

Answers for Tutorial 3

The marks to be awarded for each question are indicated in square brackets.

Problem 1 [10]

$$\text{Magnitude of electric field } E = \sqrt{1^2 + (-2)^2 + (-1)^2} = \sqrt{6} = 2.45 \text{ V m}^{-1} \quad [2]$$

$$\text{Distance moved } r = \sqrt{3^2 + 2^2 + (-1)^2} = \sqrt{14} = 3.74 \text{ m} \quad [2]$$

$$\text{Work done } W = \vec{F} \cdot \vec{r} = q\vec{E} \cdot \vec{r} \quad [2]$$

$$\text{Hence } W = 7 \times 10^{-9} \times (3 \times 1 + 2 \times (-2) + (-1) \times (-1)) = 0 \quad [2]$$

No work done because movement of charge is perpendicular to direction of E field [2]

Problem 2 [10]

$$\text{Vector describing particle's motion } \vec{r} = \vec{r}_2 - \vec{r}_1 \quad [2]$$

$$\vec{r} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ -3 \end{pmatrix} \text{ m} \quad [2]$$

$$\text{Work done } W = q\vec{E} \cdot \vec{r} = 9 \times 10^{-6} \times (-4 + 0 + 3) = -9 \times 10^{-6} \text{ J} \quad [2]$$

$$\text{Angle } \theta \text{ from } \cos \theta = \frac{\vec{E} \cdot \vec{r}}{|\vec{E}||\vec{r}|} = \frac{-4 + 0 + 3}{\sqrt{4 + 4 + 1}\sqrt{4 + 0 + 9}} = -0.0925 \quad [3]$$

$$\text{Hence } \theta = 1.66 \text{ rad or } 95.1^\circ \quad [1]$$

Problem 3 [5]

$$\text{Torque } \vec{\tau} = \vec{p} \times \vec{E} \quad [2]$$

$$\vec{p} \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -1 & 2 & 1 \\ 3 & 1 & -1 \end{vmatrix} = \begin{pmatrix} -3 \\ 2 \\ -7 \end{pmatrix} \text{ Nm} \quad [3]$$

Problem 4 [10]

$$\text{Potential energy } U_1 = qV(1,2,0) = 3 \times 10^{-3} \times (1^2 + 2 \times 1 \times 2 + 2 - 1 \times 0^2) = 21 \times 10^{-3} \text{ J} \quad [4]$$

$$\text{Potential energy } U_2 = qV(0,3,1) = 3 \times 10^{-3} \times (0^2 + 2 \times 0 \times 3 + 3 - 0 \times 1^2) = 9 \times 10^{-3} \text{ J} \quad [4]$$

$$\text{Change in potential energy } \Delta U = U_2 - U_1 = 9 \times 10^{-3} - 21 \times 10^{-3} = -12 \times 10^{-3} \text{ J} \quad [2]$$

Problem 5 [15]

$$\text{Electric field } \vec{E} = -\nabla V = \begin{pmatrix} -\frac{\partial}{\partial x} 240y \\ -\frac{\partial}{\partial y} 240y \\ -\frac{\partial}{\partial z} 240y \end{pmatrix} = \begin{pmatrix} 0 \\ -240 \\ 0 \end{pmatrix} \text{Vm}^{-1} \quad [3]$$

The electric field is uniform, has a magnitude of 240 Vm^{-1} and points in the -ive y direction [2]

Change in potential energy

$$\Delta U = q(V(0.3,0.3,0.1) - V(0.1,0.2,0.3)) = 7 \times 10^{-3} \times (240 \times 0.3 - 240 \times 0.2) = 0.168 \text{ J} \quad [2]$$

$$\text{Work done } W = q\vec{E} \cdot (\vec{r}_3 - \vec{r}_1) \quad [2]$$

$$W = 7 \times 10^{-3} \begin{pmatrix} 0 \\ -240 \\ 0 \end{pmatrix} \cdot \left(\begin{pmatrix} 0.3 \\ 0.3 \\ 0.1 \end{pmatrix} - \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \end{pmatrix} \right) = 7 \times 10^{-3} \begin{pmatrix} 0 \\ -240 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0.2 \\ 0.1 \\ -0.2 \end{pmatrix} = -0.168 \text{ J} \quad [3]$$

Hence the work done appears as the change in potential energy of the system [1]

The work done is independent of the path taken, so the result must be the same despite the intermediate step [2]

The maximum total mark for this Tutorial is 50.