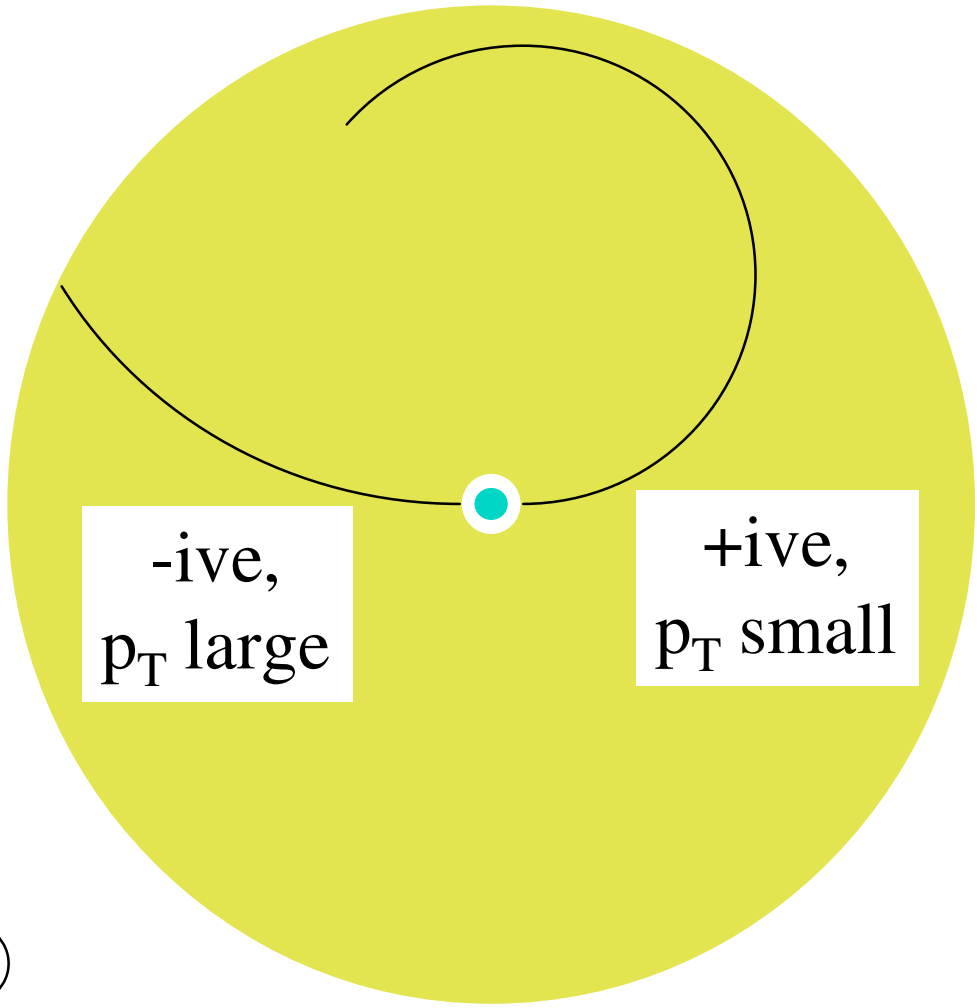
Measuring masses and momenta

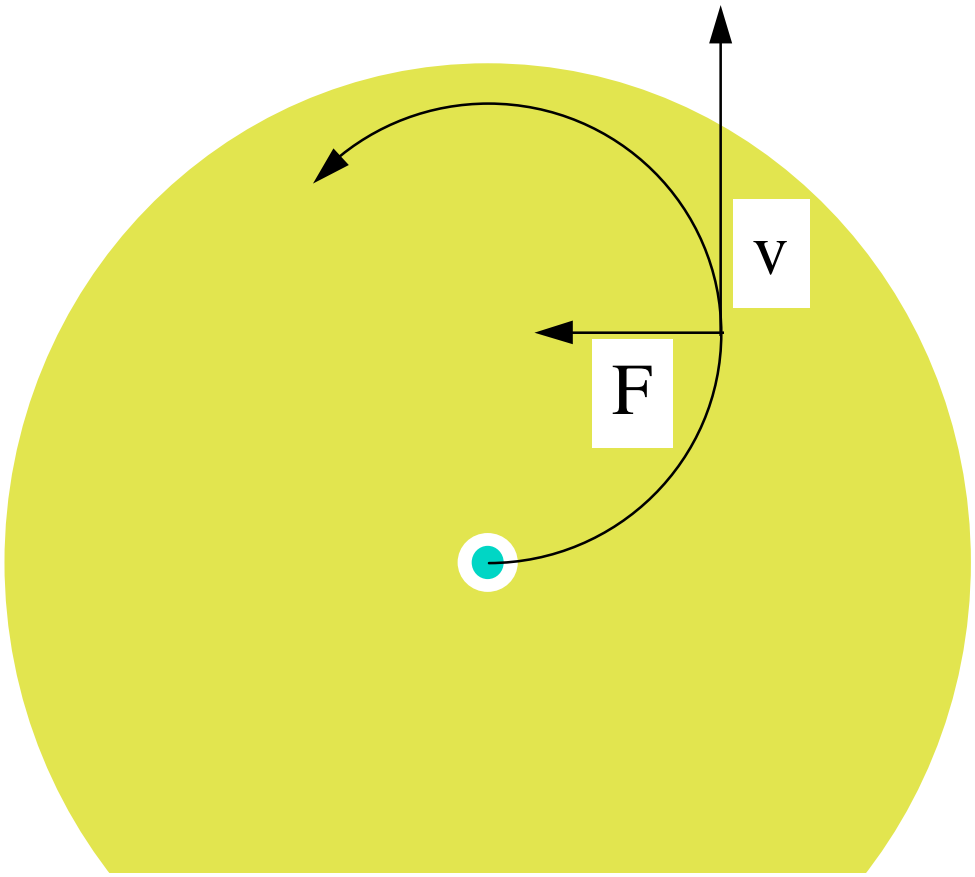
- Measuring charged particle momenta.
- Using a particle's energy and momentum to determine its mass.

Measuring charged particle momenta.

- Lorentz force due to magnetic field parallel to beam makes path helical.
- Radius of curvature gives transverse momentum.



Motion of Charged Particle in a Magnetic Field



B-field

- Charge Q, magnetic field strength B.
- Velocity v, normal to magnetic field.
- Particle experiences force F, normal to directions of field and velocity.
- Magnitude of force
- Path of particle is circle.

Radius of Curvature

- Centripetal force $F = mr\omega^2 = m\frac{v^2}{r}$
- Equate Lorentz force and centripetal force

$$BQv = \frac{mv^2}{r}$$

$$\Rightarrow BQ = \frac{mv}{r} = \frac{p}{r}$$

$$\text{hence } p = BQr$$

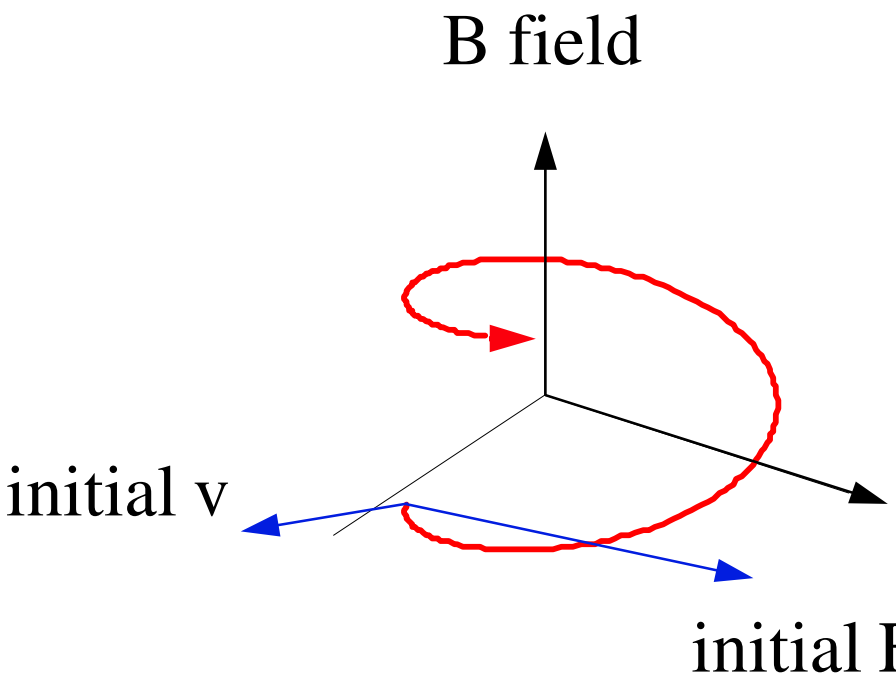
- Relax constraint that v normal to B

$$p_T = BQr$$

- Derivation non-relativistic, but relativistic treatment gives same result.

Motion of Charged Particle in a Magnetic Field

- Full expression for Lorentz force is $\vec{F} = Q\vec{v} \times \vec{B}$ path helix.



- Path still circle if look along B field.

Special Relativity

- $E = mc^2$ is probably best known of all formulae!
- Classically, (kinetic) energy and momentum related

  $T = \frac{1}{2}mv^2$ or $T = \frac{p^2}{2m}$.
- SR tells us that energy, momentum and mass are related

  $E^2 = p^2c^2 + m^2c^4$.
- Choose units eV, 1 eV = 1.6×10^{-19} J:
  - GeV for energy.
  - GeV/c for momentum.
  - GeV/c^2 for mass.
- Above becomes $E^2 = p^2 + m^2$.

Charged Particles in Magnetic Fields, Part 2

- We had $p = BQr$ (kg m s^{-1}).
- What is this using GeV?
- Multiply p by c to get energy

$$pc = BQrc \text{ (J)}$$

$$= \frac{BQrc}{1.6 \times 10^{-19}} \text{ (eV).}$$

- But $Q = 1.6 \times 10^{-19}$ C for e, μ etc.

$$pc = Brc \text{ (eV).}$$

- Now $c = 3 \times 10^{8}$ m s^{-1} so

$$pc = Br \cdot 3 \times 10^{8} \text{ (eV)}$$

$$= \frac{Br \cdot 3 \times 10^{8}}{1 \times 10^{9}} \text{ (GeV)}$$

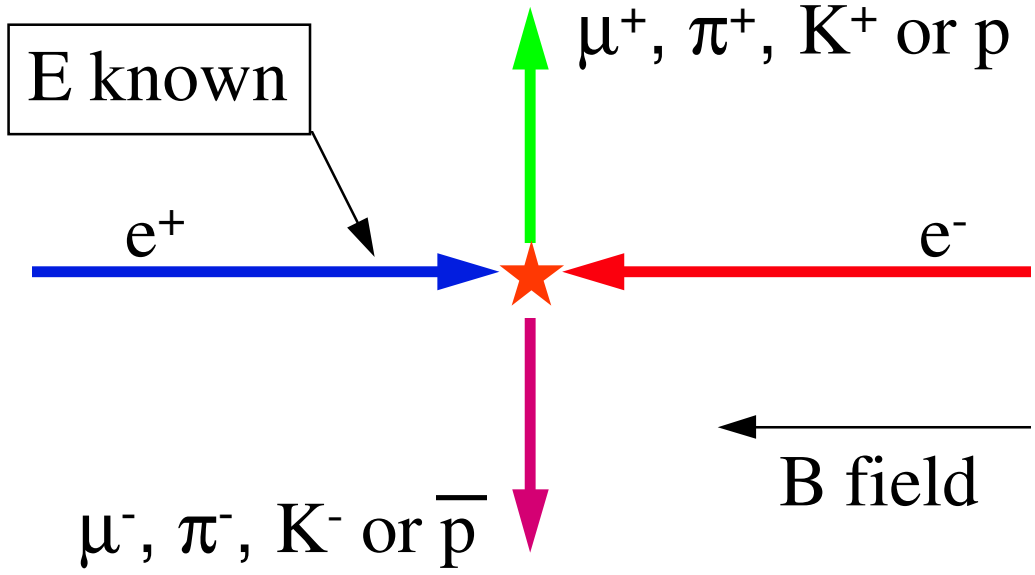
$$= 0.3Br \text{ (GeV)}$$

- Finally, express p in units of GeV / c

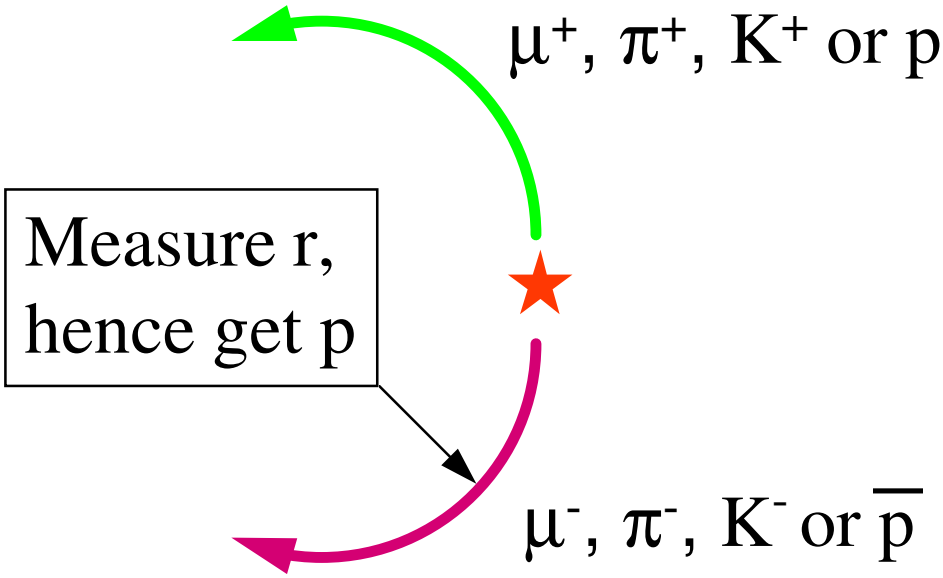
$$p = 0.3Br \text{ (GeV / c)}$$

PC Exercise 2

- Use PC to do following "thought" experiment, side view.



- End on view



PC exercise 2 cont.

- Using known energy (from energy conservation) and momentum (from r measurement) calculate mass.

  $m = \sqrt{E^2 - p^2}$

- Compare with known particle masses, can you identify the particles?