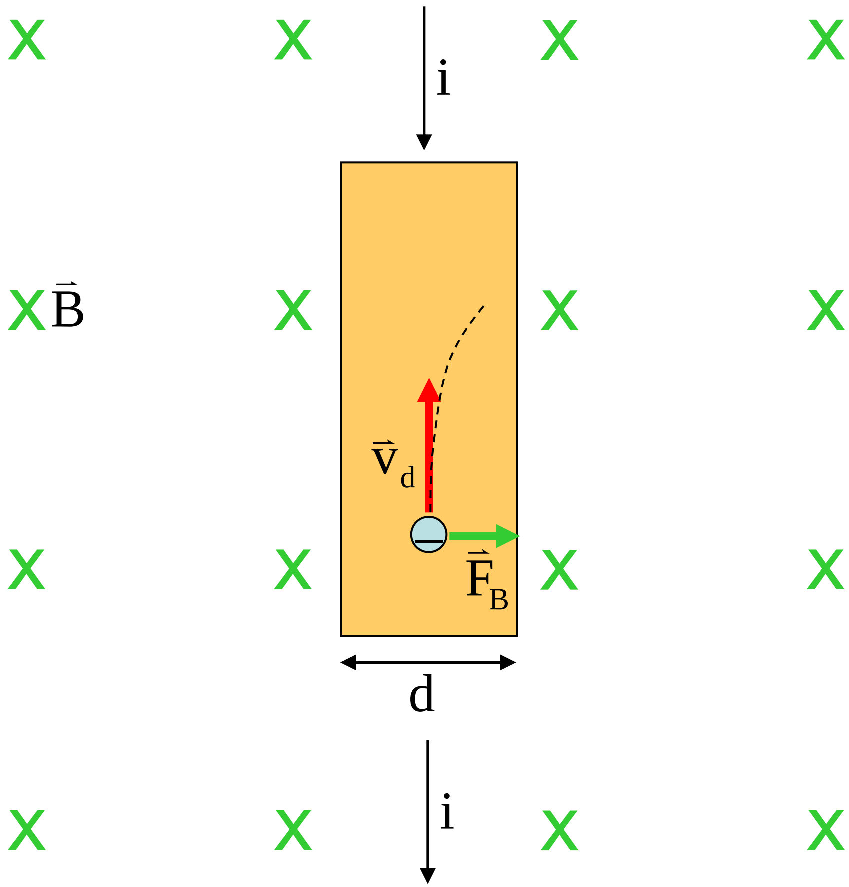
# Lecture 13

- In this lecture we will look at:
  - The Hall Effect.
  - The Cathode Ray Tube.
  - The force on a current carrying wire and the electric motor.
  - Magnetic dipole moments.
  - The galvanometer.

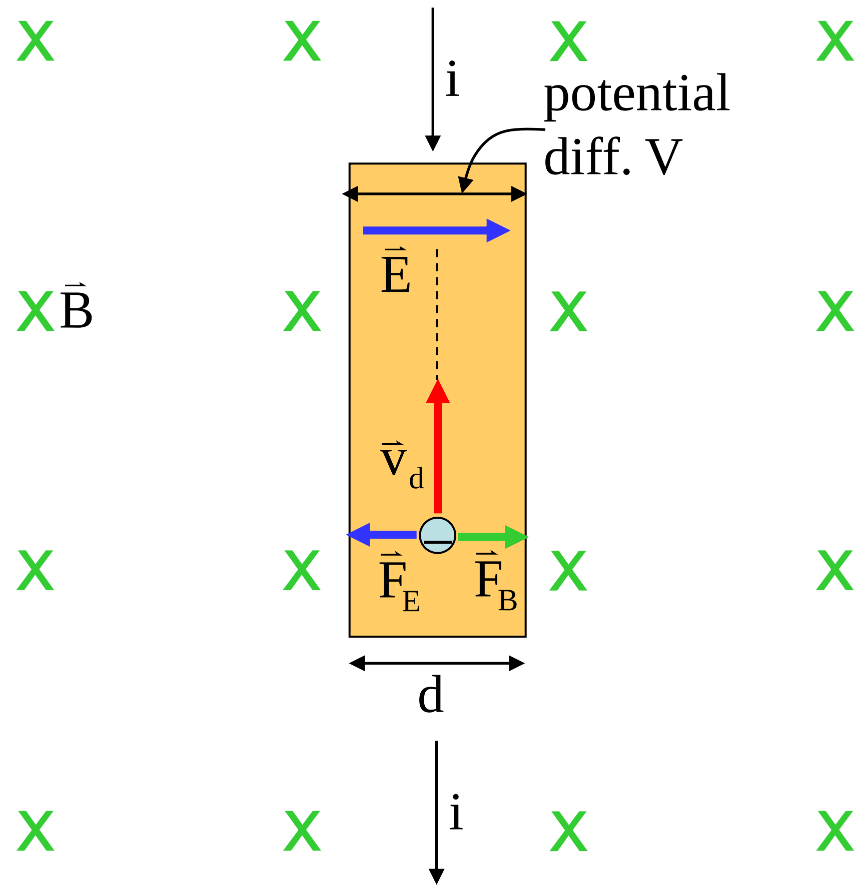
- After this lecture, you should be able to answer the following questions:
- How can the Hall Effect be used to determine the density of charge carriers in a conductor?
- Explain the functioning of a Cathode Ray Tube.
- Show how the force on a wire carrying a current in a magnetic field, $\vec{F}_B = i\vec{L} \times \vec{B}$, is related to the force on the charge carriers in the wire, $\vec{F} = q\vec{v} \times \vec{B}$.

# Crossed Electric and Magnetic Fields: Hall Effect

- The Hall Effect.
- Consider a copper strip carrying a current i in a magnetic field.




- Electrons forced to RHS of strip, establishing E field which opposes magnetic force; at equilibrium:



# The Hall Effect

- At equilibrium, the Hall potential difference, V, across the strip gives rise to the field E = V/d.
- Equating the magnetic and electric forces:

$$F_E = F_B \Rightarrow e\frac{V}{d} = ev_d B.$$

- We know $v_d = \frac{J}{ne} = \frac{i}{neA}$.
- Here, A is the cross-sectional area of the strip and J = i/A.
- if the strip's thickness is t, $A = t\,d$, so

$$e\frac{V}{d} = e\frac{i}{neA}B$$

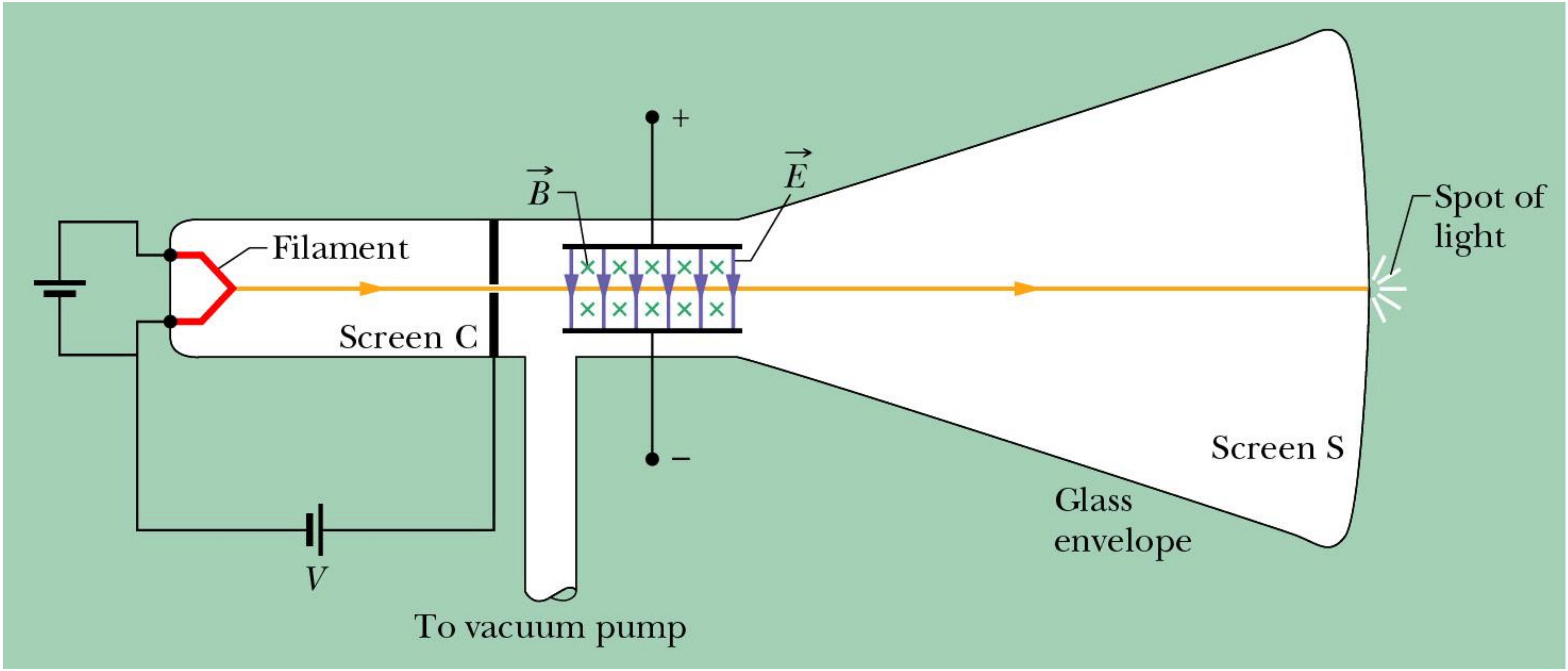
$$\Rightarrow n = \frac{id}{etdV}B = \frac{Bi}{Vte}.$$

- Hence n, the number of charge carriers per unit volume, can be measured.
- The Hall Effect can also be used to measure magnetic field strengths, rewrite expression

$$B = \frac{nVte}{i} \qquad [13.1]$$

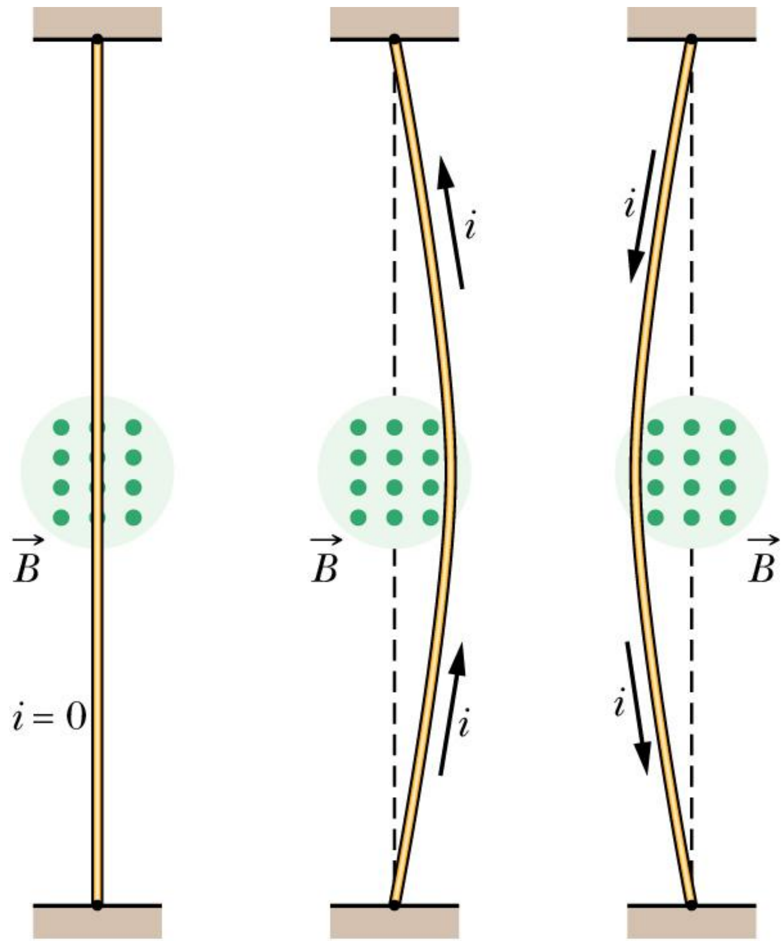
- By moving the probe through a B field in the direction opposite to the drift of the charge carriers and adjusting the speed until the Hall potential is zero, $v_d$ can be measured.
- Can the Hall Effect be used to determine the sign of the charge carriers?

# Crossed Fields: Cathode Ray Tube

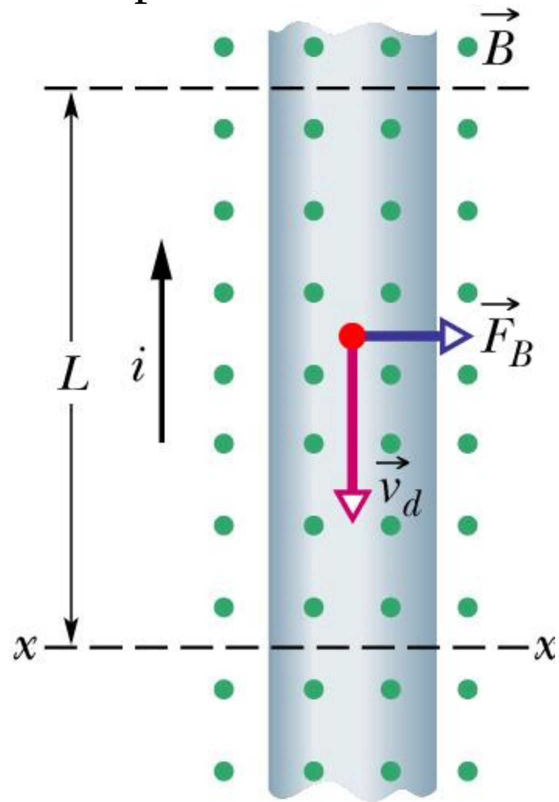


# Force on a Current Carrying Wire

- Consider flexible wire passing between poles of magnet:

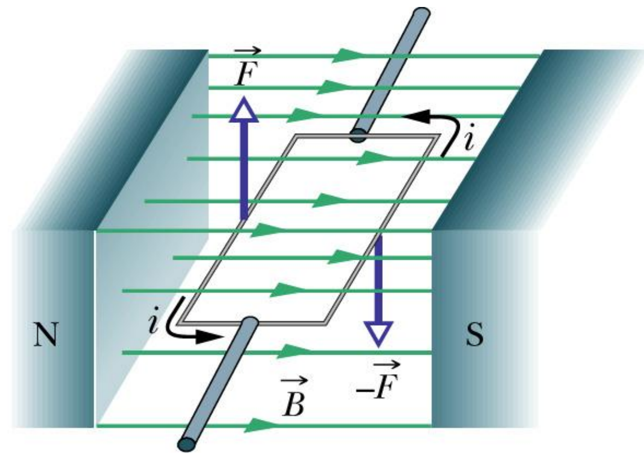



- No current, no deflection; current upwards, deflection to right; downwards deflection to left.
- Close-up of section of wire:



# Force on a Current Carrying Wire: Electric Motor

- Force on moving charge, $\vec{F} = q\vec{v} \times \vec{B}.$
- For wire of length L: $$q = i\,t = i\frac{L}{v_d}.$$
- Hence, for wire $\vec{F}_B = i\,\vec{L} \times \vec{B}$ [13.2]
- If orientation of wire in B field changes, must calculate force for elements of wire... $d\vec{F}_B = i\,d\vec{L} \times \vec{B}$ ...and integrate over wire
- Now calculate torque on loop:

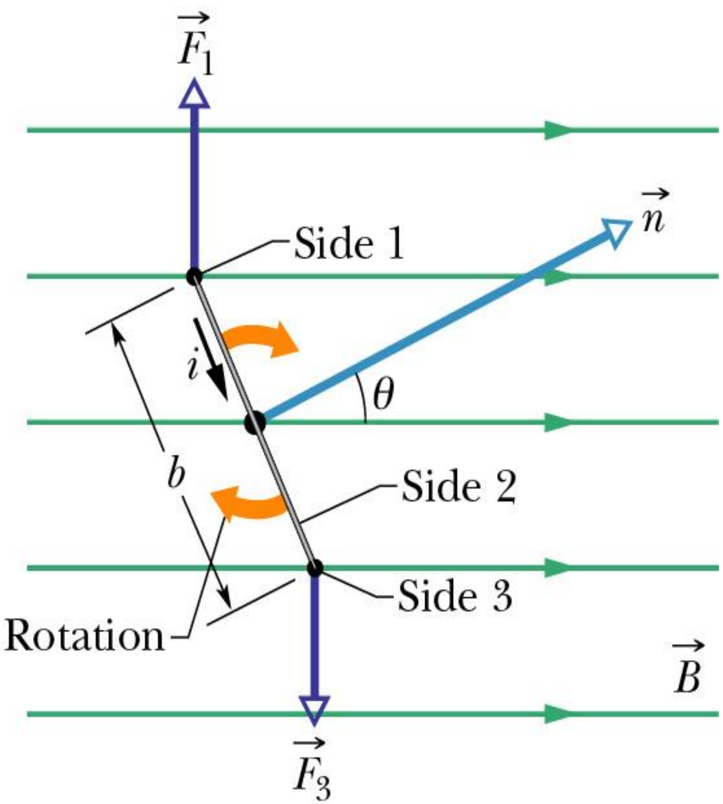



- Forces $\vec{F}$ and $-\vec{F}$ cause torque on loop.
- No torque due to forces on ends of loop.
- Calculate torque, $\tau_1$, due to current in long sides of (single) loop.
- Length of each long side is a.

# Electric Motor

- $\tau_1 = iaB\frac{b}{2}\sin\theta + iaB\frac{b}{2}\sin\theta$

  $= iabB\sin\theta.$



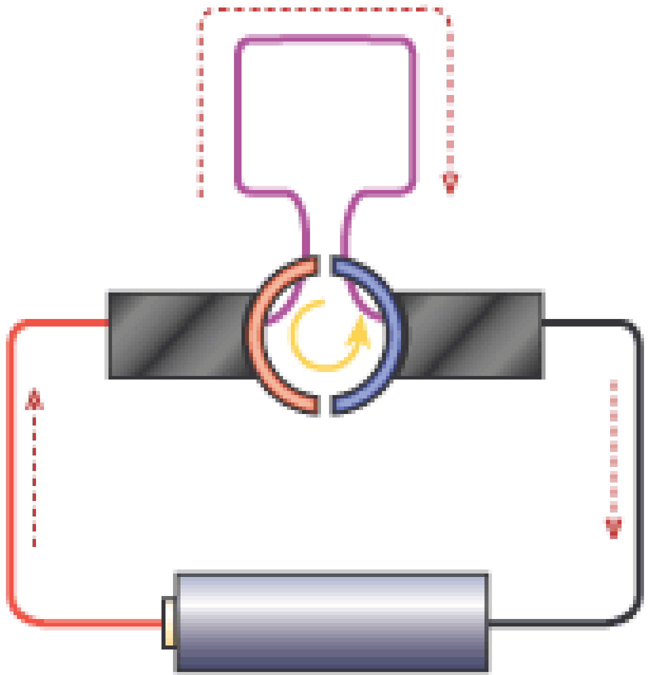

- Replace single loop with coil of N turns.
- Then have N times torque calculated for one loop.

  $\tau = Ni(ab)B\sin\theta.$
- Replace the length and width of the coil (a and b) by the area A = ab.

  $\tau = (NiA)B\sin\theta$ [13.3]
- Note, torque will tend to align $\vec{n}$ along $\vec{B}$.
- Formula applies to all shapes of flat coil.
- In a motor, the current must be reversed as $\vec{n}$ lines up with $\vec{B}$ to ensure the torque tends to keep the coil turning.

# Commutator for Electric motor

- Flipping of current performed by "commutator":



- Current flows in one direction through circuit containing battery.
- "Brushes" at end of loop in magnetic field contact with alternate ends of loop as this rotates, flipping the direction of the current in the loop.
- Direction of torque doesn't change.

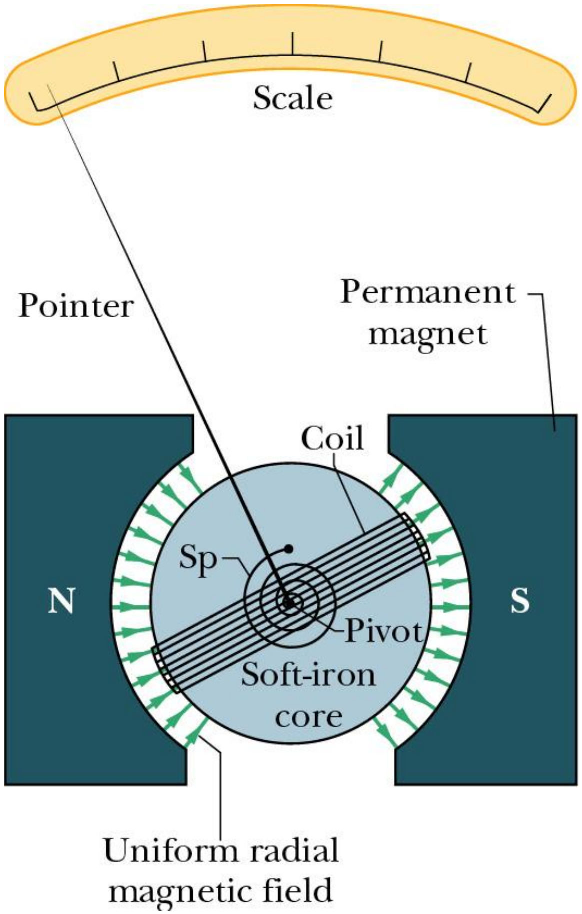
# Magnetic Dipole Moment

- The quantity (N i A), with direction $\hat{n}$, is termed the magnetic dipole moment vector:
  $|\vec{\mu}| = N\,i\,A$ [13.4]
- Units A m$^2$.
- Can then rewrite equation for torque on loop: $\vec{\tau}_B = \vec{\mu} \times \vec{B}$ [13.5]
- C.f. expression for torque on dipole in E field: $\vec{\tau}_E = \vec{p} \times \vec{E}$.
- Similarly to electric case, potential energy of magnetic dipole in B field given by:
  $U = -\vec{\mu} \cdot \vec{B}$ [13.6]

- Example:
- A square loop has N = 100 turns. The area of the loop is 4 cm$^2$ and it carries a current I = 10 A. It makes an angle of 30$^o$ with a B field of strength 0.8 T. Find the magnetic moment of the loop and the torque.
- $\mu = NiA = 100 \times 10 \times 4 \times 10^{-4}$
  $= 0.4\,\mathrm{Am}^2.$
- $|\vec{\tau}| = |\vec{\mu} \times \vec{B}| = \mu B \sin\phi$
  $= 0.4 \times 0.8 \times \sin 30^\circ$
  $= 0.16\,\mathrm{Nm}.$

# Galvanometer

- Use magnetic dipole induced by current in loop in uniform B field to measure current.




- Spring (“Sp”) used to provide torque opposite to that due to current.
- Equilibrium position when torque due to current equal and opposite to that due to spring.
- Hence ammeter: may need small resistance in parallel with galvanometer to adjust full scale deflection to appropriate range.
- Voltmeter can be constructed by adding large resistance in series with galvanometer.