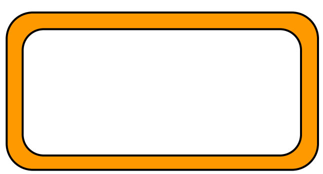
Lecture 9

- In this lecture we will look at:
  - Charges and currents.
  - Current density.
  - The drift speed of charge carriers.
  - Resistance and resistivity.
  - Ohm's Law.
  - Conduction in metals.
  - Power in electric circuits.
  - Semiconductors and superconductors.

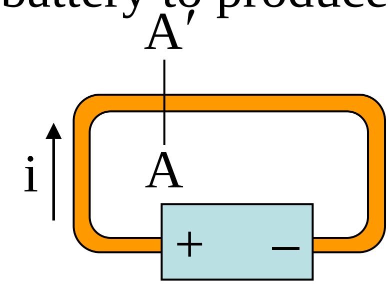
- After this lecture, you should be able to answer the following questions:
- The drift speed of the electrons carrying an electric current in a copper wire is typically only about 10^{-3} ms^{-1}. Why then does a light come on almost instantaneously when the switch is depressed even though it is 5 m away from the bulb?
- What is the difference between resistance and resistivity?
- Give three formulae for the power dissipated in a resistor R across which there is a potential V and through which a current i is flowing.

Charges and Currents

- Move from consideration of electrostatics to study of electric currents and their effects.
- Consider first steady ("direct") currents.
- For current to flow must be a net flow of charge.
- Electrons in an isolated loop of copper wire travel at about 10^6 ms^{-1}, but no net flow of electrons, so no current.



- Add battery to produce current:



- Charges move because electric field, E, established in copper and charges (electrons) feel force, F = Eq.
- Current through plane (e.g. $A - A'$):

$$i = \frac{dq}{dt} \qquad [9.1]$$

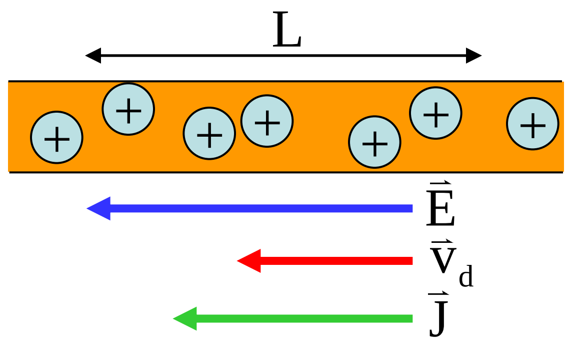
- If i constant, $i = q/t$ [9.2]
- Charge from current through:

$$q = \int dq = \int_0^t i\,dt' \qquad [9.3]$$

- If i constant, $q = i\,t$ [9.4]

Current Density and Drift Speed of Charge Carriers

- Unit of current is ampere ($A = C\,s^{-1}$).
- Current direction taken to be direction in which +ive charge travels.
- Arrows often used to indicate current direction, but current is scalar, not vector quantity.
- Current density $\vec{J}$ is vector quantity with direction given by that of the velocity of the moving +ive charges (or opposite to direction of velocity of –ive charges).
- Magnitude of $\vec{J}$ is current per unit area.
- Units of current density, $A\,m^{-2}$.
- Hence calculate current from current density: $i = \int \vec{J} \cdot d\vec{A}$ [9.5]
- If current is uniform and perpendicular to area A, $J = i/A$ [9.6]
- In metals, current is due to drift of electrons.



- If there are n charge carriers per unit volume, sum of charge of carriers in length L is $q = nALe$.
- This moves through plane in wire in time $t = L/v_d$.
- Hence $i = q/t = nAv_de$.

Drift Speed, Resistance and Resistivity

- Solve for v_d to get:

$$v_d = \frac{i}{nAe} = \frac{J}{ne} \qquad [9.7]$$

- In vector form: $\vec{J} = ne\,\vec{v}_d$.
- Note, v_d typically 10^{-3} m s^{-1}!
- Current through conductor related to potential difference across it through resistance, defined by:

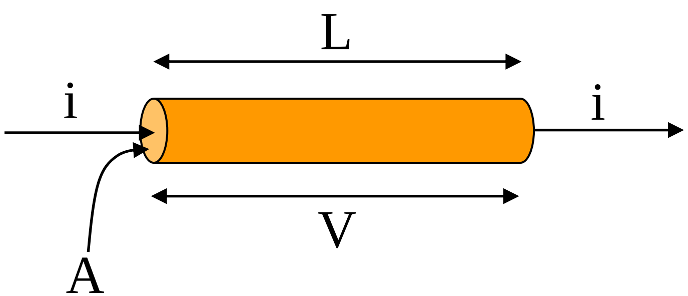
$$R = \frac{V}{i} \qquad [9.8]$$

- Unit ohm ($\Omega = V\,A^{-1}$).
- i = V/R, so increasing R reduces current: "resistance" aptly named!
- Resistance is property of an object (a particular component in a circuit).

- Resistivity, ρ, is property of material.
- Defined by: $\rho = \frac{E}{J}$ [9.9]
- Units $\frac{V/m}{A/m^2} = \frac{V}{A}\,m = \Omega\,m$.
- In vector form, $\vec{E} = \rho\vec{J}$.
- Eqn.s for ρ only for isotropic materials.
- Conductivity σ is reciprocal of resistivity, conductance G reciprocal of resistance.
- Units of σ are S m^{-1}, of G are S (S = Siemens or mho = Ω^{-1}).
- From above, $\vec{J} = \sigma\vec{E}$ and $G = \frac{i}{V}$

Calculating Resistance from Resistivity

- Calculate the resistance of a length L of wire of cross-sectional area A and resistivity ρ.

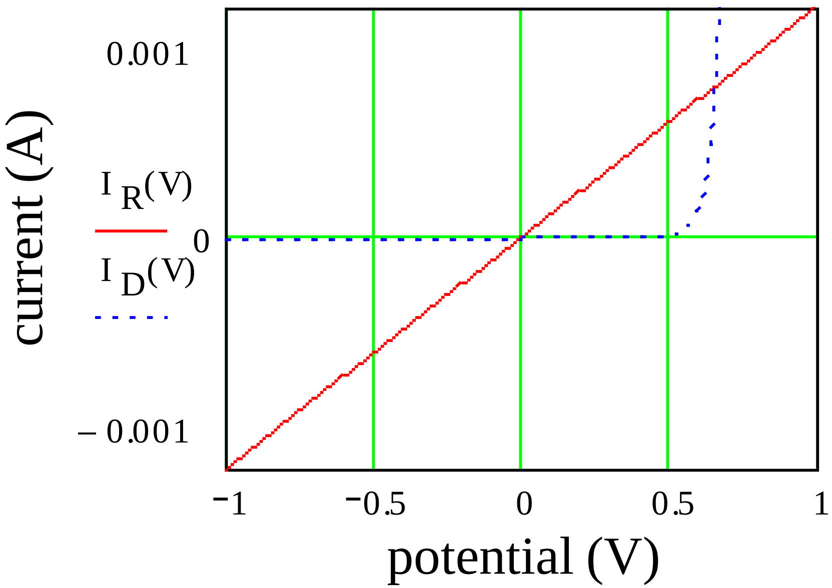


- Assume E field and current density uniform throughout wire.
- Hence E = V/L and J = i/A.
- $\rho = \frac{E}{J} = \frac{V/L}{i/A} = R\frac{A}{L}.$
- So we have $R = \frac{\rho L}{A}$ [9.10]

- Resistivity varies with temperature, T.
- For metals, the variation is fairly linear over a broad range of T.
- Choose a reference temp. T_0 at which the resistivity is ρ_0.
- (Usually T_0 = 293 K, ~ room temp.)
- Can then write:
  $\rho - \rho_0 = \alpha\rho_0(T - T_0)$ [9.11]
- The quantity α is the temperature coefficient of resistance.
- For copper:
  - $\rho_0 = 1.69 \times 10^{-8}\ \Omega\,m$.
  - $\alpha = 4.3 \times 10^{-3}\ K^{-1}$.

Ohm's Law

- Contrast the behaviour of the current through two devices illustrated below:



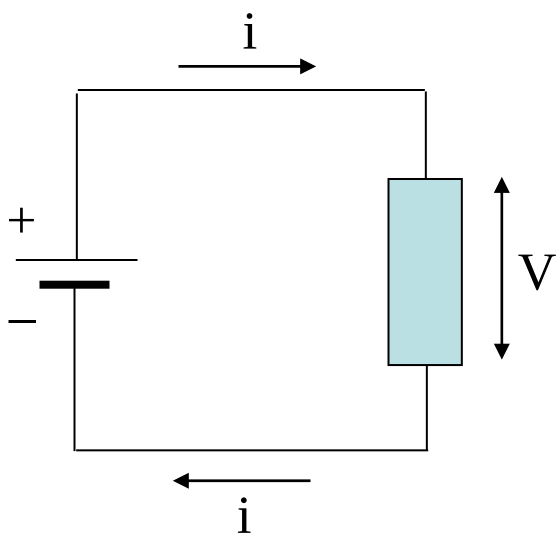
- For device "R", the current is proportional to the voltage.
- For device "D", the current is small until V > 0.6 V at which point it increases sharply.
- "R" obeys Ohm's law, "D" does not.
- Ohm's law states: The current through a device is directly proportional to the potential difference applied to the device.
- This holds for a resistance, as i = V/R with R the same for all V.
- (Note that V = iR holds even for devices that do not obey Ohm's law, where R is the resistance at that potential: devices obey Ohm's law when the same value of R holds for all potential differences!)

Conduction in Metals and Ohm's Law

- Charge transport in metals occurs as (some) electrons are free to move.
- These have speeds of about 10^6 ms^{-1} and "bounce around" inside the metal, colliding with atoms on average every τ seconds.
- If an electric field is applied, the electrons experience a force F = eE and hence an acceleration
$$a = \frac{F}{m_e} = \frac{eE}{m_e}.$$
- Between collisions they acquire a drift speed due to the E field of
$$v_d = \frac{eE}{m_e}\tau.$$
- Hence $\vec{J} = ne\vec{v}_d = ne\frac{e\vec{E}}{m_e}\tau = \frac{ne^2\vec{E}}{m_e}\tau.$
- But $\vec{J} = \frac{\vec{E}}{\rho}$ so $\rho = \frac{m_e}{ne^2\tau}.$
- As $v_d << 10^6$ ms^{-1}, τ is approximately independent of E and therefore so is ρ.
- Hence metals obey Ohm's law.

Power in Electric Circuits

- Consider a device connected to a battery:



- A charge dq passes through the device in a time dt with dq = i dt.
- In passing through the device, the charge moves through a potential difference of V.
- Hence, the potential energy decreases by dU = V dq = i dt V.
- The power dissipated in the device (rate of energy transfer) is thus

$$P = \frac{dU}{dt} = iV \qquad [9.12]$$

- Unit of power is the Watt (W = V A).
- Combining with the expressions V = i R and i = V / R we get:

$$P = i^2 R \qquad [9.13]$$

$$\text{and } P = \frac{V^2}{R} \qquad [9.14]$$

Superconductors and Semiconductors

- Superconductivity is observed in some materials, e.g. mercury.
- For these materials, below a certain critical temperature (about 4 K for mercury), the resistance drops to zero.

- Semiconductors, like silicon, have a resistivity that is between that of metals and insulators.
- In metals, some of the "outer" electrons are only loosely bound and can be caused to move through the metal by the application of an E field.
- In insulators all the electrons are tightly bound.

- In semiconductors, some electrons can be freed by thermal energy.
- Hence the resistivity of a semi-conductor is a strong function of its temperature.

Property	Copper	Silicon
Type of material	Metal	Semiconductor
Charge carrier density n (m^{-3})	9×10^{28}	9×10^{16}
Resistivity ρ (Ωm)	2×10^{-8}	3×10^{3}
Temp. Coeff. of resistance α (K^{-1})	$+2 \times 10^{-3}$	-70×10^{-3}