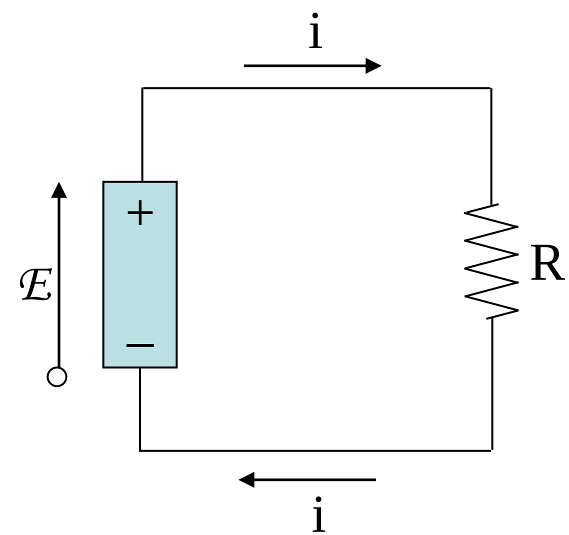
Lecture 10

- In this lecture we will look at:
  - Electromotive force.
  - Calculating the current in a circuit using the energy and potential methods.
  - Internal resistance.
  - Resistances in parallel.
  - Resistances in multi –loop circuits.

- After this lecture, you should be able to answer the following questions:
- State Kirchoff's voltage and current rules: what alternative names are used for these rules?
- What is the current in a circuit in which resistors of 3 Ω and 5 Ω are connected in parallel with an emf of 9 V which has an internal resistance of 0.5 Ω?
- Describe how the loop and junction rules can be used to determine the current in a circuit consisting of a network of resistors and emfs.

Electromotive Force

- In order to make charge carriers flow round a circuit, an "electromotive force" or emf is needed.
- The job of a device providing an emf is to do work on the charges, pushing them through the resistance of the circuit.
- Common emf devices are:
  - Batteries.
  - Solar cells.
  - Generators.
- All of these convert energy (chemical, light, mechanical) into electrical energy.
- Consider circuit with emf $\mathcal{E}$:

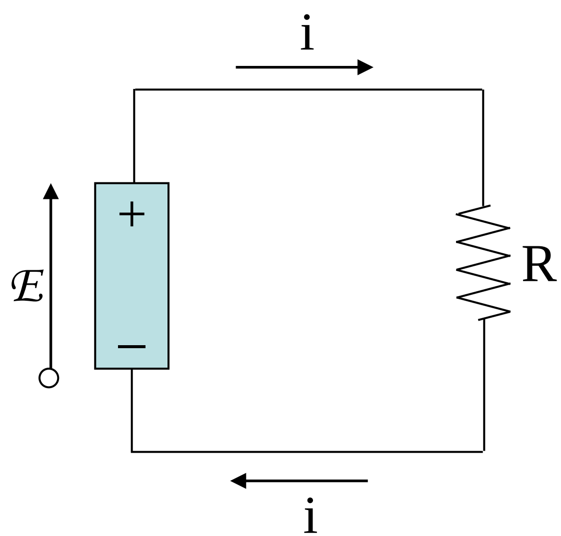


- Within the emf, +ive charges move from low potential to high potential against the electric field.
- The emf does an amount of work dW in moving a charge dq, define $\mathcal{E}$ as:

$$\mathcal{E} = \frac{dW}{dq} \text{ (units J C}^{-1} = \text{V).}$$

Calculating the Current in a Circuit – Energy Method

- Consider a simple loop circuit containing an ideal emf and a resistor, connected with wires of negligible resistance:

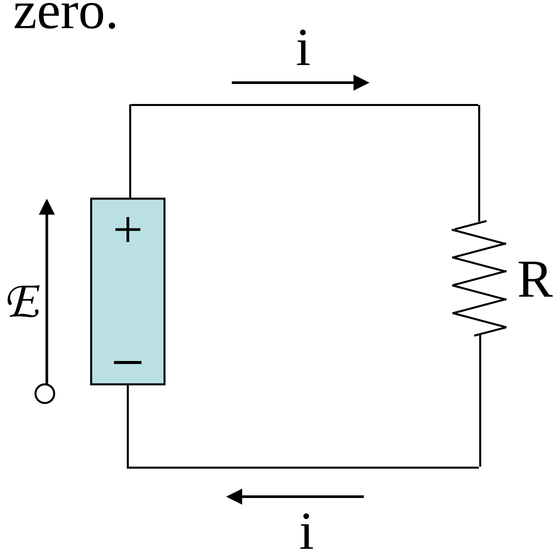


- We know $P = i^2R$, so in an interval dt, an amount of energy $i^2R\,dt$ will appear in the resistor.
- During this time, a charge $dq = i\,dt$ will have moved through the emf.
- The emf will have done an amount of work $dW = \mathcal{E}\,dq = \mathcal{E}\,i\,dt$ on the charge.
- The work done by the emf must equal the energy appearing in the resistor.
- Hence $\mathcal{E} = iR$.
- Solving for the current:

$$i = \frac{\mathcal{E}}{R} \qquad [10.1]$$

Current – Potential Method or Kirchoff's Voltage Rule

- Start at any point in the circuit and work round it, adding all the potential differences you come across.
- Kirchoff's voltage rule (or loop rule): The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.



- Start at –ive terminal of emf, which we assume has a potential V_.
- Move to +ive terminal, potential difference is $\mathcal{E}$.
- Move along wires, no potential change as these have negligible resistance.
- Move through the resistor, potential difference is –iR.
- Move along wires, no potential change as these have negligible resistance.
- Arrive back at point with potential V_.
- From Kirchoff's law:

$$V_- + \mathcal{E} - iR = V_- \Rightarrow \mathcal{E} - iR = 0$$

$$\Rightarrow i = \frac{\mathcal{E}}{R}$$

Internal Resistance

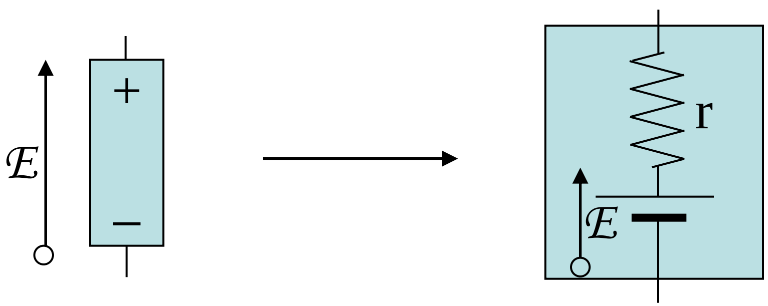
- For our ideal emf: $\mathcal{E} = iR$.
- Any real emf device has an internal resistance r, so

$$\mathcal{E} = iR + ir \qquad [10.2]$$

iR: potential across external R

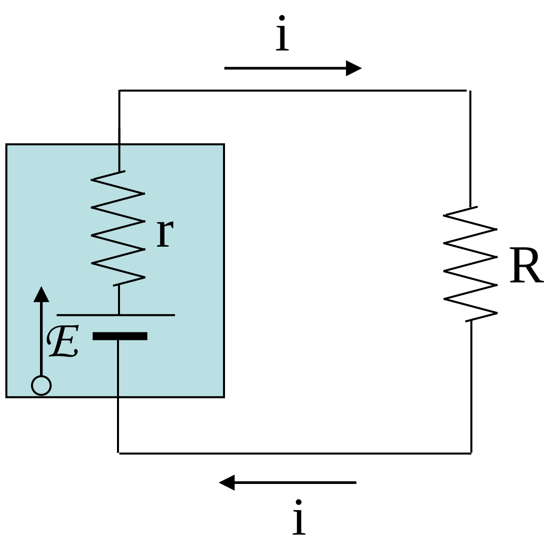
ir: potential across internal r

- An ideal emf will always have a potential across its terminals equal to its nominal value (e.g. 12 V for an ideal 12 V battery).
- A real emf will only have its nominal potential across its terminals when no current is flowing.



Resistance in More Complex Circuits

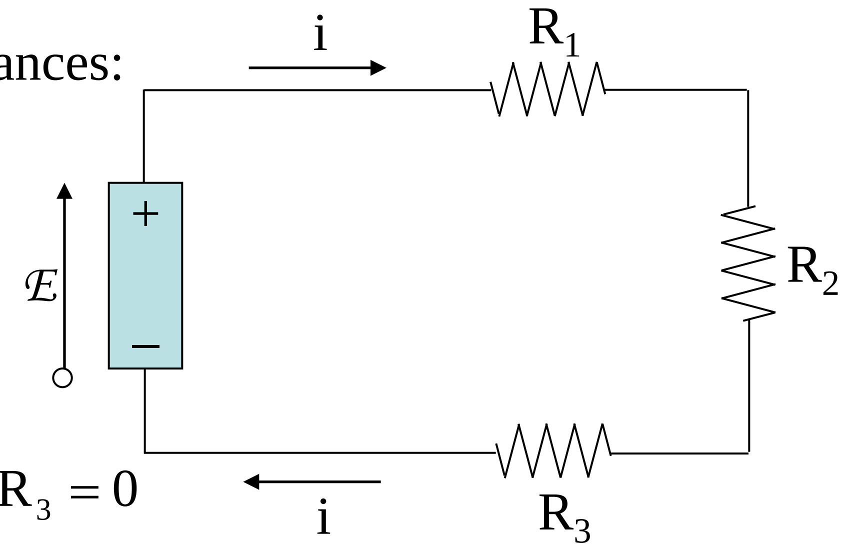
- Add internal resistance:



- Now get:

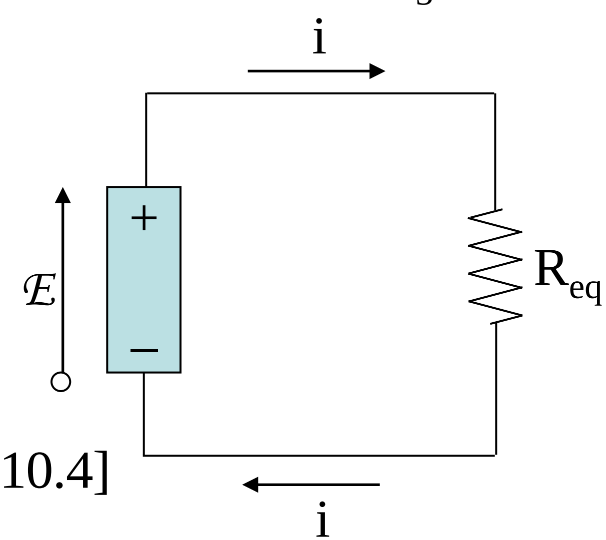
$$V_- + \mathcal{E} - ir - iR = V_-$$
$$\Rightarrow \mathcal{E} - i(r+R) = 0$$
$$\Rightarrow i = \frac{\mathcal{E}}{r+R} \qquad [10.3]$$

- Add other resistances:
- An identical current i flows through each resistance.



- Hence

$$\mathcal{E} - iR_1 - iR_2 - iR_3 = 0$$
$$\Rightarrow i = \frac{\mathcal{E}}{R_1 + R_2 + R_3}.$$

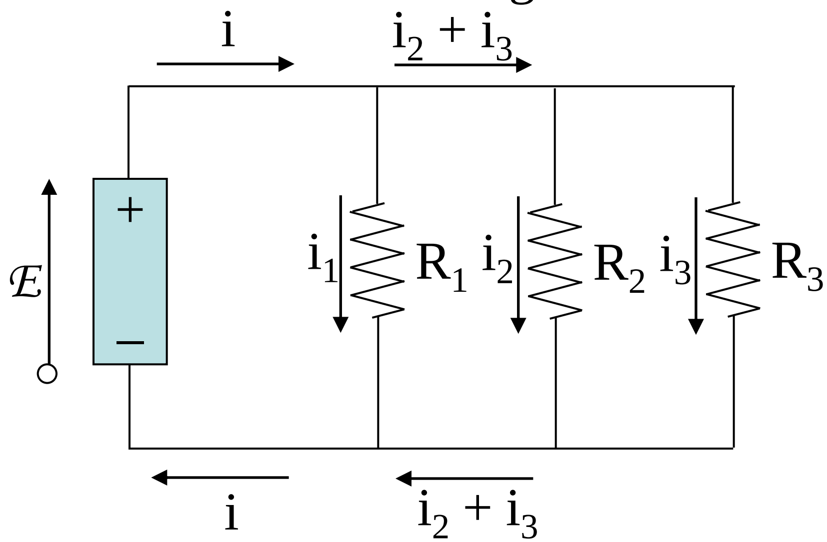


- The combination of resistors can be replaced by an equivalent resistance

$$R_{eq} = R_1 + R_2 + R_3 \qquad [10.4]$$

Resistors in Parallel – Kirchoff's Current Rule

- Consider the following circuit:



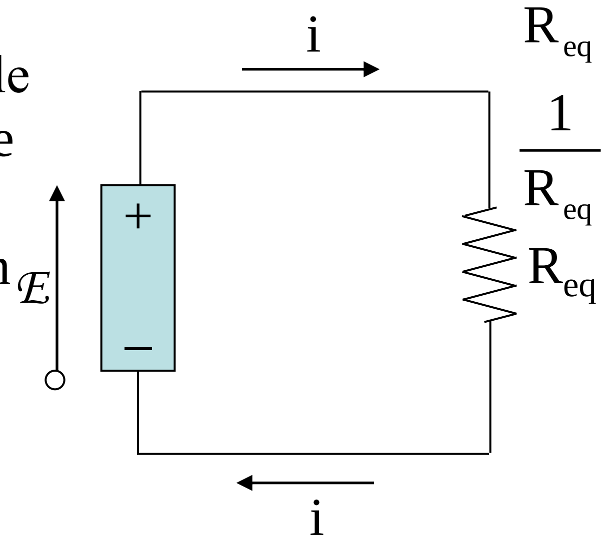
- We can see Kirchoff's current rule (or junction rule) must apply: The sum of the currents entering any junction must be equal to the sum of the currents leaving the junction.
- This tells us: $i = i_1 + i_2 + i_3$.
- We also know that: $i_1 = \mathcal{E}/R_1$, $i_2 = \mathcal{E}/R_2$ and $i_3 = \mathcal{E}/R_3$.
- Substituting these in the above:

$$i = i_1 + i_2 + i_3 = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2} + \frac{\mathcal{E}}{R_3}.$$

- The equivalent resistance for parallel resistors can then be found from:

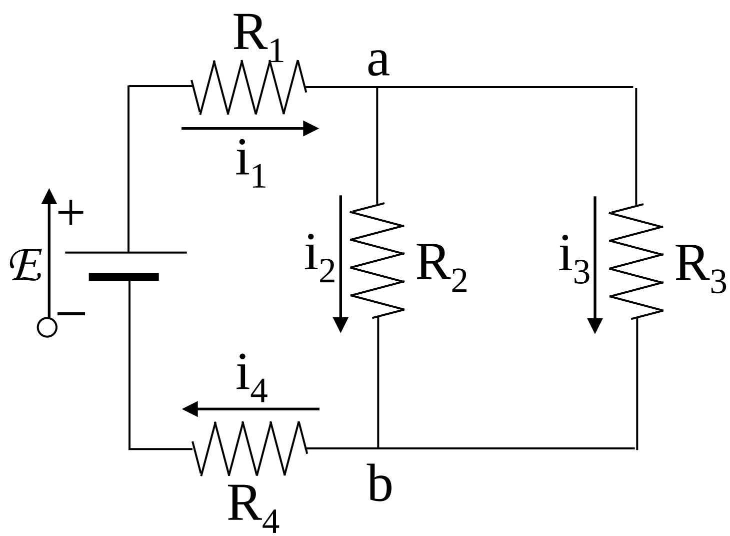
$$\frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2} + \frac{\mathcal{E}}{R_3}, \text{i.e.}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad [10.5]$$



Multi-loop Circuits

- Consider the circuit:



- Use loop rule LH loop:
  $\mathcal{E} = i_1R_1 + i_2R_2 + i_4R_4 \quad (1).$
- Now for RH loop:
  $i_2R_2 = i_3R_3 \quad (2).$
- For outermost loop:
  $\mathcal{E} = i_1R_1 + i_3R_3 + i_4R_4 \quad (3).$
- Now use junction rule at point a:
  $i_1 = i_2 + i_3 \quad (4).$
- And at point b:
  $i_4 = i_2 + i_3 \quad (5).$
- Now have 5 equations for 4 unknowns, solve for i_1, i_2, i_3 and i_4.
- From (2) $i_2 = i_3 R_3/R_2$ or $i_3 = i_2 R_2/R_3$.
- From (4) $i_1 = i_3 \dfrac{R_3}{R_2} + i_3 = i_3\left(\dfrac{R_2 + R_3}{R_2}\right)$.
- From (4) and (5) $i_4 = i_1$,
  but also $i_4 = i_3(R_3/R_2 + 1)$.
- From (1) and above:

$$\mathcal{E} = i_1R_1 + i_3R_2\frac{R_3}{R_2} + i_1R_4.$$

Multi-loop Circuits

- Substituting for i_3 and rearranging:

$$\mathcal{E} = i_1 R_1 + i_1 \frac{R_2}{R_2 + R_3} \frac{R_2 R_3}{R_2} + i_1 R_4$$
$$= i_1 R_1 + i_1 \frac{R_2}{R_2 + R_3} R_3 + i_1 R_4$$
$$= i_1 \left(\frac{R_1(R_2 + R_3) + R_2 R_3 + R_4(R_2 + R_3)}{R_2 + R_3} \right)$$
$$\Rightarrow i_1 = \frac{\mathcal{E}(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3 + R_2 R_4 + R_3 R_4}.$$

- Strategy: use junction rule at every junction to give minimum number of unknown currents.
- Apply loop rule around same number of different loops as there are unknown currents.
- Solve equations for currents.
- For some circuits there may be better (simpler) strategies...
- E.g. here can find equivalent resistance due to R_2 and R_3 as these are in parallel:

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_2 + R_3}{R_2 R_3}.$$

- Total equivalent resistance is then

$$R_{eq} = R_1 + R_{23} + R_4$$
$$= R_1 + \frac{R_2 R_3}{R_2 + R_3} + R_4$$
$$= \frac{R_1(R_2 + R_3) + R_2 R_3 + R_4(R_2 + R_3)}{R_2 + R_3}$$

- Hence total current is

$$i_1 = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3 + R_2 R_4 + R_3 R_4}.$$