Answers to lecture problems – lectures 20

Lecture 20

Slide 1

Write x^3+2x in terms of Legendre polynomials by using their orthonormality.

Use:

$P_1 = x.$

$P_3 = \tfrac{1}{2}(5x^3 - 3x).$

Write $f(x) = x^3 + 2x = aP_1 + bP_3.$

We have:

$$\int_{-1}^{1} P_1(x) f(x)\,dx = \frac{2}{2\times 1+1}a = \frac{2}{3}a.$$

But also:

$$\int_{-1}^{1} P_1(x) f(x)\,dx = \int_{-1}^{1} x f(x)\,dx = \int_{-1}^{1} x(x^3+2x)\,dx = \int_{-1}^{1} (x^4+2x^2)\,dx = 2\left(\frac{x^5}{5}+\frac{2x^3}{3}\right)_0^1 = \frac{26}{15}$$

Hence:

$$\frac{2}{3}a = \frac{26}{15} \Rightarrow a = \frac{13}{5}.$$

$$\int_{-1}^{1} P_3(x) f(x)\,dx = \frac{2}{2\times 3+1}b = \frac{2}{7}b.$$

But also:

$$\int_{-1}^{1} P_3(x) f(x)\,dx = \int_{-1}^{1} x f(x)\,dx = \int_{-1}^{1} \tfrac{1}{2}(5x^3-3x)(x^3+2x)\,dx = \int_{-1}^{1} \tfrac{1}{2}(5x^6+7x^4-6x^2)\,dx$$

$$= \left(\frac{5x^7}{7}+\frac{7x^5}{5}-\frac{6x^3}{3}\right)_0^1 = -\frac{94}{35}$$

Hence:

$$\frac{2}{7}b = -\frac{94}{35} \Rightarrow b = -\frac{47}{5}.$$