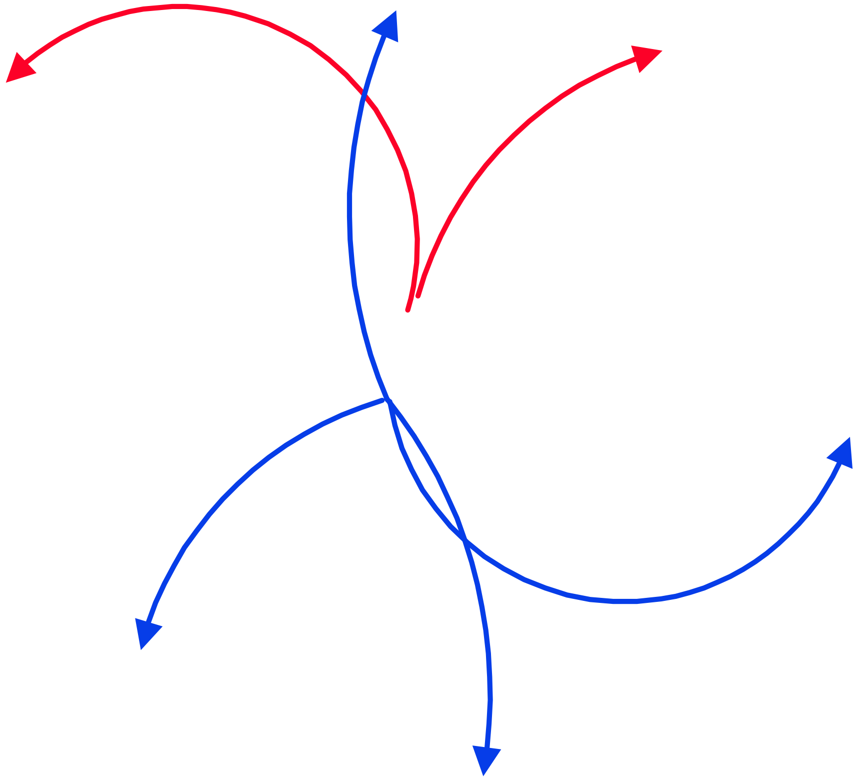
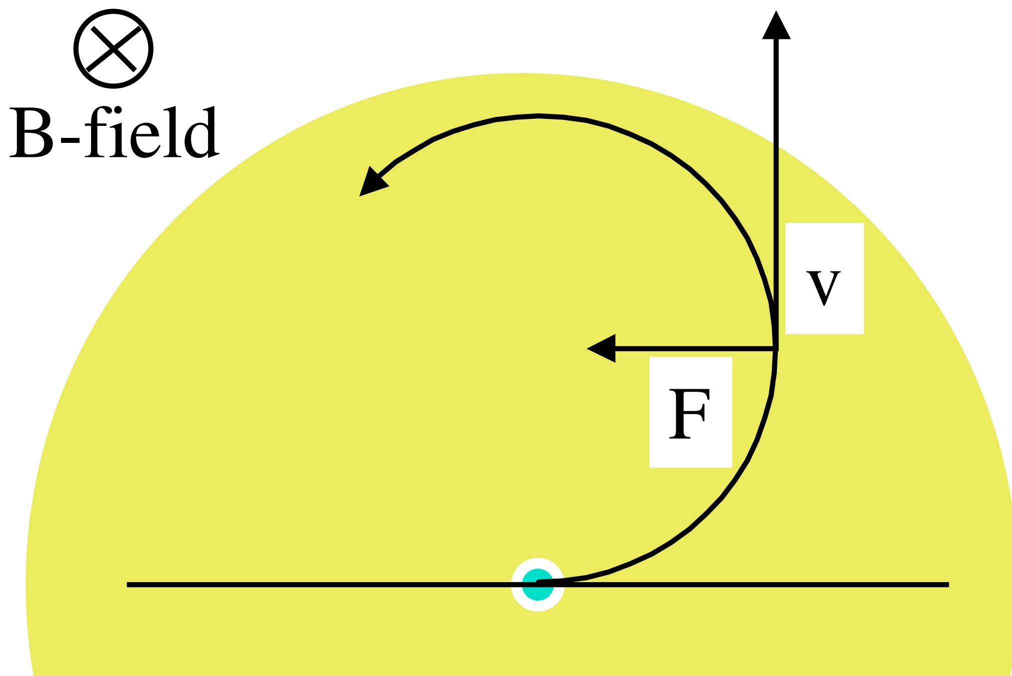
Measuring masses and momenta



- Measuring charged particle momenta.
- Momentum and Special Relativity.
- Kinetic energy in a simple accelerator.
- Total energy, mass and momentum.
- Measuring masses.

Motion of Charged Particle in a Magnetic Field



- Charge Q, magnetic field strength B.
- Velocity v, normal to magnetic field.
- Lorentz force F, normal to directions of B and v.
- Magnitude of force $F = QvB$.
- Circular path.

Measuring momentum

- Centripetal force $F = mr\omega^2 = m\dfrac{v^2}{r}$.

- Equate Lorentz and centripetal forces

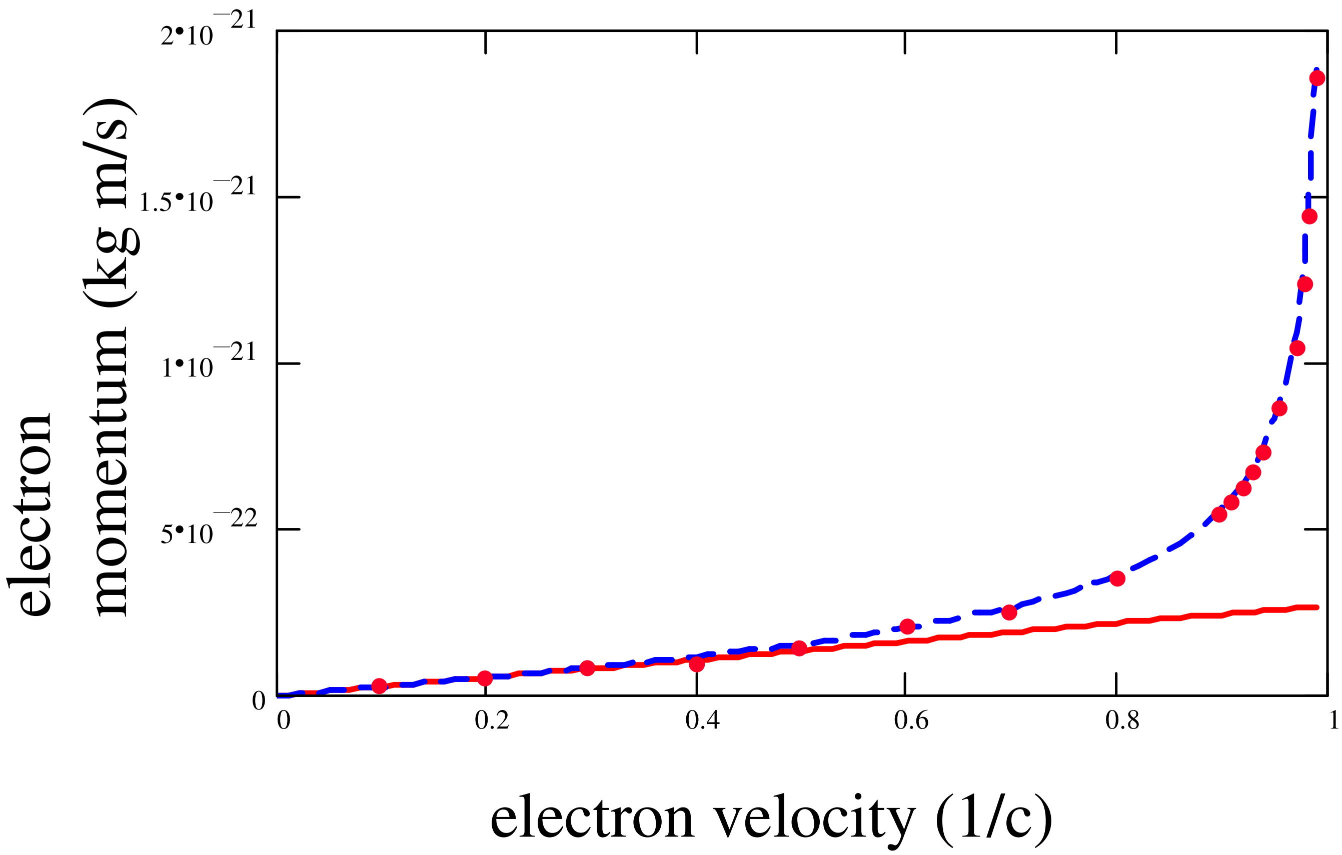
$$BQv = \frac{mv^2}{r}$$

$$\Rightarrow BQ = \frac{mv}{r} = \frac{p}{r}$$

hence $\boxed{p = BQr}$

Momentum and Special Relativity

- Some measurements:

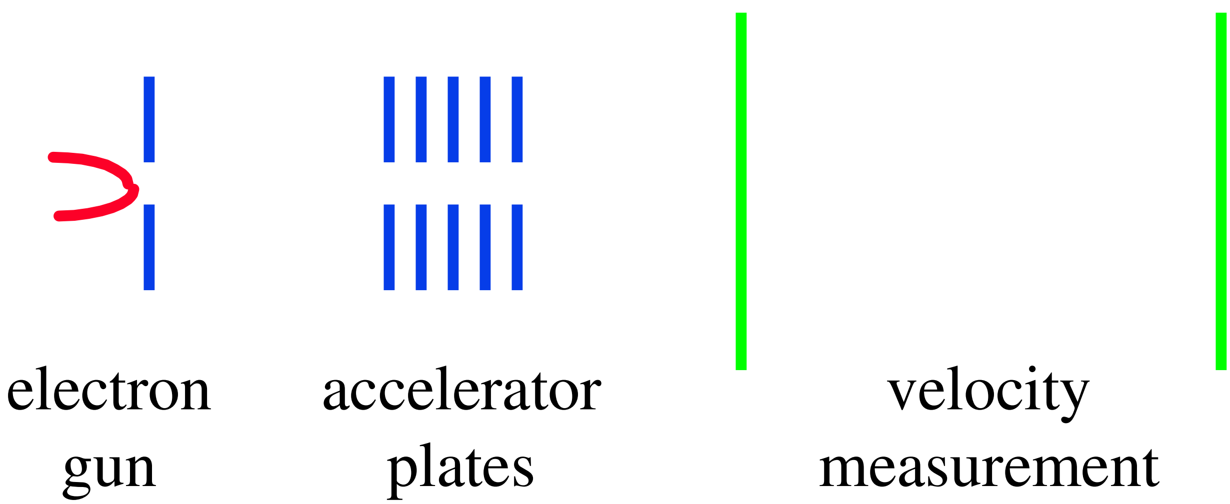


- Must redefine momentum to keep law of conservation of momentum.
- Relativistic momentum:

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}}.$$

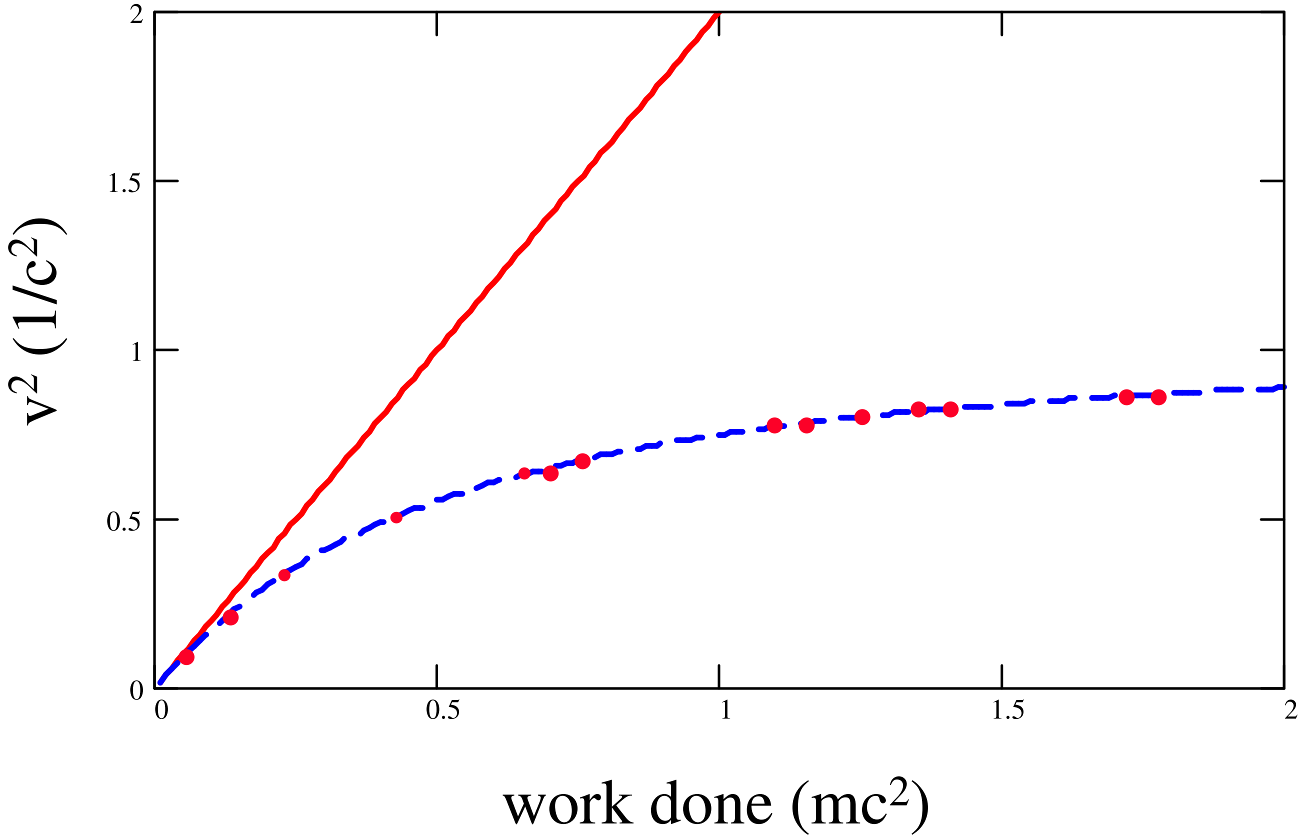
Kinetic energy in a simple accelerator

- Remember two rules:
  - Work done = ______ x __________.
  - Change in K.E. = work done.
- Build an accelerator to check this:



Kinetic energy cont.

- More measurements:



- If want to keep K.E. = work done, define:

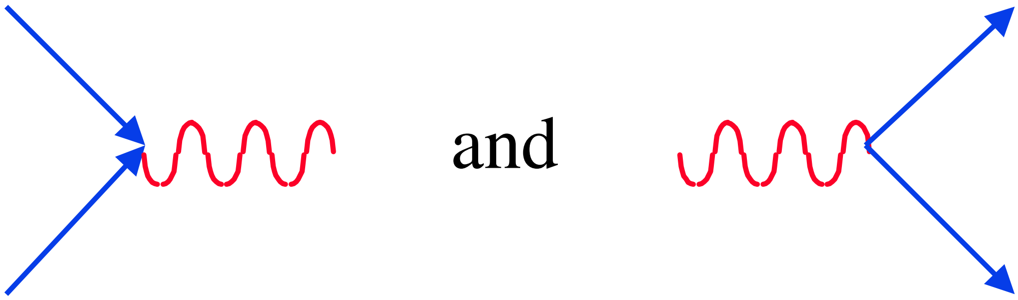
$$T = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2$$

$$= (\gamma - 1)mc^2$$

where $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$.

Energy and mass

- Have seen:



so know $E = mc^2$ (ignoring K.E.).

- Putting it all together:

$$E = T + mc^2$$

$$T = (\gamma - 1)mc^2$$

$$p = mv\gamma.$$

- Remembering $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$,

can show that

$$E^2 = p^2c^2 + m^2c^4.$$

An aside, units

- We have $p = BQr\ (\mathrm{kg\ m\ s^{-1}})$.
- Multiply p by c to get energy

$$pc = BQrc\ (\mathrm{J})$$

$$= \frac{BQrc}{1.6\times10^{-19}}\ (\mathrm{eV}).$$

- But $Q = 1.6 \times 10^{-19}$ C for e, μ etc.

$$pc = Brc\ (\mathrm{eV}).$$

- Now $c = 3 \times 10^{8}\ \mathrm{m\ s^{-1}}$ so:

$$pc = Br\cdot 3\times10^{8}\ (\mathrm{eV})$$

$$= \frac{Br\cdot 3\times10^{8}}{1\times10^{9}}\ (\mathrm{GeV})$$

$$= 0.3Br\ (\mathrm{GeV})$$

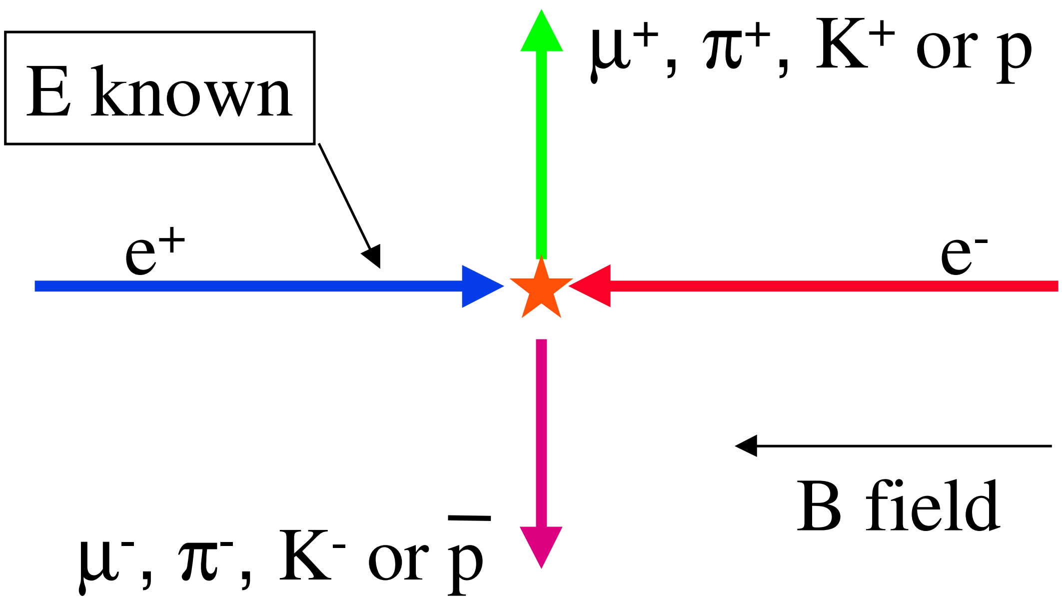
- Finally, express p in units of GeV / c

$$p = 0.3Br\ (\mathrm{GeV}/c)$$

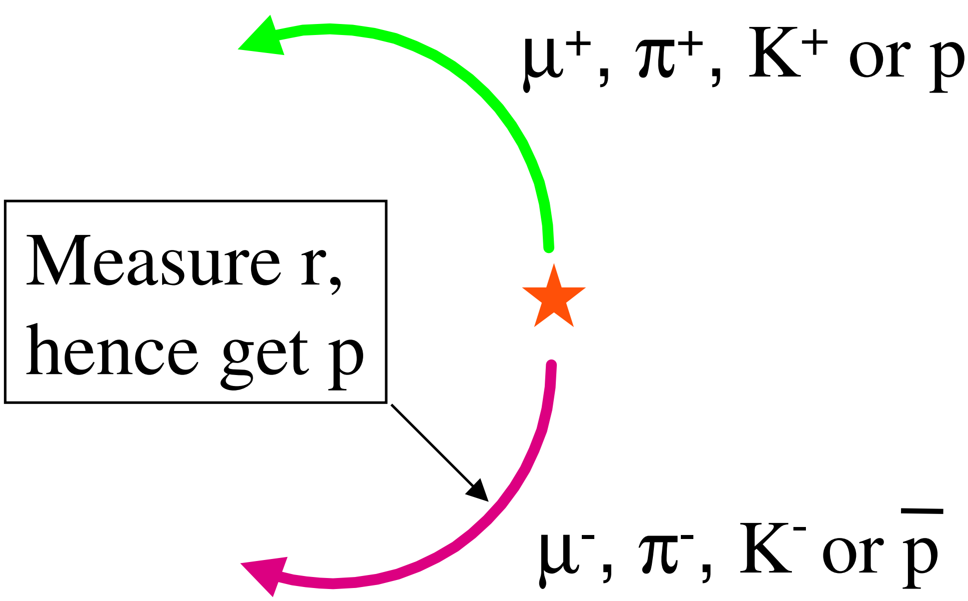
PC Exercise 2

- Use PC to do following experiment:

<u>Side view</u>



<u>End view</u>



PC exercise 2 cont.

- Using known energy (from energy conservation) and momentum (from r measurement) calculate mass:
  $m = \sqrt{E^2 - p^2}$.
- Compare with known particle masses, can you identify the particles?