Lecture 3

- This lecture, we will look at:
  - Torque on dipole in electric field: the vector product
  - Potential energy of dipole in electric field: the scalar product
  - Electric field due to a continuous charge distribution
  - Flux of an electric field

- After this lecture, you should be able to answer the following questions:
- The dipole moment of a water molecule is 6.2×10^{-30} Cm. What is the maximum torque on a water molecule in an electric field of strength 1000 NC^{-1}?
- Derive the electric field on the symmetry axis of a circular ring of radius R carrying a uniformly distributed charge of magnitude q.
- What is "electric flux" and in what units is it measured?

Torque on Dipole in Electric Field

- Can represent torque using vectors.
- Torque has magnitude $\tau = pE\sin\theta$ and direction given by "right-hand screw" rule: torque is normal to $\vec{p}$ and $\vec{E}$ with sense of screw turned from direction of $\vec{p}$ to that of $\vec{E}$:

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad [3.1]$$

- In terms of Cartesian coordinates:

$$\vec{p} \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ p_x & p_y & p_z \\ E_x & E_y & E_z \end{vmatrix}$$

$$= (p_y E_z - p_z E_y)\hat{x} - (p_x E_z - p_z E_x)\hat{y} + (p_x E_y - p_y E_x)\hat{z}$$

Potential Energy of Dipole in Electric Field

- Can associate potential energy with dipole in (uniform) E field.
- Choose zero of potential energy to be when $\vec{p}$ is normal to $\vec{E}$.

$$\Delta U = -W = -\int_{\theta}^{\pi/2} \tau \, d\theta'$$

$$= -\int_{\theta}^{\pi/2} pE\sin\theta' \, d\theta'$$

$$= pE\cos\pi/2 - pE\cos\theta = -pE\cos\theta.$$

- Can generalise this to vector equation

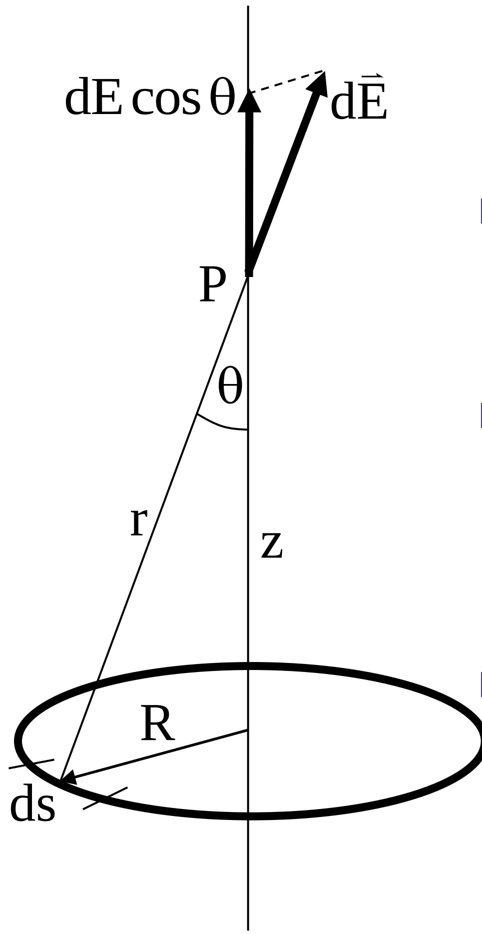
$$U = -\vec{p} \cdot \vec{E} \qquad [3.2]$$

- In Cartesian coordinates:

$$U = -(p_x E_x + p_y E_y + p_z E_z).$$

Electric Field due to Continuous Charge Distribution

- Continuous charge distribution can be line, ring, sheet, volume...
- Divide distribution into elemental charges dq.
- Calculate field $d\vec{E}$ due to dq using symmetry to simplify the problem where possible.
- Integrate over charge distribution.
- Consider example – field due to a ring of charge, linear charge density λ.



- Element ds, charge dq = λds, produces field $d\vec{E}$ at P:

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,ds}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,ds}{z^2+R^2}.$$

- Horizontal components cancel (symmetry) and...

- $$\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{z^2+R^2}}.$$

so $$dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}\,ds$$

- Total field:

$$E = \int dE\cos\theta$$

$$= \frac{z\lambda}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}\int_0^{2\pi R} ds$$

Electric Field due to Continuous Charge Distribution

- Hence

$$E = \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0 (z^2 + R^2)^{3/2}}.$$

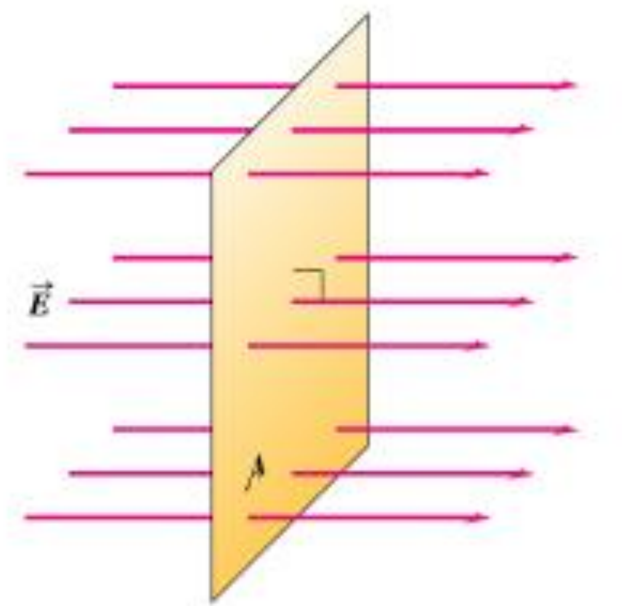
- Now λ is charge per unit length of ring, so $\lambda(2\pi R) = q$, the total charge on the ring.
- We can re-write the expression for the field due to a ring of charge:

$$E = \frac{qz}{4\pi\varepsilon_0 (z^2 + R^2)^{3/2}}.$$

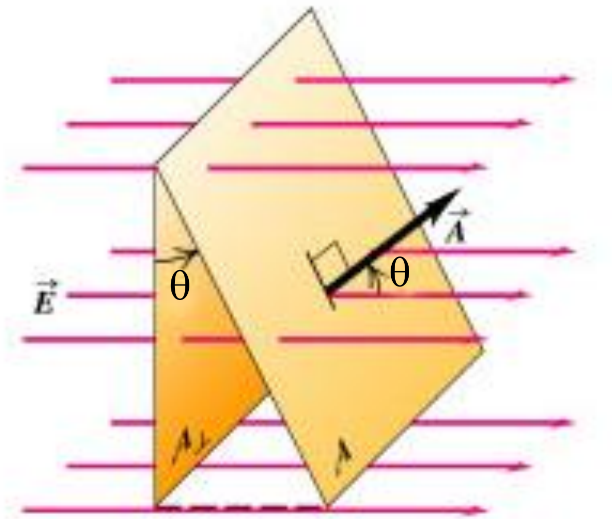
- Similar calculations can be done for other simple charge distributions (continuous sheet or sphere of charge, line of charge...).
- Situations in which there is less symmetry may well have to be solved numerically using a computer.

Flux of an Electric Field

- Can think of electric field lines as "flowing" through space.
- The flux, Φ, of the E field is the number of lines through an area A (recall density of lines is proportional to field strength).



- Must take account of fact that area may not be perpendicular to field.



- Hence, define:
  $\Phi = EA\cos\theta$ [3.3]
  where θ is the angle between $\vec{E}$ and $\vec{A}$.
- In terms of vectors:
  $\Phi = \vec{E}\cdot\vec{A}$ [3.4]

Flux of an Electric Field

- If electric field varies, consider elements of area: $\Delta\Phi = \vec{E} \cdot \Delta\vec{A}.$
- Then get total flux through a surface by adding up all the elements:
  $\Phi = \sum \vec{E} \cdot \Delta\vec{A}.$
- Becomes integral as elements taken to be infinitely small:
  $$\Phi = \int_S \vec{E} \cdot d\vec{A} \qquad [3.5]$$
- For a closed surface:
  $$\Phi = \oint_S \vec{E} \cdot d\vec{A}.$$