Prostate Model

Theory

Model PSA diffusion using Fick's laws. In one dimension, the first law is:

$$J = -D\frac{\partial \phi}{\partial x},$$

where D is the diffusion coefficient, ϕ is the PSA density, and the diffusion flux J is the amount of PSA crossing unit area per unit time.

From this, we can derive (see here) Fick's second law:

$$\frac{\partial \phi}{\partial t} = D\frac{\partial^2 \phi}{\partial x^2}.$$

If ϕ is in a steady (or approximately steady) state, this reduces to

$$\frac{\partial^2 \phi}{\partial x^2} = 0.$$

In 3D this becomes Laplace's equation:

$$\nabla^2 \phi = 0,$$

which can be solved numerically using the relaxation method.

The requirement that ϕ be in a steady state is satisfied if the rate at which the system changes (new tumour cells are created, in the prostate tumour case) is much lower than the rate at which diffusion occurs.