describe motion. Define* U_0=0 *at* x=0*, then:*

$$E_0 = K_0 + U_0 = \tfrac{1}{2}kd^2$$

At x *we than have*

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = E_0$$

$$v = \sqrt{\frac{2E_0 - kx^2}{m}} = \sqrt{\frac{kd^2 - kx^2}{m}}$$

Using conservation of energy cont.

$$v = \sqrt{\frac{k}{m}}\sqrt{d^2 - x^2}$$

- Can now determine instantaneous power

$$P = Fv = -kx\sqrt{\frac{k}{m}}\sqrt{d^2 - x^2}$$

- Can also obtain time to reach position x

$$v = \frac{dx}{dt}$$

$$\int dt = \int \frac{dx}{v}$$

$$t = \int \frac{dx}{\sqrt{\frac{k}{m}}\sqrt{d^2 - x^2}}$$

Using conservation of energy cont.

$$t = \sqrt{\frac{m}{k}}\int \frac{dx}{\sqrt{d^2 - x^2}}$$

$$= \sqrt{\frac{m}{k}}\sin^{-1}\left(\frac{x}{d}\right) + t_0$$

Get t_0 *from condition* t=0 *when* x=d

$$t_0 = -\sqrt{\frac{m}{k}}\sin^{-1}(1)$$

$$= -\frac{\pi}{2}\sqrt{\frac{m}{k}}$$

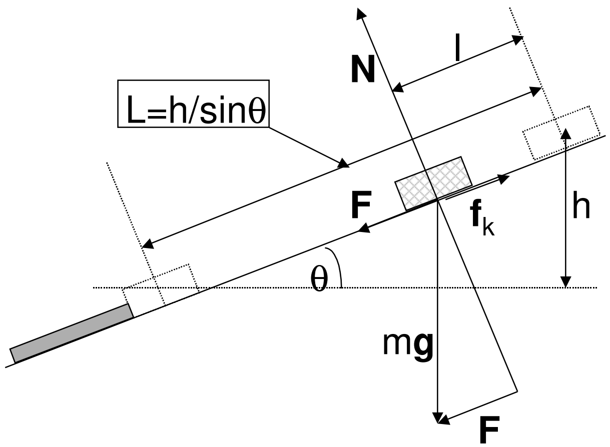
$$t = \sqrt{\frac{m}{k}}\left(\sin^{-1}\left(\frac{x}{d}\right) - \frac{\pi}{2}\right)$$

Period, T*, of resulting oscillations*

$$T = 4\sqrt{\frac{m}{k}}(\sin^{-1}(1) - \sin^{-1}(0)) = 2\pi\sqrt{\frac{m}{k}}$$

Energy Conservation and Friction

- Consider block slipping down inclined plane.



$$f_k = \mu_k N = \mu_k mg\cos\theta$$

$$F = mg\sin\theta$$

Net force down plane

$$F - f_k = mg(\sin\theta - \mu_k\cos\theta)$$

Energy conservation and friction cont.

Initial energy

$$E_i = mgh$$

Motion down slope described by:

$$a = \frac{d^2l}{dt^2} = g(\sin\theta - \mu_k\cos\theta)$$

$$v = \frac{dl}{dt} = g(\sin\theta - \mu_k\cos\theta)t$$

$$l = g(\sin\theta - \mu_k\cos\theta)\frac{t^2}{2}$$

$$t = \sqrt{\frac{2l}{g(\sin\theta - \mu_k\cos\theta)}}$$

Time to travel L

$$T = \sqrt{\frac{2L}{g(\sin\theta - \mu_k\cos\theta)}}$$

Energy conservation and friction cont.

Speed as block hits stop

$$v_L = g(\sin\theta - \mu_k \cos\theta)T$$
$$= \sqrt{2Lg(\sin\theta - \mu_k \cos\theta)}$$

Energy as block hits stop

$$E_f = \tfrac{1}{2} m v_L^{\,2}$$
$$= mLg(\sin\theta - \mu_k \cos\theta)$$
$$= mgh - \frac{mgh\mu_k \cos\theta}{\sin\theta}$$
$$= E_i\left(1 - \frac{\mu_k}{\tan\theta}\right) = E_i - E_m$$

Where the "missing energy" is

$$E_m = \frac{E_i \mu_k}{\tan\theta}$$

Energy conservation and friction cont.

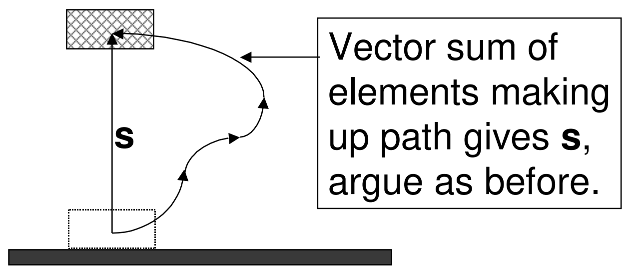
- Where did the missing energy go? Calc. work done by f_k

$$W = f_k L$$
$$= (\mu_k mg\cos\theta)\left(\frac{-h}{\sin\theta}\right)$$
$$= \frac{-E_i \mu_k}{\tan\theta}$$

- Work done by f_k appears as internal energy, kinetic energy of atoms and molecules. Results in change of mechanical (kinetic plus potential) energy of system.

Conservative and Non-Conservative Forces

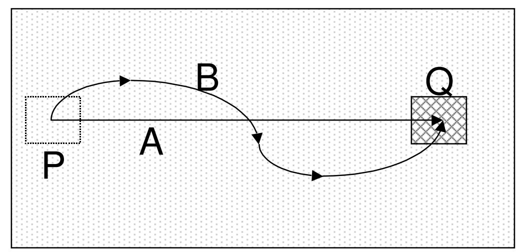
- Work done by gravity independent of path, gravity is a conservative force. It can be associated with potential



- Same applies to elastic force (try it!)

Conservative and non-conservative forces cont.

- Work done by frictional forces <u>does</u> depend on path taken. Friction not conservative, cannot associate with potential. Consider sliding book from P to Q on table, via paths A and B



- Work done along paths A and B is W_A ~(length of path A) and W_B ~(length of path B).
- $W_A < W_B$, result depends on path.

Energy Conservation- Summary

- Mechanical energy of system conserved if only conservative forces
  $$\Delta E = \Delta K + \Delta U = 0$$
- As W=-ΔU we have
  $$W = \Delta K$$
- If non-conservative forces involved, energy dissipated via these must be taken into account. Provided system isolated
  $$\Delta E = \Delta K + \Delta U + \Delta E_{int} = 0$$
- Using W_f=-ΔE_{int} we have
  $$W_f = \Delta K + \Delta U$$
- If external forces do work W_{ext} on the system then
  $$\Delta E = \Delta K + \Delta U + \Delta E_{int} = W_{ext}$$