Lecture 16

- In this lecture we will look at:
  - Electromagnetic induction.
  - Lenz's Law.
  - Induction and energy transfer.
  - Induced electric fields.
  - Faraday's Law (take two!).
  - Electric potential (take two!).

- After this lecture, you should be able to answer the following questions:
- Write down the equation that defined magnetic flux and explain what the symbols in the equation refer to.
- State Lenz's Law.
- What are eddy currents?
- A copper plate is placed at a gradient of 60° to the horizontal. A penny and a penny-sized magnet are allowed to slide down the slope. Assuming they both have similar coefficients of friction with the copper, which will travel faster? Explain your answer!

Electromagnetic Induction

- Can get B field from current (caused by E field); can we get current from B field?
- Yes, process of electromagnetic induction, requires changing magnetic flux.
- Magnetic flux defined similarly to electric flux:

  $$\Phi_B = \int \vec{B}\cdot d\vec{A} \qquad [16.1]$$

- (Recall, $d\vec{A}$ is vector, direction normal to element of area, magnitude dA.)
- If $d\vec{A} \parallel \vec{B}$ then $\vec{B}\cdot d\vec{A} = B\,dA$, and if B field uniform:

  $$\int \vec{B}\cdot d\vec{A} = B\int dA = BA$$

  so $\Phi_B = BA \qquad [16.2]$

- Unit of flux is weber, $1\,\mathrm{Wb} = 1\,\mathrm{T\,m^2}$.
- Can now state Faraday's law of induction.
- Rate of change of magnetic flux through conducting loop determines emf induced in the loop:

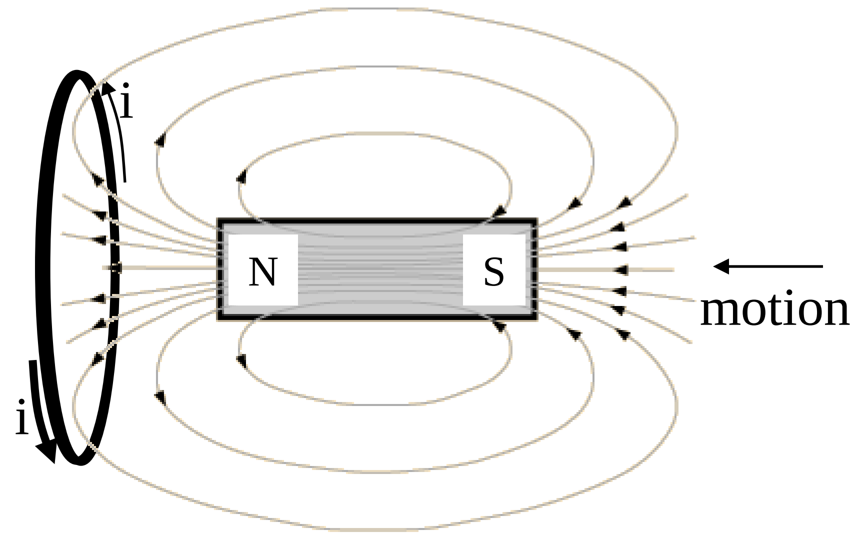
  $$\mathcal{E} = -\frac{d\Phi_B}{dt} \qquad [16.3]$$

  For N coils, $\mathcal{E} = -N\dfrac{d\Phi_B}{dt} \qquad [16.4]$

- Direction of emf from Lenz's law:
- An induced current has direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current (hence –ive sign in Faraday's law).

Lenz's Law

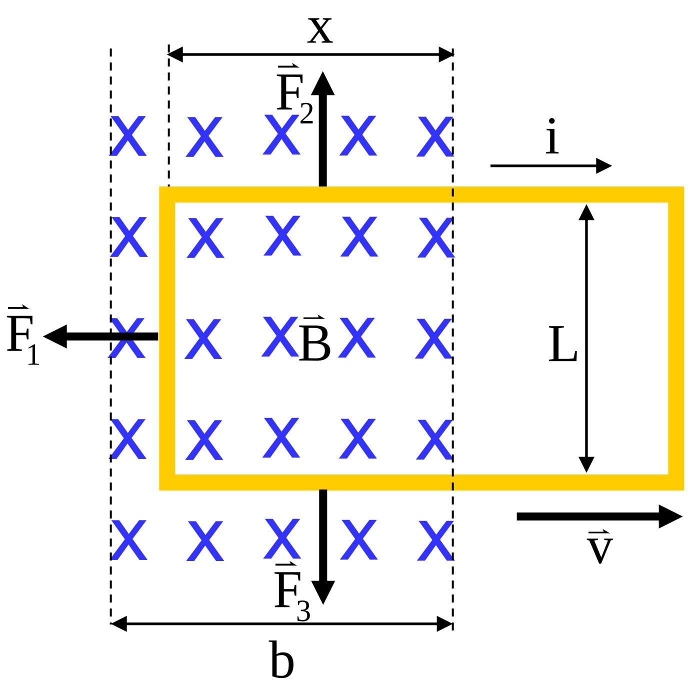
- An example of Lenz's law is provided by the following situation:



- Moving magnet towards loop induces current in loop.
- Lenz's law says that magnetic dipole caused by this current must oppose the change inducing the current.
- This means induced dipole must have north pole pointing towards north pole of magnet (like poles repel).
- Hence, from RH rule, current must be as illustrated.
  - Curl RH fingers around in direction of i.
  - Thumb gives direction of dipole (and B field).
  - Field lines run from north to south pole outside loop.
- If move magnet in opposite direction, induced magnetic dipole tries to oppose change, so south pole next to magnet which tends to attract magnet back towards loop.

Induction and Energy Transfer

- Pull a closed loop out of a magnetic field at constant velocity:



- Here, flux through loop is changing because area of loop in magnetic field is changing (decreasing).
- Flux through loop $\Phi_B = BA = BLx.$
- $|\mathcal{E}| = \frac{d\Phi_B}{dt} = \frac{d}{dt}BLx = BL\frac{dx}{dt} = BLv.$
- Cannot use Kirchoff's loop rule to work out i as cannot define a potential difference for induced emf.
- Use i = $\mathcal{E}$/R (R is resistance of loop):

$$i = \frac{BLv}{R}.$$

- $\vec{F}_2$ and $\vec{F}_3$ cancel.
- $F_1 = iLB = \frac{B^2L^2v}{R}.$
- Hence F constant if v constant.

Induction and Energy Transfer: Eddy Currents

- Rate at which work done is power:

$$P = \frac{d}{dt}Fx = F\frac{dx}{dt} = Fv.$$

- Using expression for force:

$$P = Fv = \frac{B^2L^2v^2}{R}.$$

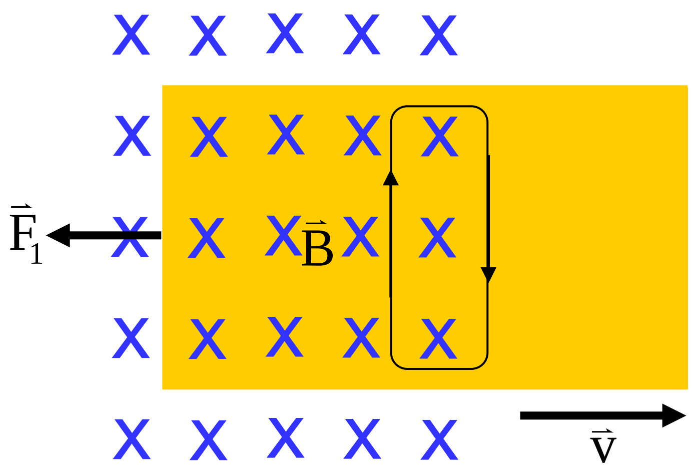
- Rate at which thermal energy appears in loop is given by:

$$P_H = i^2R = \left(\frac{BLv}{R}\right)^2 R = \frac{B^2L^2v^2}{R}.$$

- See work done on loop appears as thermal energy:

$$P_H = P.$$

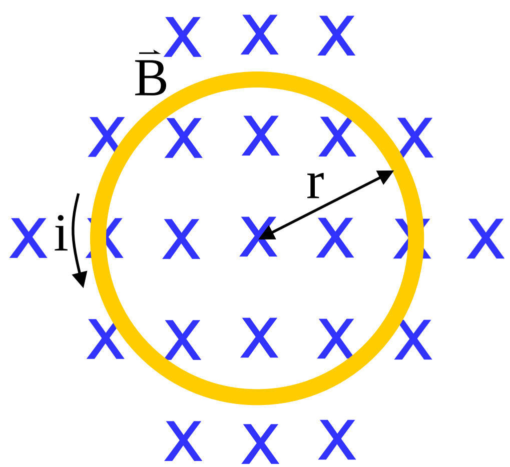
- Replace loop in previous example by copper plate:



- Swirls of current, "eddy currents", induced in copper as it is moved (represented as one current loop).
- Mechanical energy (movement) is converted into thermal energy.

Induced Electric Fields

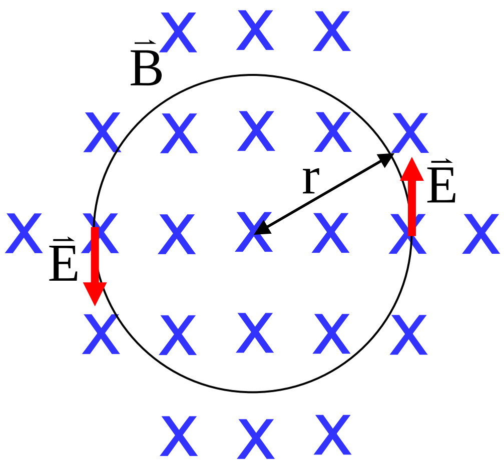
- Consider copper ring of radius r in uniform external magnetic field:



- Increase field at a steady rate, changing magnetic flux through ring and hence inducing emf (Faraday's law) and current.
- Lenz's law tells us induced current is anti-clockwise.
- If there is current, an electric field must be present to exert the force qE on the charges which are flowing to cause the current.
- Hence can restate Faraday's law: a changing magnetic field induces an electric field.
- There is nothing in this law that tells us that the copper ring must be present for a field to be induced!
- Think about the same situation without a copper ring…

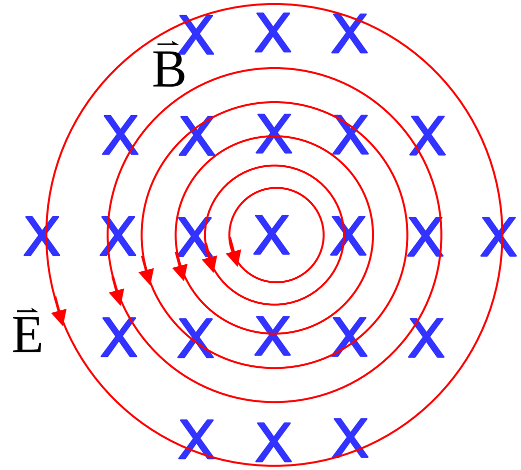
Induced Electric Fields

- Replace copper ring by hypothetical circular path:

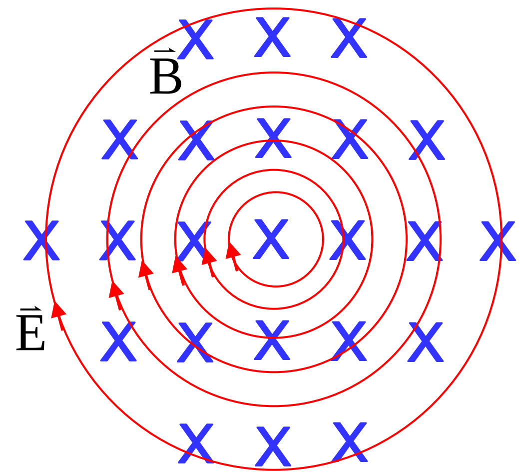


- Magnetic field increasing at constant rate dB/dt.
- Electric field induced must be tangent to ring (from symmetry).
- If choose another ring, same applies so electric field lines must be set of concentric rings.

- E field $dB/dt > 0$:

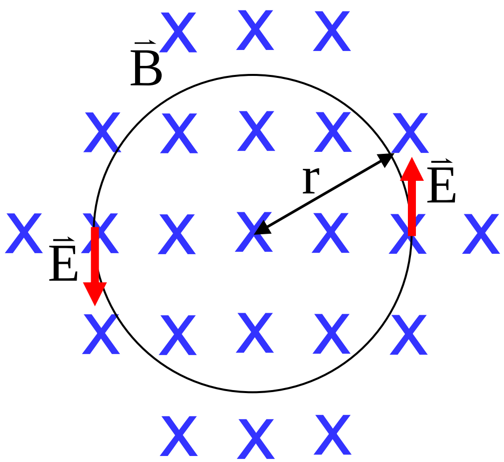


- E field $dB/dt < 0$:



Faraday's Law Reformulated

- Consider a charge q_0 moving round the circular path below:



- Work done on charge:

  $W = \oint \vec{F} \cdot d\vec{s} = q_0 \oint \vec{E} \cdot d\vec{s}.$

- Work also given by:

  $W = q_0 \mathcal{E}.$

- Equating two expressions we see:

  $q_0 \mathcal{E} = q_0 \oint \vec{E} \cdot d\vec{s} \ \ (= q_0 E(2\pi r) \text{ for loop})$

  $\Rightarrow \mathcal{E} = \oint \vec{E} \cdot d\vec{s}.$

- Combine this with expression for Faraday's law…

  $\mathcal{E} = -\frac{d\Phi_B}{dt}$

- …we get:

  $$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad [16.5]$$

- This is the mathematical expression of our restatement of Faraday's law: a changing magnetic field induces an electric field.

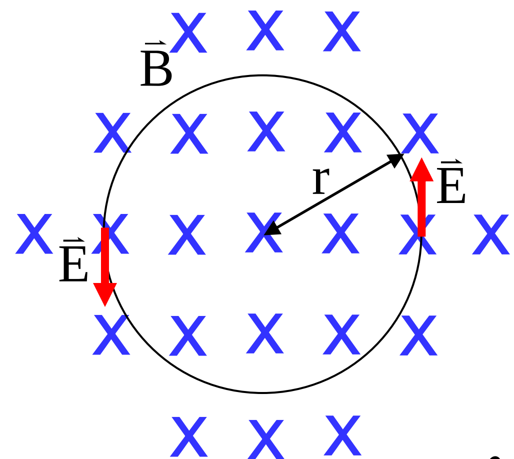
Another Look at Electric Potential

- Electric fields induced by changing magnetic fields are different to those caused by static charges.
- Induced fields lines form closed loops, those caused by charges start on +ive and finish on –ive charges.
- Electric potential has no meaning for induced E fields.
- Recall definition of electric potential:

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}.$$

- (We introduced this expression before we knew about induced electric fields!)
- See this in example opposite.

- Consider going round loop in:



- Initial, final positions same: $\oint \vec{E} \cdot d\vec{s} = 0.$
- However, because of changing B field answer should be:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}.$$

- Facit: potential undefined for E fields induced by B fields.