Department of Physics
Year 1 Tutorials
Module Phys123
Electricity and Magnetism

Answers for Tutorial 2

The marks to be awarded for each question are indicated in square brackets.

Problem 1 [5]

The E field must be directed upwards so that the electrostatic force counteracts the proton's weight. [2]

The magnitude of the E field is given by:

$$q_p E = m_p g \Rightarrow E = \frac{m_p g}{q_p} = \frac{1.67\times10^{-27}\times 9.81}{1.60\times10^{-19}} = 1.02\times10^{-7}\ \mathrm{V\,m^{-1}}. \quad [3]$$

Problem 2 [5]

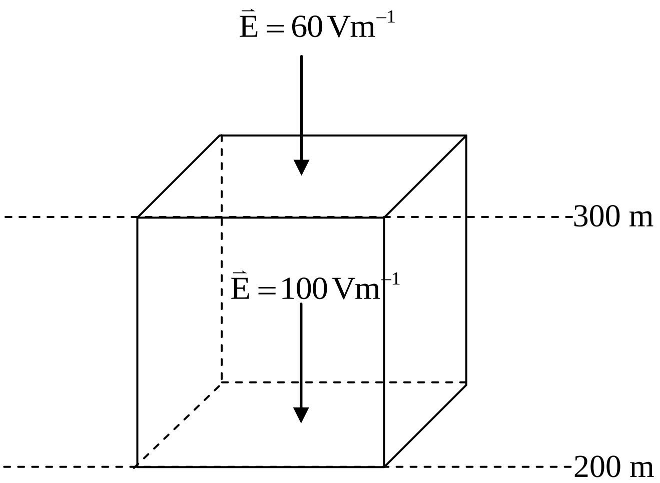
Using Gauss' law: $\Phi = \frac{q}{\varepsilon_0} = \frac{2\times10^{-6}}{8.85\times10^{-12}} = 2.26\times10^{5}\ \mathrm{NC^{-1}m^{-2}}$. [3]

Electric flux is same for cube. [1]

Flux through each of six faces is same, i.e. $\Phi_{face} = \frac{2.26\times10^{5}}{6} = 3.77\times10^{4}\ \mathrm{NC^{-1}m^{-2}}$. [1]

Results not affected by size of cube.

Problem 3 [10]



[2]

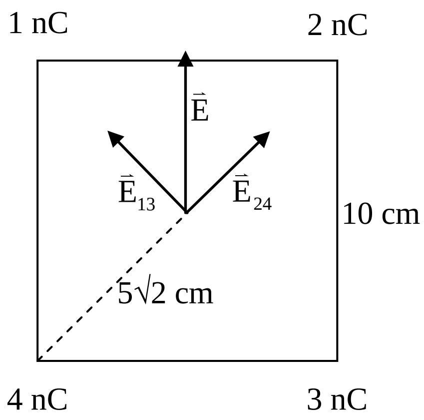
$$\Phi_{top} = \vec{E}\cdot\vec{A} = -EA = -60\times100^2 = 60\times10^4\ \mathrm{NC^{-1}m^{-2}}. \quad [2]$$

$\Phi_{bot} = \vec{E} \cdot \vec{A} = EA = 100 \times 100^2 = 100 \times 10^4\ NC^{-1}m^{-2}$. [2]

Hence, the net outward flux is $40 \times 10^4\ NC^{-1}m^{-2}$. [2]

Enclosed charge q from: $\Phi = \dfrac{q}{\varepsilon_0} \Rightarrow q = \varepsilon_0 \Phi = 8.85 \times 10^{-12} \times 40 \times 10^4 = 3.54\,\mu C.$ [2]

<u>Problem 4</u> [20]



[3]

Similar situation to that discussed in Tutorial 1, problem 2. Directions of fields due to charges of 1 nC and 3 nC and charges of 2 nC and 4 nC as illustrated above,

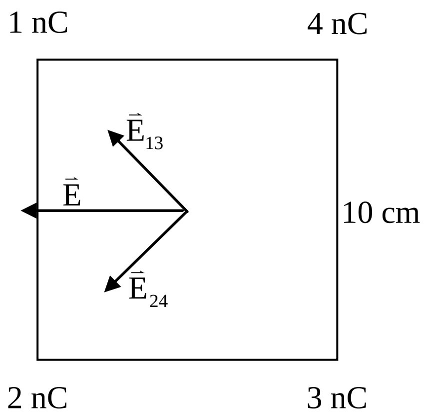
$$\left|\vec{E}_{13}\right| = \left|\vec{E}_{24}\right| = \frac{1}{4\pi\varepsilon_0} \frac{2 \times 10^{-9}}{\left(0.05\sqrt{2}\right)^2} = 3.60 \times 10^3\ Vm^{-1}. \quad [3]$$

Adding these gives the resultant field with direction as shown in the diagram. The magnitude of this field is $\left|\vec{E}\right| = \sqrt{2} \times \left|\vec{E}_{13}\right| = 5.09 \times 10^3\ Vm^{-1}$. [1]

The potential is given by

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \frac{q_4}{r_4}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{1 \times 10^{-9}}{0.0707} + \frac{2 \times 10^{-9}}{0.0707} + \frac{3 \times 10^{-9}}{0.0707} + \frac{4 \times 10^{-9}}{0.0707}\right) = 1270\,V. \quad [3]$$

If the ordering changes as shown below, then, as illustrated, the direction of the E field changes, but the magnitude of the field and the value of the potential at the centre of the square remain the same.



Place 1 nC charge first, no work must be done so no potential energy. This charge generates potential $V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$. [1]

(The value of V_1 at the position of the second charge is 89.9 V)

Then bring in 2 nC charge. Potential energy due to its position in potential of 1st charge is

$$U_{12} = q_2 V_1 = q_2 \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r} = 2\times10^{-9}\times8.99\times10^{9}\times\frac{1\times10^{-9}}{0.01} = 1.80\times10^{-7}\text{ J.} \qquad [2]$$

Choosing r_1 and r_2 appropriately, these two charges generate a potential

$$V_{12} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}\right). \qquad [1]$$

(The value of V_{12} at the position of the third charge is 243 V.)

Now bring up the third charge, potential energy is:

$$U_{123} = q_3 V_{12} = 3\times10^{-9}\frac{1}{4\pi\varepsilon_0}\left(\frac{1\times10^{-9}}{0.1414}+\frac{2\times10^{-9}}{0.1}\right) = 7.19\times10^{-7}\text{ J.} \qquad [2]$$

These three charges generate a potential $V_{123} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}+\frac{q_3}{r_3}\right)$. [1]

(The value of V_{123} at the position of charge 4 is 487 V.)

Bringing in the final charge results in a potential energy given by:

$$U_{1234} = q_4 V_{123} = 4\times10^{-9}\frac{1}{4\pi\varepsilon_0}\left(\frac{1\times10^{-9}}{0.1}+\frac{2\times10^{-9}}{0.1414}+\frac{3\times10^{-9}}{0.1}\right) = 19.5\times10^{-7}\text{ J.} \qquad [2]$$

Adding up these contributions to the potential energy gives

$$U = U_{12} + U_{123} + U_{1234} = 2.86\times10^{-6}\text{ J.} \qquad [1]$$

The maximum total mark for this Tutorial is 40.