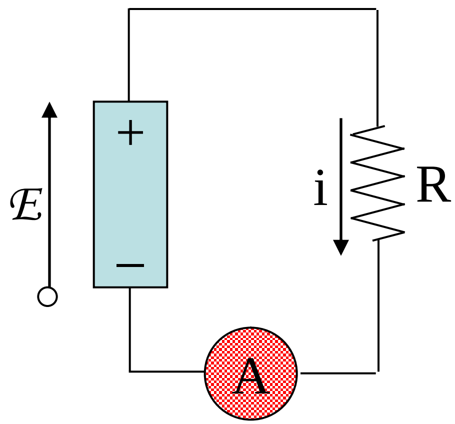
Lecture 11

- In this lecture we will look at:
  - Measuring currents
  - Measuring potentials.
  - Circuits with resistance and capacitance.

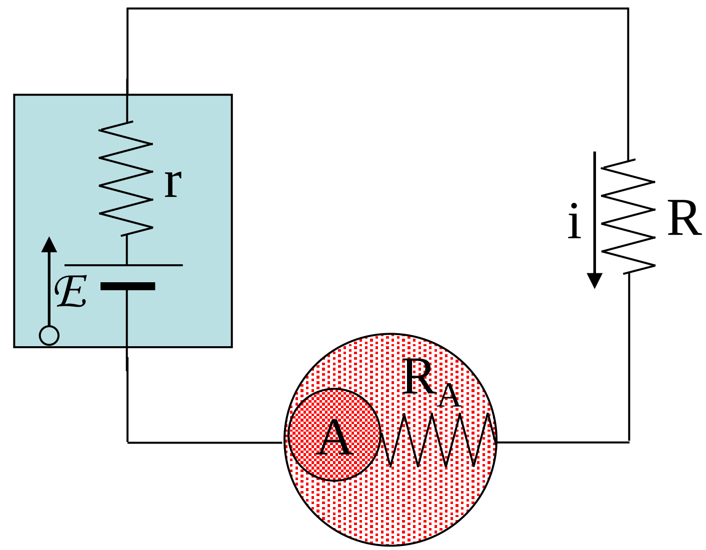
- After this lecture, you should be able to answer the following questions:
- Explain why the resistance of an ammeter must be small and the resistance of a voltmeter large.
- Write down the equation describing how the current in a series circuit containing a capacitance C, a resistance R, an emf $\mathcal{E}$ and a switch changes from the instant that the switch is closed.

Measuring Current

- The ammeter is used to measure current.
- Must be inserted in the circuit (in series) at the point at which the current is to be measured.



- The resistance of the ammeter, R_A, must be small ($R_A << R + r$) otherwise the meter causes a change in the current.

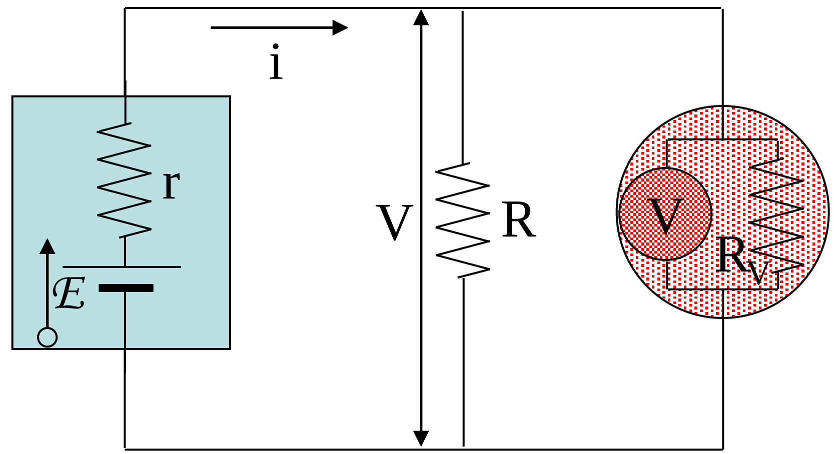


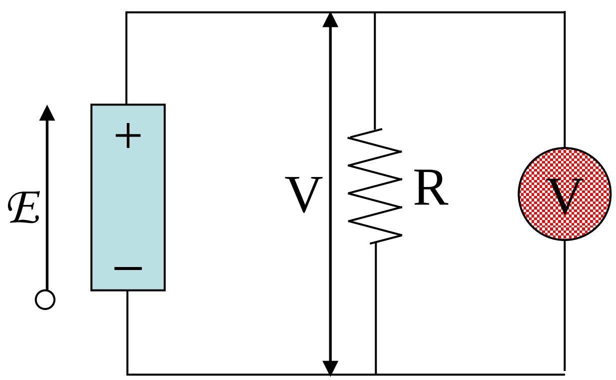
- Example, if R = 1 kΩ, r = 200 Ω, R_A = 10 Ω and ℰ = 12 V:
  - i = 12/1200 = 10 mA before insertion of ammeter...
  - ...and afterwards

$$i' = \frac{12}{1000+200+10} = 9.92\,\text{mA}.$$

Measuring Potential

- The voltmeter is used to measure potential.
- Must be connected across (in parallel with) the component over which the potential is to be measured.





- The resistance of the voltmeter, R_V, must be large ($R_V >> R + r$) otherwise the meter changes V.
- Example, if R = 1 kΩ, r = 200 Ω, R_V = 100 kΩ and $\mathcal{E}$ = 12 V:
- Before insertion of voltmeter i = 10 mA, so V = 0.01 x 1000 = 10 V.
- Afterwards the resistance seen by $\mathcal{E}$ is:

$$R_{eq} = r + \left(\frac{1}{R} + \frac{1}{R_V} \right)^{-1} = r + \frac{R\,R_V}{R + R_V}$$

$$= \frac{r(R + R_V) + R\,R_V}{R + R_V} = 1190.1\Omega.$$

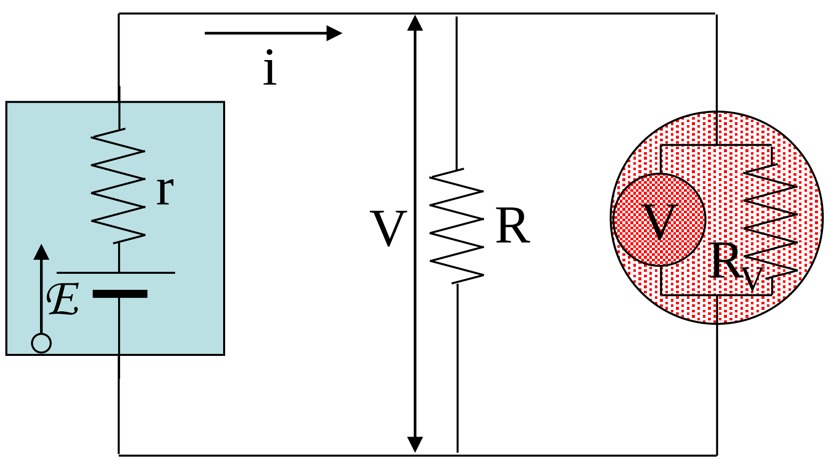
Measuring Potential

- Hence new current:

$$i' = \frac{\mathcal{E}}{R_{eq}} = \frac{12}{1190} = 10.08\,\text{mA}.$$

- The resistance over which the potential is measured is:

$$R' = \left(\frac{1}{R} + \frac{1}{R_V}\right)^{-1} = \frac{R\,R_V}{R + R_V} = 990.1\,\Omega.$$



- From these we obtain potential measured by voltmeter:

$$V' = i'R'$$

$$= \frac{\mathcal{E}}{\dfrac{r(R+R_V) + R\,R_V}{R+R_V}} \frac{R\,R_V}{R+R_V}$$

$$= \frac{\mathcal{E}\,R\,R_V}{r(R+R_V) + R\,R_V}.$$

- Numerically,

$$V' = i'R'$$

$$= 0.01008 \times 990$$

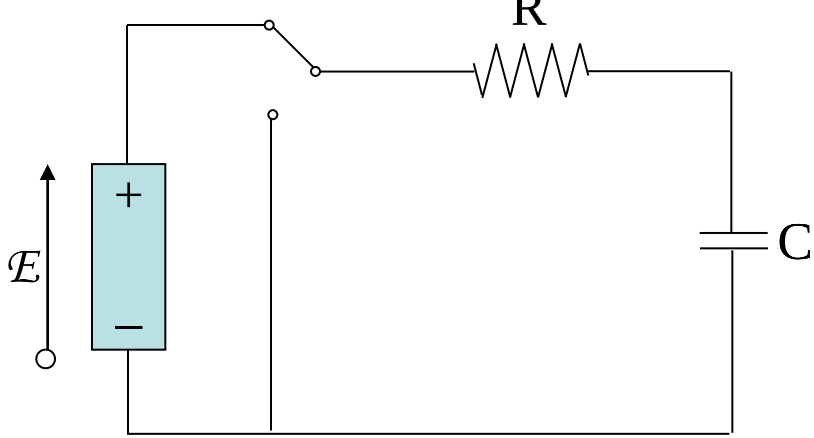
$$= 9.98\,\text{V}.$$

- Note "potential divider":

$$V_{R'} = 12 \times 990.1/1190,\ V_r = 12 \times 200/1190.$$

Circuits with Resistance and Capacitance

- First consideration of circuits in which current varies with time: charging a capacitor.



- Potential across capacitor $V_C(t) = q(t)/C$.
- Potential across resistor $V_R(t) = i(t)\,R$.
- Note these change with time!

- Applying Kirchoff's loop rule:

$$\mathcal{E} - i(t)R - \frac{q(t)}{C} = 0.$$

- Cannot solve as contains two variables, i(t) and q(t).

- However: $i = \frac{dq}{dt}$.

- Hence, $R\frac{dq}{dt} + \frac{q}{C} = \mathcal{E}$ [11.1]

- Try solution:
$$q = C\mathcal{E}\left(1 - \exp(-t/RC)\right).$$

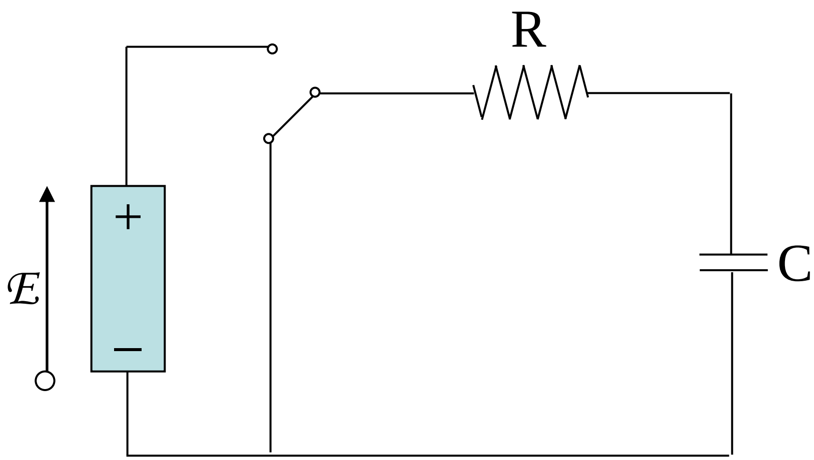
- Differentiating:
$$\frac{dq}{dt} = \frac{\mathcal{E}}{R}\exp(-t/RC)$$

Circuits with Resistance and Capacitance

- Substitute into equation to check have solution.
- $$R\frac{dq}{dt}+\frac{q}{C}$$
  $$= R\frac{\mathcal{E}}{R}\exp\left[-\frac{t}{RC}\right]+\frac{1}{C}C\mathcal{E}\left(1-\exp\left[-\frac{t}{RC}\right]\right)$$
  $$= \mathcal{E}\exp\left[-\frac{t}{RC}\right]+\mathcal{E}-\mathcal{E}\exp\left[-\frac{t}{RC}\right]$$
  $$= \mathcal{E}, \text{QED.}$$
- The quantity $\tau = RC$ is the time constant of the circuit: units seconds.
- Using $q = CV$ see solution can be written:
  $$V_C = \frac{q}{C} = \mathcal{E}\left(1-\exp(-t/RC)\right) \qquad [11.2]$$
- The differential w.r.t. time is the current through the circuit:
  $$i(t) = \frac{dq}{dt} = \frac{\mathcal{E}}{R}\exp(-t/RC) \qquad [11.3]$$
- Check get expected limits:
  - $q(0) = 0$.
  - $V(0) = 0$.
  - $i(0) = \mathcal{E}/R$, (i.e. maximum current when switch first closed).
  - $q(\infty) = C\mathcal{E}$.
  - $V(\infty) = \mathcal{E}$.
  - $I(\infty) = 0$.

Circuits with Resistance and Capacitance

- Now think about discharging capacitor:



- Using Kirchoff's loop rule:

$$i(t)R + \frac{q(t)}{C} = 0 \text{ or}$$

$$R\frac{dq}{dt} + \frac{q}{C} = 0 \qquad [11.4]$$

- Try solution $q = q_0 \exp(-t/RC)$.
- Differentiating and substituting we get:

$$R\frac{dq}{dt} + \frac{q}{C}$$

$$= -R\frac{q_0}{RC}\exp\left[-\frac{t}{RC}\right] + \frac{1}{C}q_0 \exp\left[-\frac{t}{RC}\right]$$

$$= 0, \text{ QED.}$$

- Voltage across C, current in circuit:

$$V = \frac{q}{C} = \frac{q_0}{C}\exp(-t/RC)$$

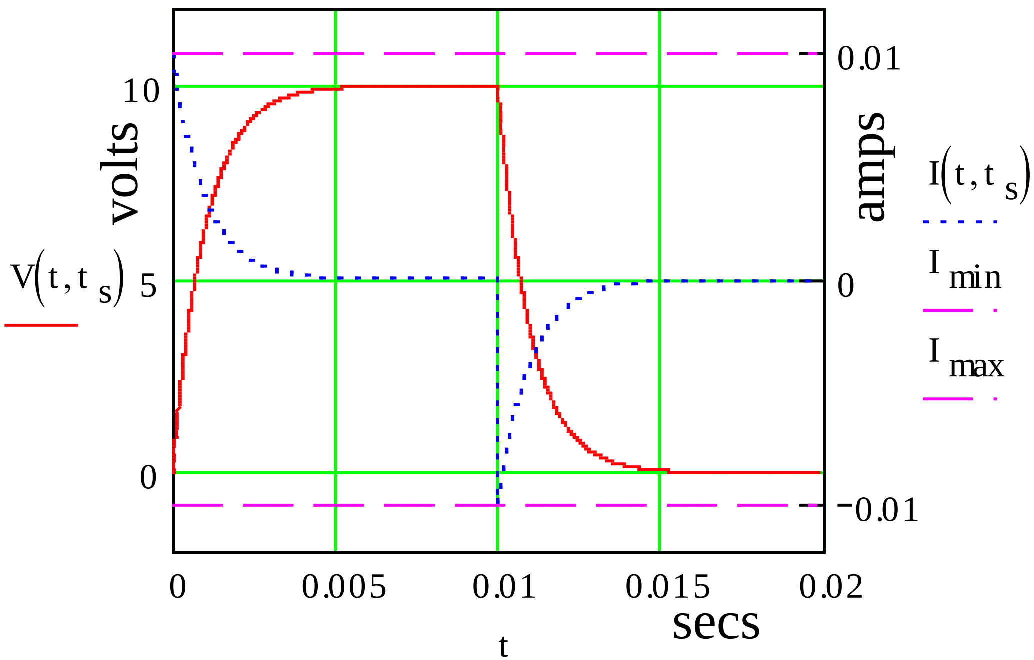
$$= V_0 \exp(-t/RC) \qquad [11.5]$$

$$i = \frac{dq}{dt} = -\frac{q_0}{RC}\exp(-t/RC)$$

$$= i_0 \exp(-t/RC) \qquad [11.6]$$

Circuits with Resistance and Capacitance

- Look at example:
  - R = 1 kΩ, C = 1 μF so τ = 1 ms.
  - $\mathcal{E}$ = 10 V.
  - Charge for 10 ms, then discharge:



- If flip switch before charge "complete" (here at t = 2τ):