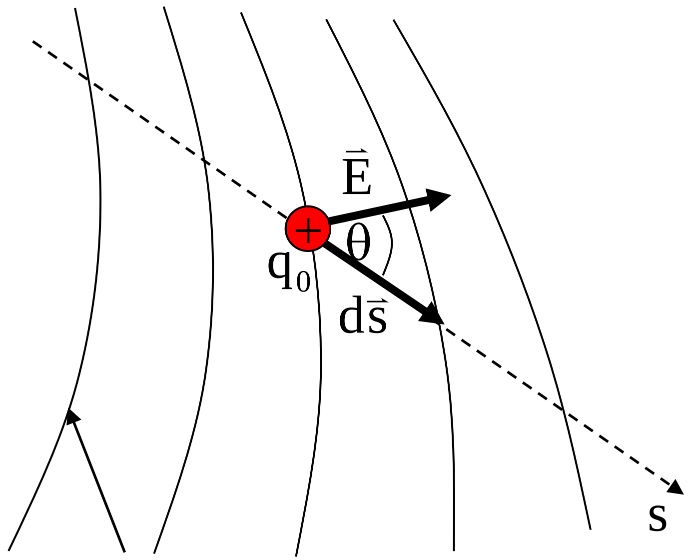
Lecture 6

- This lecture, we will look at:
  - Calculating the electric field from the electric potential.
  - The potential of a charged isolated conductor.

- After this lecture, you should be able to answer the following questions:
- Determine the electric field due to an isolated charge from the expression for the electric potential.
- Describe the electric field and potential inside and outside a charged isolated conductor.
- Explain why you could survive a lighting strike if you were inside a car.

Calculating $\vec{E}$ from V

- We know how to find the potential from the electric field, now look at getting the field from the potential.



- Charge q_0 moves from one equipotential to next, step $d\vec{s}$ along s axis.

- Work done by field is related to change in potential energy...
  $$dW = -dU = -q_0 dV.$$
- ...but also to force and distance:
  $$dW = q_0 \vec{E} \cdot d\vec{s} = q_0 E \cos\theta \, ds.$$
- Equating these two expressions:
  $$-q_0 dV = q_0 E \cos\theta \, ds.$$
- Hence $E\cos\theta = -\dfrac{dV}{ds}$
- But $E\cos\theta$ is the component of $\vec{E}$ along the s axis, so we can write:
  $$E_s = -\frac{\partial V}{\partial s}.$$

An aside – Partial Derivatives

- Consider a function of two variables, f(x,y).
- The partial derivatives of this function w.r.t. x and y are defined by:

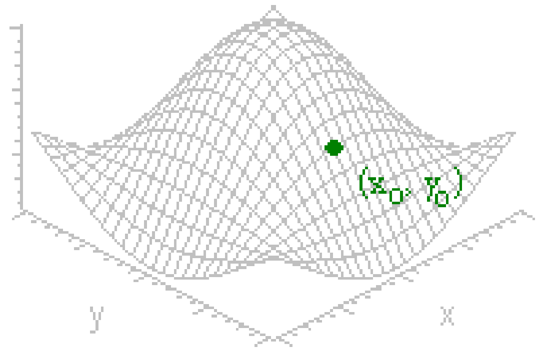
$$\frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

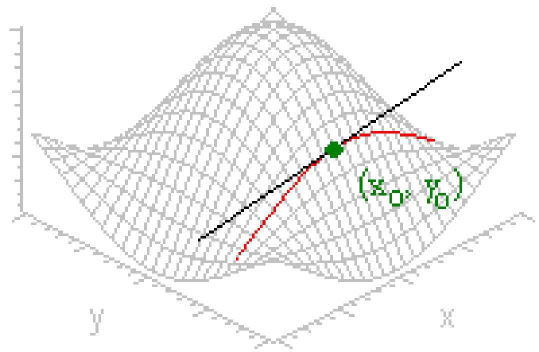
$$\frac{\partial f}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$$

- Example: $f(x,y) = xy^2$.

$$\frac{\partial f}{\partial x} = y^2, \ \frac{\partial f}{\partial y} = 2xy.$$

- Geometrically, consider z = f(x,y) shown opposite:





- Keep $y = y_0$, then $z = f(x,y_0)$ traces out the red curve shown.
- The slope of this curve at (x_0,y_0) is given by $\frac{\partial z(x_0, y_0)}{\partial x}$ or $\left.\frac{\partial z}{\partial x}\right|_{x_0, y_0}$.

Calculating $\vec{E}$ from V

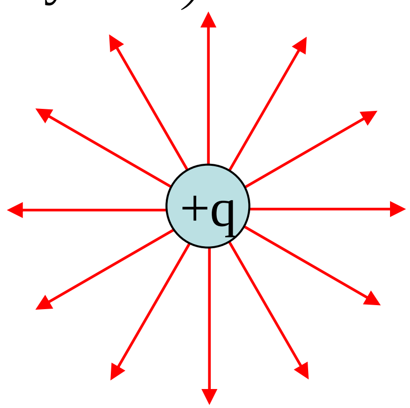
- Taking s to be the x, y and z axes in turn, we have:

$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y} \text{ and } E_z = -\frac{\partial V}{\partial z}.$$

- See units of E field also V m^{-1}.
- More succinctly, the electric field is given by the (negative) gradient of the potential:

$$\vec{E} = -\nabla V \equiv -\left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right) \qquad [6.1]$$

- Consider example of point charge again.



- We have $V = \frac{-1}{4\pi\varepsilon_0}\frac{q}{r}$

$$= \frac{-1}{4\pi\varepsilon_0}\frac{q}{\sqrt{x^2+y^2+z^2}}.$$

- Calculate E field using our prescription:

$$\frac{\partial V}{\partial x} = \frac{q}{4\pi\varepsilon_0}\frac{\partial}{\partial x}(x^2+y^2+z^2)^{-1/2}$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{-1}{2}(x^2+y^2+z^2)^{-3/2}\times 2x$$

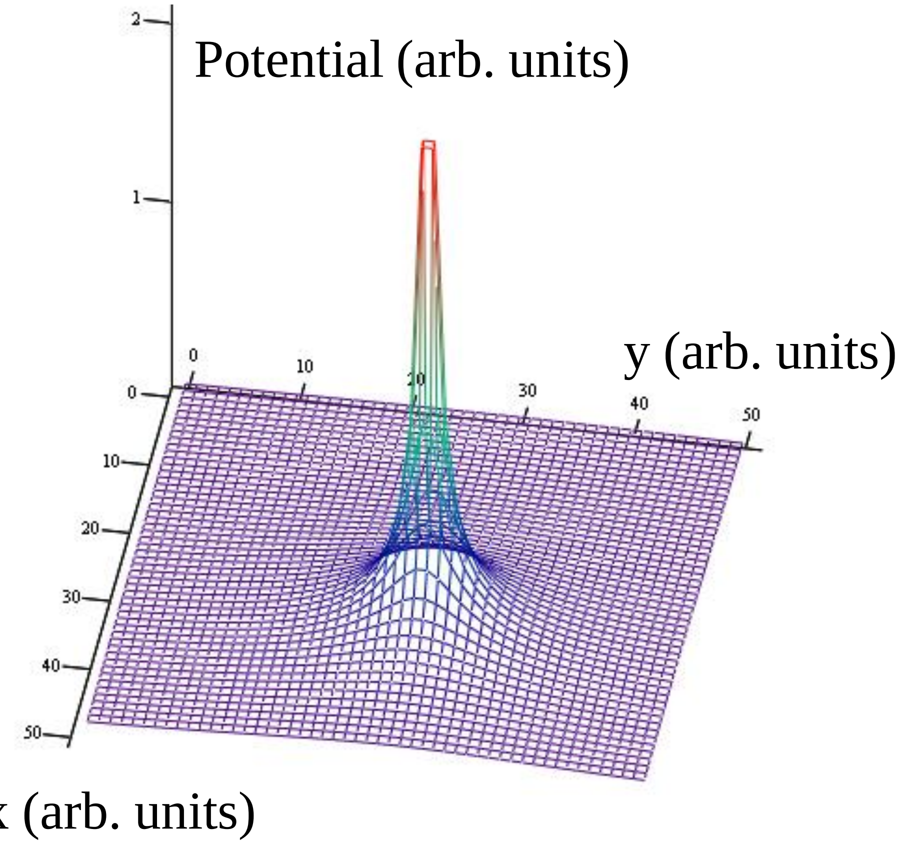
$$= \frac{-1}{4\pi\varepsilon_0}\frac{q}{x^2+y^2+z^2}\frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$= \frac{-1}{4\pi\varepsilon_0}\frac{q}{r^2}\frac{x}{r}.$$

Calculating $\vec{E}$ from V

- Doing the same for y and z we have:
$$\frac{\partial V}{\partial y} = \frac{-1}{4\pi\varepsilon_0}\frac{q}{r^2}\frac{y}{r} \text{ and } \frac{\partial V}{\partial z} = \frac{-1}{4\pi\varepsilon_0}\frac{q}{r^2}\frac{z}{r}.$$
- Hence:
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\left(\frac{x}{r},\frac{y}{r},\frac{z}{r}\right).$$
- Now x/r = r cos θ_{xr} is the component of the radius vector in the x direction, y/r that in the y direction and z/r that in the z direction, so we see:
- $$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \text{ and...}$$
- ...the E field is directed radially away from the charge, as expected.

- The gradient vector is in the direction of the maximum variation of the potential.
- Can see this in 2D:



Potential of a Charged Isolated Conductor

- We have shown that $\vec{E} = 0$ inside a conductor (and that the charge sits on the outer surface of the conductor).
- Using

$$\vec{E} = -\nabla V \equiv -\left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right),$$

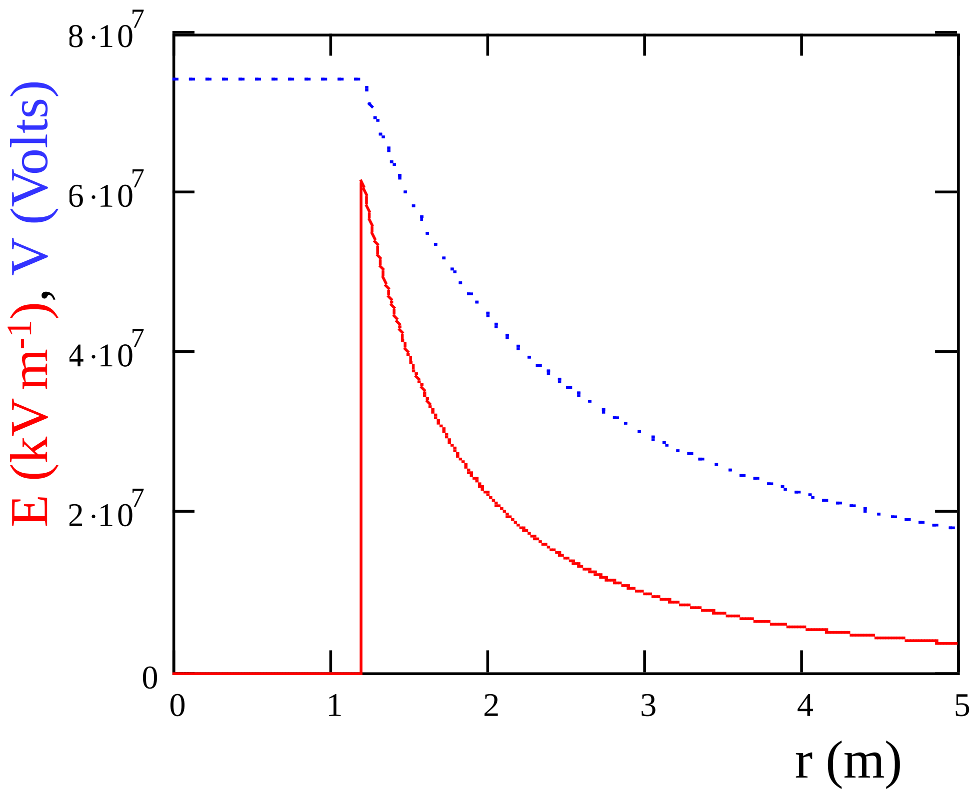
we see:

$$\left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right) = (0,0,0).$$

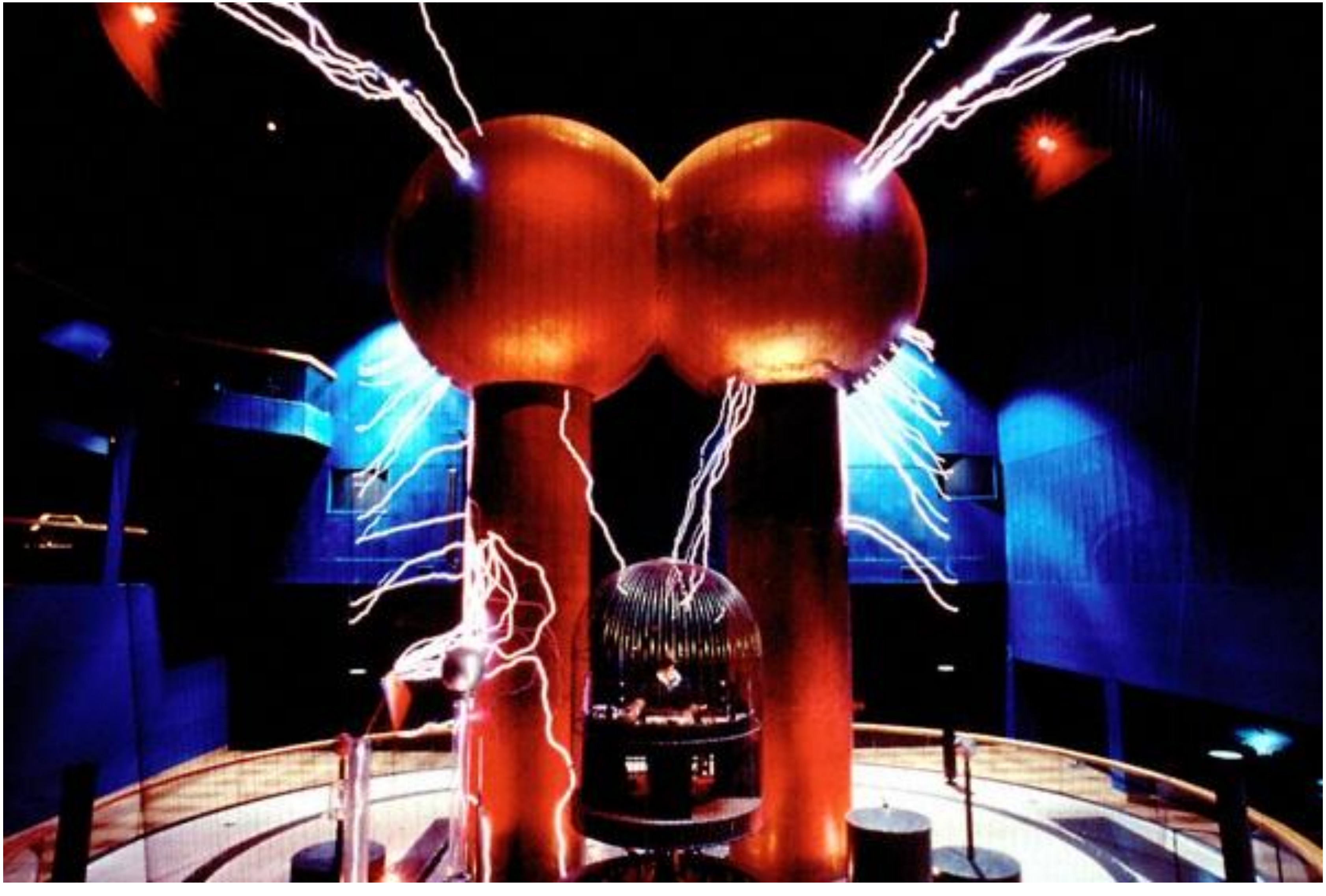
- Hence V = const. (Remember that

$$\frac{\partial}{\partial x}\text{const.} = 0,\ \frac{\partial}{\partial y}\text{const.} = 0,\ \text{etc.})$$

- Example, E field and potential inside and outside a conducting sphere, radius 1.2 m, carrying charge 10 mC:



Field and Potential in Conductor: Faraday Cage



Corona Discharge