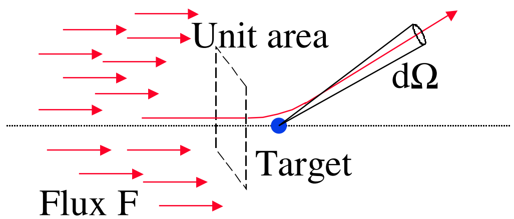
PHYS 488
Particles in Matter

- This lecture, Basics:
  - Cross section, e.g. Rutherford scattering.
- Muons passing through matter:
  - Mean free path.
  - Multiple Coulomb scattering.
  - Energy loss formula.
- Next Lecture, Particle physics detectors
  - Tracking detectors
  - Electromagnetic calorimeters
    - » A simple shower model
  - Hadronic calorimeters
  - Muon and neutrino detection
- Why use Monte Carlo programs?
- Some references.

Cross section

- Differential cross section is the average number of incident particles scattered into $d\Omega$ per unit flux per target particle:



$$\frac{d\sigma(E,\Omega)}{d\Omega} = \frac{1}{F}\frac{dN_{scat}}{d\Omega}.$$

- Total cross section for all angles:

$$\sigma(E) = \int d\Omega \frac{d\sigma}{d\Omega}.$$

Rutherford scattering

- For a spinless particle, charge $z|e|$, scattering off nucleus with atomic number Z:

$$\frac{d\sigma}{d\Omega} = \frac{z^2 Z^2 \left(e^2/4\pi\varepsilon_0\right)^2}{4p^2c^2\beta^2 \sin^4 \theta/2}.$$

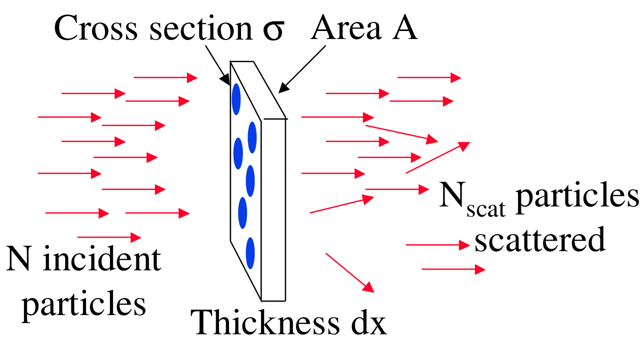
- Some questions:
  - What is total Rutherford scattering cross section?
  - Why does it have this value?
  - Why is the total cross section less than the above value for particles passing through material?

Muon travelling through matter

- Can interact electromagnetically with nucleii.
  - How often (Rutherford scattering)?
  - What happens?
- Can interact electromagnetically with atomic electrons.
  - How often?
  - What happens?

Prob. of scattering off nucleii in slab

- Thin slab, n scattering centres per unit vol:



- Prob. of interaction for each muon is total "area" of scattering centres divided by area of slab:

$$\frac{n\sigma A dx}{A} = n\sigma dx.$$

- No. scattered $N_{scat} = Nn\sigma dx.$

Mean free path

- Change in number of beam particles in depth dx is given by:

$$dN = -N_{scat} = -Nn\sigma dx$$

$$\frac{dN}{dx} = -Nn\sigma.$$

- For large depths $\frac{dN}{N} = -n\sigma dx$

$$N = N_0 \exp[-n\sigma x]$$
$$= N_0 \exp[-x/\lambda].$$

- Where the mean free path $\lambda = \frac{1}{n\sigma}.$

Mean free path cont.

- Do we have to worry about effects of Rutherford scattering?
- Typical value of λ?

$$n = \frac{\text{no. of nucleii}}{\text{volume}} = \frac{N_A \rho}{A}$$

$$\text{for carbon} \approx \frac{6\times 10^{23}\times 2}{12} = 10^{23}\,\text{cm}^{-3}$$

$$\sigma \approx \text{atomic area} \approx \pi(10^{-8}\,\text{cm})^2$$

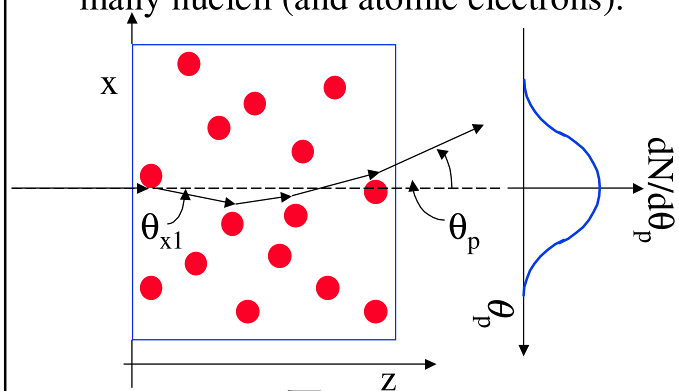
$$\approx 10^{-16}\,\text{cm}^2.$$

$$\lambda = \frac{1}{n\sigma} \approx \frac{1}{10^{23}\times 10^{-16}} = 10^{-7}\,\text{cm}.$$

- We do have to worry!

Multiple Coulomb scattering

- Effect on direction of scattering off many nucleii (and atomic electrons).



- In plane $\theta_p = \sum \theta_{xi}$.
- Central limit theorem: θ_p distribution is gaussian (large non-gaussian tails!).
- Width (σ) given by:

$$\theta_0 = \frac{13.6\,\text{MeV}}{\beta c p} z \sqrt{\frac{x}{X_0}}\left[1 + 0.038 \ln\left(\frac{x}{X_0}\right)\right].$$

Multiple Coulomb scattering cont.

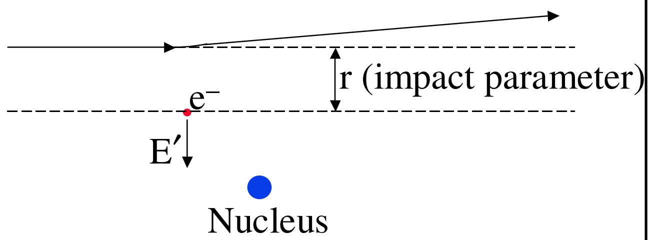
- That is, the distribution of angles θ_p is:
  $$\frac{1}{\sqrt{2\pi}\theta_0}\exp\left(-\frac{1}{2}\frac{\theta_p^{\,2}}{\theta_0^{\,2}}\right)d\theta_p.$$
- Radiation length (mean distance over which high energy electron beam loses all but 1/e of its energy):
  $$X_0 = \frac{716.4\,\mathrm{g\,cm^{-2}}\,A}{Z(Z+1)\ln\left[287/\sqrt{Z}\right]}.$$
- Beware units!
  - Calculated using above formula, X_0 is in units of g cm^{-2}.
  - If need X_0 as true length, divide by density of material, e.g. for iron: $X_0 = 13.8$ g cm^{-2}, $\rho = 7.9$ g cm^{-2} $\Rightarrow$ $X_0 = 1.8$ cm.

Multiple Coulomb scattering cont.

- (Small angle) scattering in y plane independent of scattering in x plane.
- Width of distribution of space scattering angle $\theta_s^{\,2} = \theta_{0x}^{\,2} + \theta_{0y}^{\,2} = 2\theta_0^{\,2}$.
- Central limit theorem doesn't apply to rare large angle scatters (c.f. Geiger and Marsden's observations), so above does not describe occasional large angle scatters!
- Exercise
  Use MathCad to plot distribution of space scattering angles of 1 GeV muon after it has travelled through 1 cm of iron.

Energy loss formula

- Calculate energy lost in collisions with atomic electrons:



- Impulse to electron (momentum change):

$$p = \int F dt = \langle F \rangle \langle t \rangle.$$

- Assume $\langle F \rangle = \dfrac{ze^2}{4\pi\varepsilon_0 r^2}$ and $\langle t \rangle = \dfrac{2r}{\beta c}$.

Energy loss formula cont.

- If v ~ c, lateral Coulomb force increased by factor γ, distance Lorentz contracted so:

$$p \approx \frac{\gamma z e^2}{2\pi\varepsilon_0 r^2}\frac{r}{\gamma\beta c}.$$

- Energy of electron is thus:

$$E' = \frac{p^2}{2m_e} \approx \frac{z^2 e^4}{8\pi^2 \varepsilon_0{}^2 r^2 \beta^2 c^2 m_e} \qquad (1)$$

- See energy loss to nucleii insignificant!
- Expressing (1) in terms of r and differentiating:

$$r^2 = \frac{z^2 e^4}{8\pi^2 \varepsilon_0{}^2 m_e \beta^2 c^2 E'}$$

$$r\,dr = \frac{z^2 e^4}{16\pi^2 \varepsilon_0{}^2 m_e \beta^2 c^2}\frac{dE'}{E'^2} \qquad (2)$$

Energy loss formula cont.

- Prob. of finding an electron with impact parameter between r and r + dr in distance dx:
  $$dP = Z\frac{N_A \rho}{A} 2\pi r\, dr\, dx.$$
- Putting $r\,dr$ from (2) into this gives probability of collision with transfer of energy between E' and $E' + dE'$ to atomic electron:
  $$\Phi\, dE' dx \approx \frac{\pi z^2 e^4}{8\pi^2 \varepsilon_0{}^2 m_e \beta^2 c^2} \frac{Z N_A \rho}{A} \frac{dE'}{E'^2} dx.$$
- Rewrite using:
  $$r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2} = 2.8 \times 10^{-13}\,\mathrm{cm}.$$
- $$\Phi\, dE' dx \approx \frac{2\pi Z N_A \rho}{A} r_e{}^2 \frac{z^2 m_e c^2}{\beta^2} \frac{dE'}{E'^2} dx \qquad (3)$$

Energy loss formula cont.

- Useful result: $\frac{2\pi Z N_A}{A} r_e{}^2 = 0.307 \frac{Z}{A} g^{-1} cm^2.$
- Energy loss to electrons from (3):
  $$-\frac{dE}{dx} \approx \int_{E'_{min}}^{E'_{max}} E' \Phi\, dE'$$
  $$\approx \frac{2\pi Z N_A \rho}{A} r_e{}^2 \frac{z^2 m_e c^2}{\beta^2} \int_{E'_{min}}^{E'_{max}} \frac{dE'}{E'}.$$
- Minimum energy ~ average ionization potential:
  $E'_{min} \approx I \approx 13.5\,\mathrm{eV} \times Z.$
- Maximum energy is max that can be transferred to electron in collision:
  $E'_{max} \approx 2 m_e c^2 \beta^2 \gamma^2.$

Energy loss formula cont.

- Putting this all together:

$$-\frac{dE}{dx} \approx \frac{2\pi Z N_A \rho}{A} r_e^{\,2} \frac{z^2 m_e c^2}{\beta^2} \ln\left[\frac{2 m_e c^2 \beta^2 \gamma^2}{I}\right].$$

- Correct and more complete "Bethe-Bloch" formula in Handout.
- Useful approximation:

$$-\frac{dE}{dx} \approx 0.307 \frac{Z}{A} \rho \frac{z^2}{\beta^2}\left(\ln\left[\frac{2 m_e c^2 \beta^2 \gamma^2}{I}\right] - \beta^2 \right).$$

- <u>Exercise</u>
  Using the above approximation write a method to calculate the energy loss per unit length of a muon travelling through iron. Compare the results you get with those plotted in the handout; think about the units!