Answers to lecture problems – lectures 14…15

Lecture 14

Slide 1

Show that $\int_{-\pi}^{\pi} \sin mt \sin nt \, dt = 0$ if $m \neq n$.

Use $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin a \sin\beta$.

Then we have:

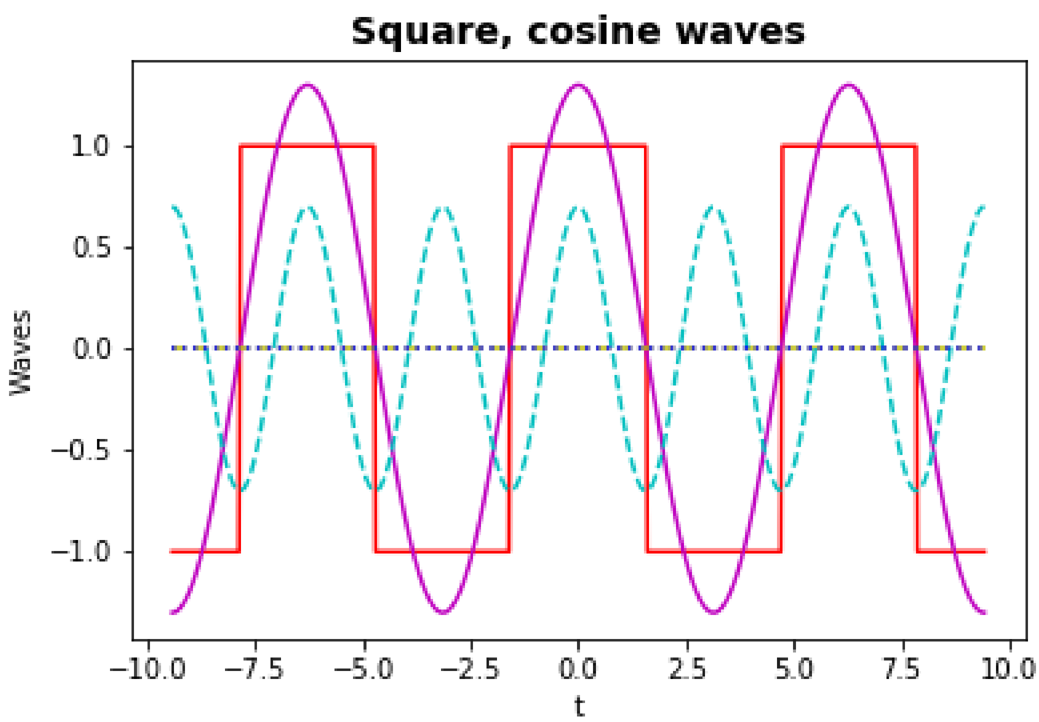
$$\tfrac{1}{2}\left(\cos(n-m)t - \cos(n+m)t\right) = \tfrac{1}{2}\left(\cos nt \cos mt + \sin nt \sin mt\right) - \tfrac{1}{2}\left(\cos nt \cos mt - \sin nt \sin mt\right)$$
$$= \sin nt \sin mt.$$

This gives:

$$\int_{-\pi}^{\pi} \sin nt \sin mt \, dt = \int_{-\pi}^{\pi} \tfrac{1}{2}\left(\cos(n-m)t - \cos(n+m)t\right) dt$$
$$= \frac{1}{2}\left(\frac{\sin(n-m)t}{n-m} - \frac{\sin(n+m)t}{n+m}\right)_{-\pi}^{\pi}$$
$$= 0$$

Slide 4

Adding a cosine wave of frequency two does not give a better approximation. It flattens the negative peak (good!) and sharpens the positive one (bad) if its coefficient is positive (as shown by the dotted pale blue lines in the graph below).



If the cos(2t) term has a negative coefficient it flattens the positive peak (good!) and sharpens the negative one (bad).

Lecture 15

Slide 1

Fourier series for function:

$f(t) = -t$ for $-1 \le t < 1$,

$f(t+2) = f(t)$ for all t.

Function is odd to a_0 and all a_n are zero.

Coefficients as for example in lecture, but with sign change!

$$
\begin{aligned}
b_n &= 2\int_0^1 -t \sin n\pi t \, dt \\
&= 2\int_0^1 t \, d\left(\frac{\cos n\pi t}{n\pi}\right) \\
&= \left.\frac{2t\cos n\pi t}{n\pi}\right|_0^1 - 2\int_0^1 \frac{\cos n\pi t}{n\pi} dt \\
&= \frac{2\cos n\pi}{n\pi} - \left.\frac{2}{n^2\pi^2}\sin n\pi t\right|_0^1 \\
&= \frac{2\cos n\pi}{n\pi} \\
&= 2\frac{(-1)^n}{n\pi}
\end{aligned}
$$

Function plus first few terms of Fourier series illustrated below: