Department of Physics
Year 1 Tutorials
Module Phys123
Electricity and Magnetism

Answers for Tutorial 5

The marks to be awarded for each question are indicated in square brackets.

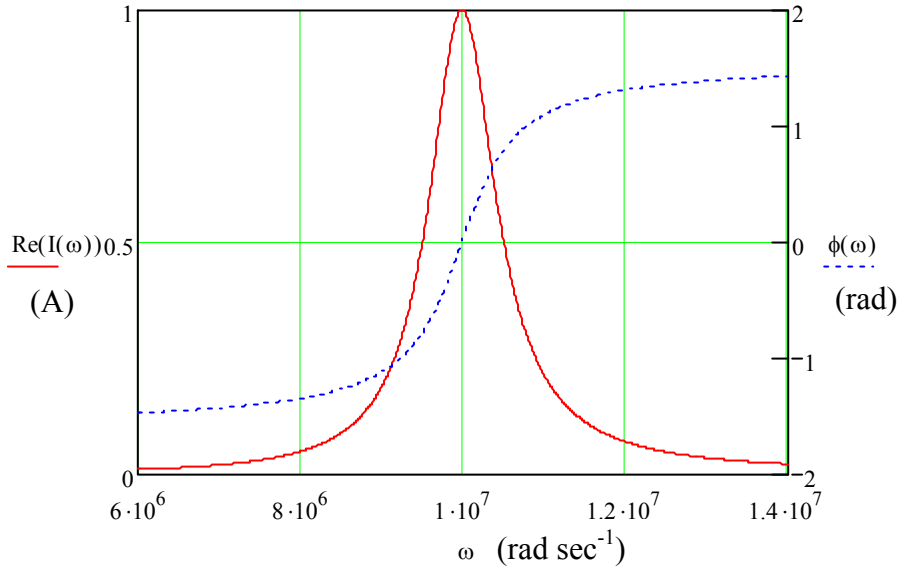
Problem 1 [5]
I = 5 A, so $I_{rms} = 5/\sqrt{2} = 3.54$ A. [2]
Average power $P = I_{rms}^2 R = 5^2/2 \times 10 = 125$ W. [3]

Problem 2 [10]
$\mathcal{E} = 30$ V.
f = 1 kHz, so $\omega = 2\pi f = 6.28 \times 10^3$ rad/sec
a) $I = \mathcal{E}/R = 30/50 = 0.6$ A. [1]
$P = I_{rms}^2 R = (0.6/\sqrt{2})^2 \times 50 = 9$ W. [1]
b) For inductor, $I = \mathcal{E}/\omega L = 30/(6.28 \times 10^3 \times 0.25) = 0.0191$ A. [2]
No power dissipated in inductor. [2]
c) For capacitor, $I = \mathcal{E}/(1/\omega C) = 30 \times 6.28 \times 10^3 \times 1.5 \times 10^{-6} = 0.283$ A. [2]
No power dissipated in capacitor. [2]

Problem 3 [5]
a) The current leads the voltage, i.e. the current peaks before the voltage. [1]
b) As the current leads the voltage, the phase ϕ in the expression $i = I \sin(\omega t - \phi)$ is negative. As this phase is given by $\phi = \tan^{-1}\left((\omega L - 1/\omega C)/R\right)$, the capacitive reactance $1/\omega C$ is larger than the inductive reactance ωL: the circuit is predominantly capacitive.
(Alternatively, i leads v, so from the mnemonic CiviL the circuit is capacitively dominated, the same conclusion is reached from the mnemonic ELi is positively the iCE man!) [2]
c) The phase is negative below the resonant frequency and positive above, so from the above, the frequency is below the resonant frequency. [2]

Problem 4 [15]
a) $X_C = 1/\omega_d C = 1/(2 \times 10^6\ 1 \times 10^{-9}) = 500\ \Omega$. [2]
$X_L = \omega_d L = 2 \times 10^6 \times 10 \times 10^{-6} = 20\ \Omega$. [2]
b) $\phi = \tan^{-1}\left(\dfrac{\omega_d L - 1/\omega_d C}{R}\right) = \tan^{-1}\left(\dfrac{20 - 500}{10}\right) = -88.8^\circ$ or -1.55 rad. [2]
Phase is negative, so current leads voltage by 88.8°
c) $Z = \sqrt{R^2 + (\omega_d L - 1/\omega_d L)^2} = 480\,\Omega$ (contribution of resistance negligible). [2]

d) Sketch of shape of resonance.



[3]

The current i has a maximum value of I = Ɛ/R = 10/10 = 1 A, [2]

at the resonant frequency $\omega = 1/\sqrt{LC} = 1/\sqrt{1\times10^{-9}\times10\times10^{-6}} = 10^7 \text{ rad sec}^{-1}$. [2]

Maximum possible number of marks for this tutorial is 35.