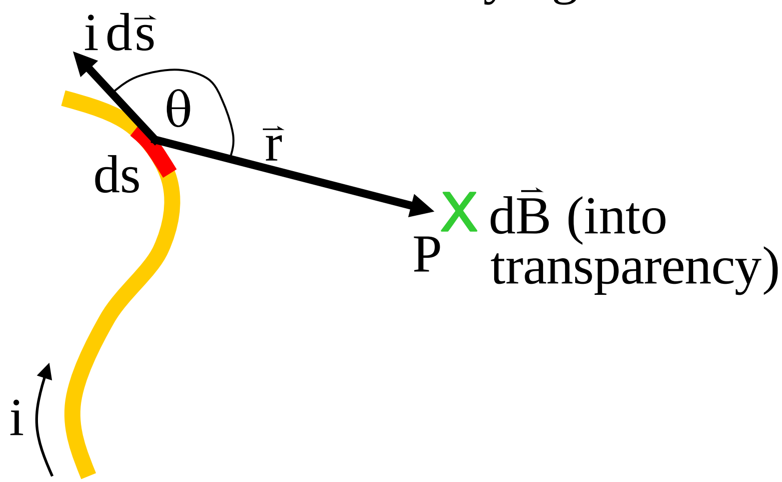
Lecture 14

- In this lecture we will look at:
  - Magnetic fields due to currents.
  - Magnetic fields due to a long straight wire.
  - Magnetism and relativity.
  - Magnetic field due to an arc and a loop.
  - Force between two parallel currents.
  - The Rail Gun.

- After this lecture, you should be able to answer the following questions:
- What is the magnitude and direction of the magnetic field a distance R from a long straight current-carrying wire?
- Determine the strength and the direction of the force between two long parallel wires, separated by a distance of 1 mm, which are each carrying a current of 6 A but which is flowing in opposite directions in each of the wires.

Magnetic Fields due to Currents

- Consider a current carrying wire:



- Magnitude of B field at point P due to current-length element $i\,d\vec{s}$ is,

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}.$$

- The permeability constant (or the permeability of free space) μ_0 is defined to be exactly:

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T\,m\,A^{-1}}$$
$$\approx 1.26 \times 10^{-6}\ \mathrm{T\,m\,A^{-1}}.$$

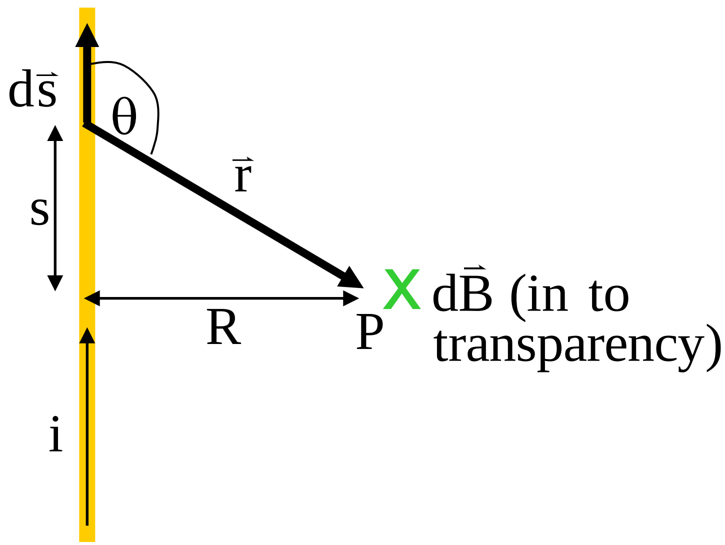
- Writing the expression for the field in terms of a cross product allows the direction to be determined:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s} \times \vec{r}}{r^3} \qquad [14.1]$$

- This expression is known as the Biot-Savart Law.

Magnetic Field due to a Long Straight Wire

- Consider long straight wire:



- $$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$

- Integrating: $B = 2\int_0^\infty dB = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{\sin\theta}{r^2}ds.$

- Using the relationships $r^2 = s^2 + R^2$ and
$$\sin\theta = \sin(\pi-\theta) = \frac{R}{\sqrt{s^2+R^2}}.$$

- We get: $B = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{R}{(s^2+R^2)^{\frac{3}{2}}}ds.$

- Using the result:
$$\int \frac{dx}{(x^2+a^2)^{\frac{3}{2}}} = \frac{x}{a^2(x^2+a^2)^{\frac{1}{2}}}$$

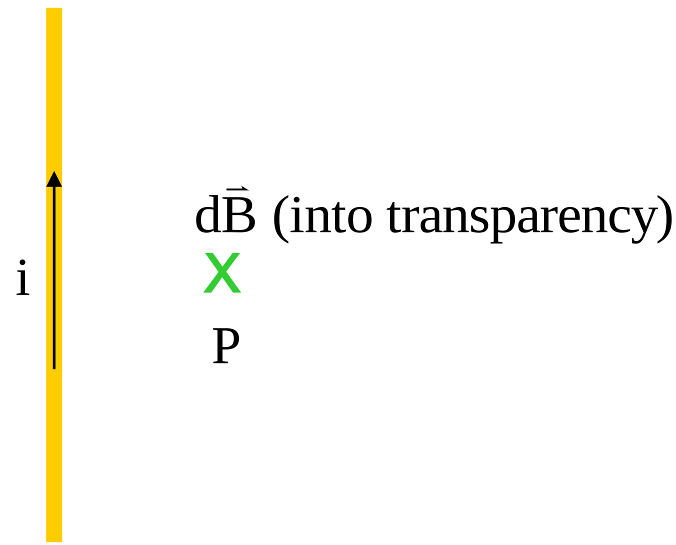
- The integral becomes:
$$B = \frac{\mu_0 i}{2\pi R}\left.\frac{s}{\sqrt{s^2+R^2}}\right|_0^\infty$$
$$= \frac{\mu_0 i}{2\pi R} \qquad [14.2]$$

Direction of Field due to Current in Wire

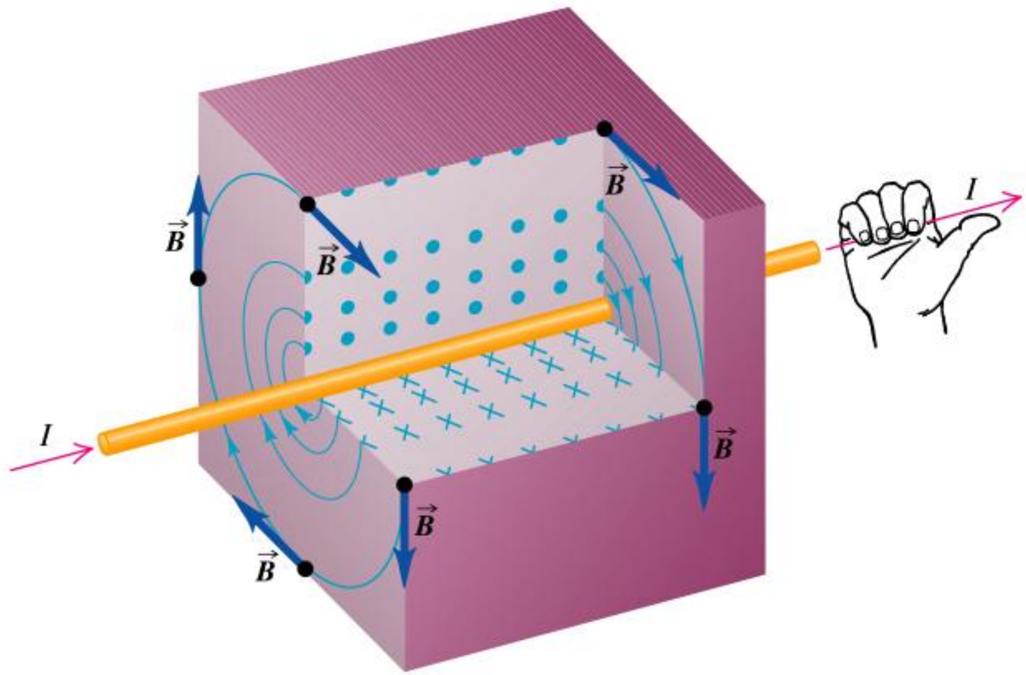
- From vector expression for Biot-Savart law...

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\vec{r}}{r^3}.$$

...know direction of B field, e.g:

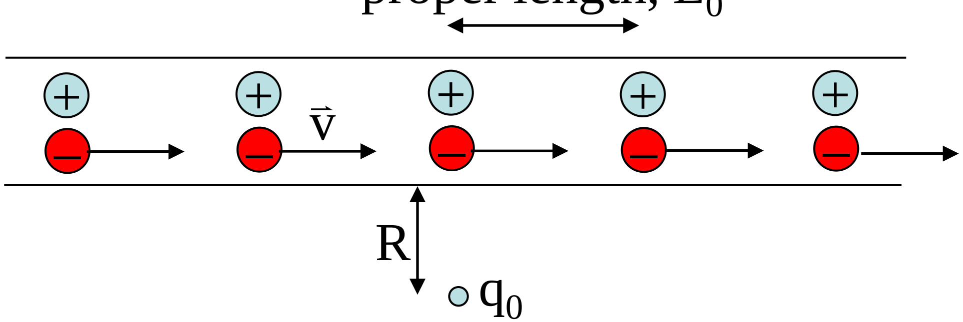


- This leads to (yet another) version of the right-hand rule.



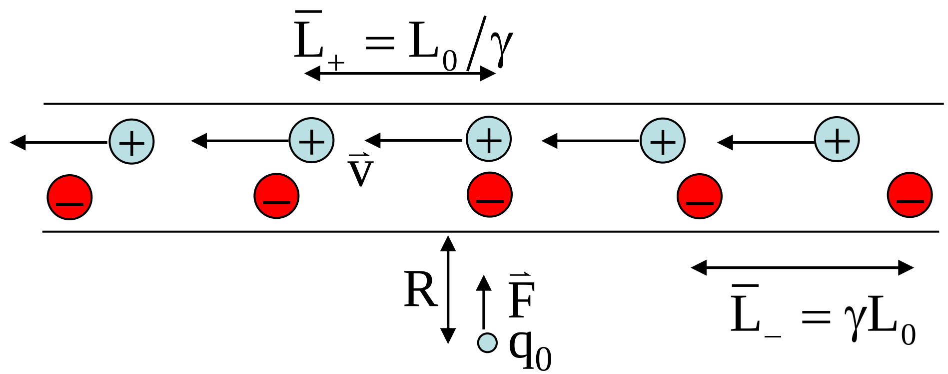
Magnetism and Relativity

- Consider a stationary test charge, q_0, next to a wire with a current flowing in it:



- Have total +ive charge +q and –ive charge –q in section of wire.
- In this situation, the test charge experiences no force (the net charge of the wire is zero).
- Now move the test charge along the wire with the same velocity as the charge carriers in the wire.

- Look at the wire in the rest frame of the test charge:



- Positive charges now moving w.r.t. q_0, separation Lorentz contracted.
- Negative charges were moving w.r.t. q_0, no longer Lorentz contracted, so must now be "uncontracted".
- $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}} = \left(1 - \dfrac{v^2}{c^2}\right)^{-\frac{1}{2}}$.

Magnetism and Relativity

- The linear charge density in the frame of the test charge is thus:

$$\lambda = \frac{q}{L_+} - \frac{q}{L_-} = \frac{\gamma q}{L_0} - \frac{q}{\gamma L_0}$$

$$= \frac{q}{L_0}\left(\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} - \left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}\right)$$

$$\approx \frac{q}{L}\left(\left(1+\frac{v^2}{2c^2}\right) - \left(1-\frac{v^2}{2c^2}\right)\right)$$

$$\approx \frac{q}{L_0}\frac{v^2}{c^2} \text{ (for } v << c\text{).}$$

- The (electrostatic) force in this frame is $\left|\vec{F}_E\right| = q_0\left|\vec{E}\right|$ and is directed away from the wire (for +ive q_0).

- Now the E field due to a continuous line of charge is:

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}.$$

- We see:

$$\left|\vec{F}_E\right| = q_0 \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R} = q_0 \frac{1}{2\pi\varepsilon_0}\frac{qv^2}{RL_0c^2}.$$

- However, $i = qv/L_0$, so

$$\left|\vec{F}_E\right| = q_0 v \frac{1}{2\pi\varepsilon_0 c^2}\frac{i}{R}$$

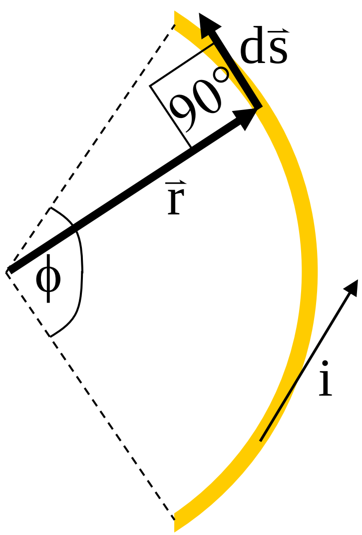
$$= q_0 v \frac{\mu_0}{2\pi}\frac{i}{R} = \left|\vec{F}_B\right| \text{ (in wire frame)}$$

using $\varepsilon_0 c^2 = 1/\mu_0$, where μ_0 is the permeability of free space.

- Hence, $B = \mu_0 i/2\pi R.$

Magnetic Field due to Circular Arc and Loop

- Calculate field at centre of arc, radius of curvature R:



- Direction always out of transparency (right hand rule).

- $dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 90^\circ}{R^2}.$
- Now ds = R dϕ, so

$$B = \int dB = \int_0^{\phi} \frac{\mu_0}{4\pi}\frac{iR}{R^2}\,d\phi'$$

$$= \frac{\mu_0 i}{4\pi R}\int_0^{\phi} d\phi'$$

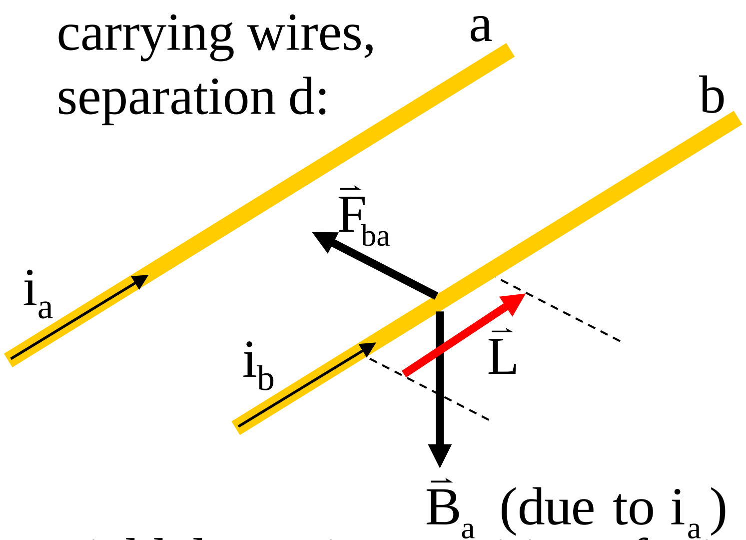
$$= \frac{\mu_0 i}{4\pi R}\phi.$$

- Note, ϕ in radians!
- For complete circle (loop of wire), ϕ = 2π, so:

$$B = \frac{\mu_0 i(2\pi)}{4\pi R} = \frac{\mu_0 i}{2R} \qquad [14.3]$$

Force Between Two Parallel Currents

- Consider two parallel current-carrying wires, separation d:



- Field due to i_a at position of wire b is:

$$B_a = \frac{\mu_0 i_a}{2\pi d}.$$

- Force produced on current in length L of wire b due to this field:

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}.$$

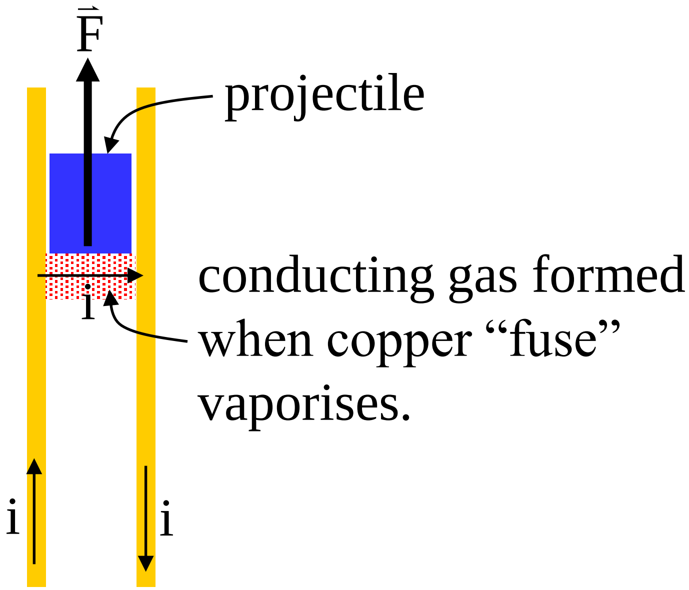
- This gives:

$$F_{ba} = i_b L B_a \sin 90^\circ$$
$$= \frac{\mu_0 L i_a i_b}{2\pi d} \qquad [14.4]$$

- The two wires attract if current parallel.
- If currents anti-parallel, magnitude of force same but is repulsive.
- Definition of ampere:
  The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length and negligible cross section, placed one metre apart in vacuum, would produce on each of these conductors a force of magnitude 2×10^{-7} N.

Rail Gun

- Device in which magnetic field can rapidly accelerate projectile.



- Right hand rule shows that current in rails causes B field between them which is directed into transparency.

- This B field causes a force on the current in the gas which pushes it upwards.
- The gas accelerates the projectile upwards.