Lecture 3: Electrons in semiconductors II

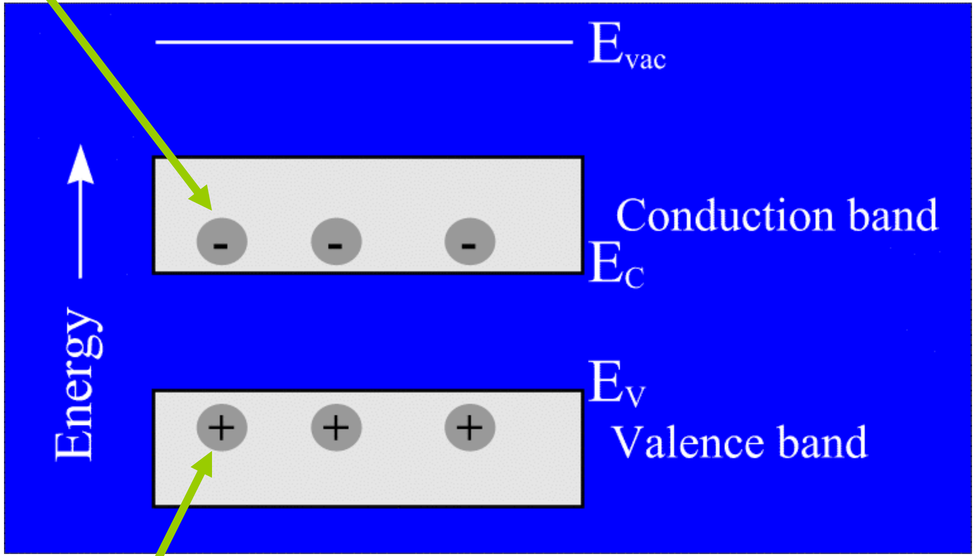
- Mobile charge carriers
  - Density of states
  - Law of mass action
  - Intrinsic Fermi level
  - Intrinsic carrier concentration
- Doping of semiconductors

Intrinsic and Extrinsic Semiconductors

- In **intrinsic semiconductors** the electron density is equal to the hole density (n_i and p_i).
- An intrinsic semiconductor is one that contains a relatively small amount of impurities compared with thermally generated electrons and holes.
- An **extrinsic semiconductor**, conversely, is a material which contains large amounts of impurities.

Mobile charge carriers

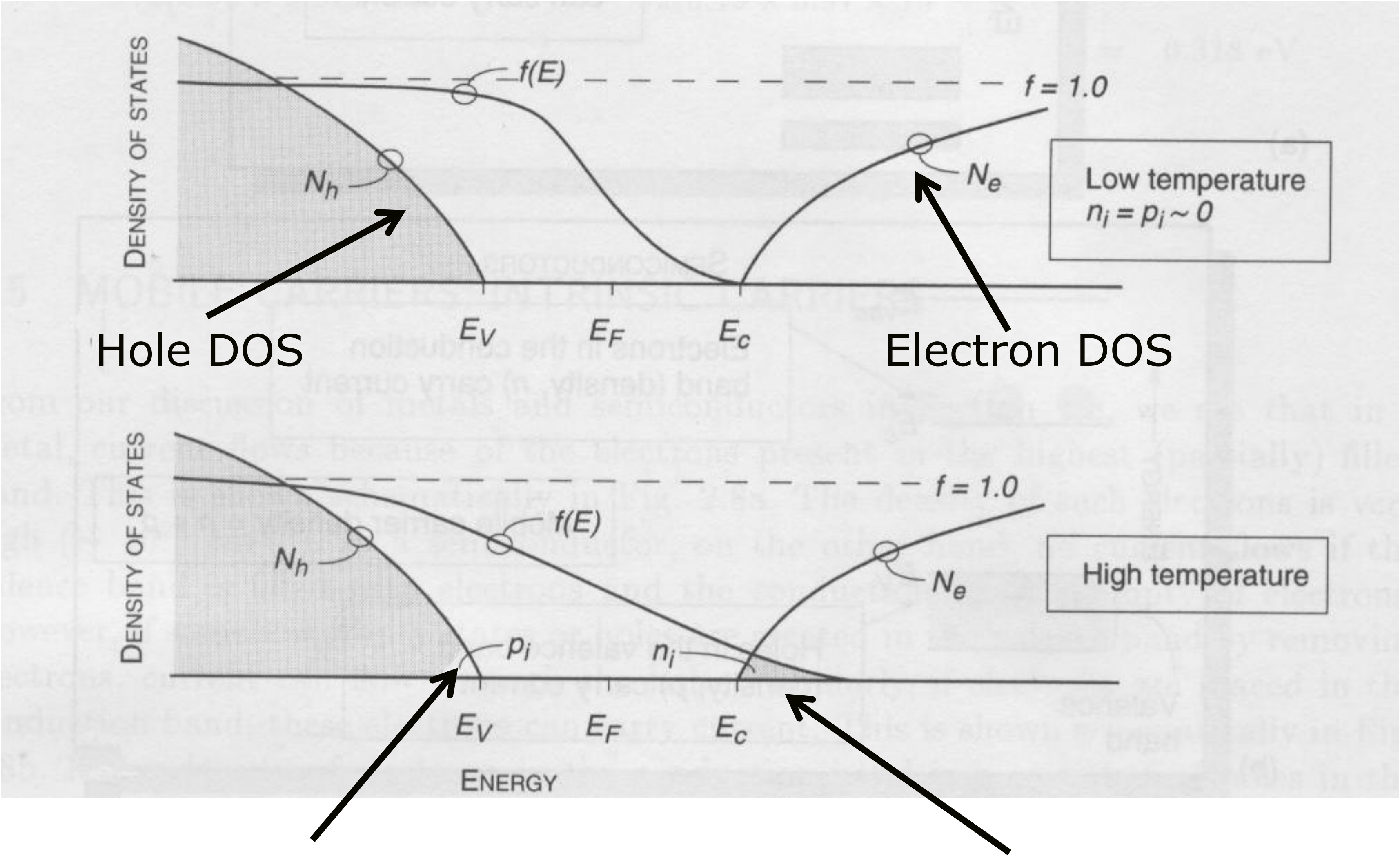
Electrons in the conduction band (density, n) carry current



Holes in the valence band (density,p) carry current

- **Mobile carrier density** = $n + p$

Density of states and Fermi function



Hole DOS

Electron DOS

Hole concentration

Electron concentration

Mobile charge carriers

- The intrinsic carrier concentration refers to the electrons (holes) present in the conduction (valence) band of a pure semiconductor.
- It is dependent on the magnitude of the bandgap and the temperature as well as the effective masses.
- The concentration of electrons in the conduction band is:

$$n = \int_{E_C}^{\infty} N_e(E) f(e) dE$$

- $N_e(E)$ is the electron density near the conduction band edge, $f(E)$ is the Fermi function

Mobile charge carriers

- The concentration of electrons in the conduction band can hence be written:

$$n = \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2} \exp\left(\frac{E_F}{k_BT}\right)\int_{E_C}^{\infty}(E-E_C)^{1/2}\exp(-E/k_BT)\,dE$$

$$= 2\left(\frac{m_e^* k_B T}{2\pi\hbar^2}\right)^{3/2} \exp[(E_F - E_C)/k_BT]$$

$$= N_C \exp[(E_F - E_C)/k_BT]$$

Effective density of states at the conduction bandedge

$$\boxed{N_C = 2\left(\frac{m_e^* k_B T}{2\pi\hbar^2}\right)^{3/2}}$$

Mobile charge carriers

- The carrier concentration is known when E_F is calculated.
- To find the intrinsic carrier concentration required finding the hole concentration p as well.
- Then hole distribution function is:

$$f_h = 1 - f_e$$

- Then:

$$p = 2\left(\frac{m_h^* k_B T}{2\pi\hbar^2}\right)^{3/2} \exp[(E_v - E_F)/k_B T]$$

$$= N_v \exp[(E_v - E_F)/k_B T]$$

The law of mass action

- **In intrinsic semiconductors the electron concentration is equal to the hole concentration**, an electron in the conduction band leaves a hole in the valence band.

$$np = 4\left(\frac{k_B T}{2\pi\hbar^2}\right)^3 (m_e^* m_h^*)^{3/2} \exp(-E_g / k_B T)$$

- The product *np* is independent of the position of the Fermi level and is dependent only on the temperature and intrinsic properties of the semiconductor.
- This is known as the **Law of mass action**
- If *n* increases, *p* must decrease. → $\mathbf{np=n_i^2}$
- For the intrinsic case $n=n_i=p=p_i$, we take the square root of the above equation:

$$n_i = p_i = 2\left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2} (m_e^* m_h^*)^{3/4} \exp(-E_g / 2k_B T)$$

Intrinsic Fermi level

- Setting *n=p* we can obtain the Fermi level position. The intrinsic Fermi level denoted by E_{fi}

$$\exp(2E_{Fi}/k_B T) = (m_h^*/m_e^*)^{3/2}\exp((E_C + E_v)/k_B T)$$

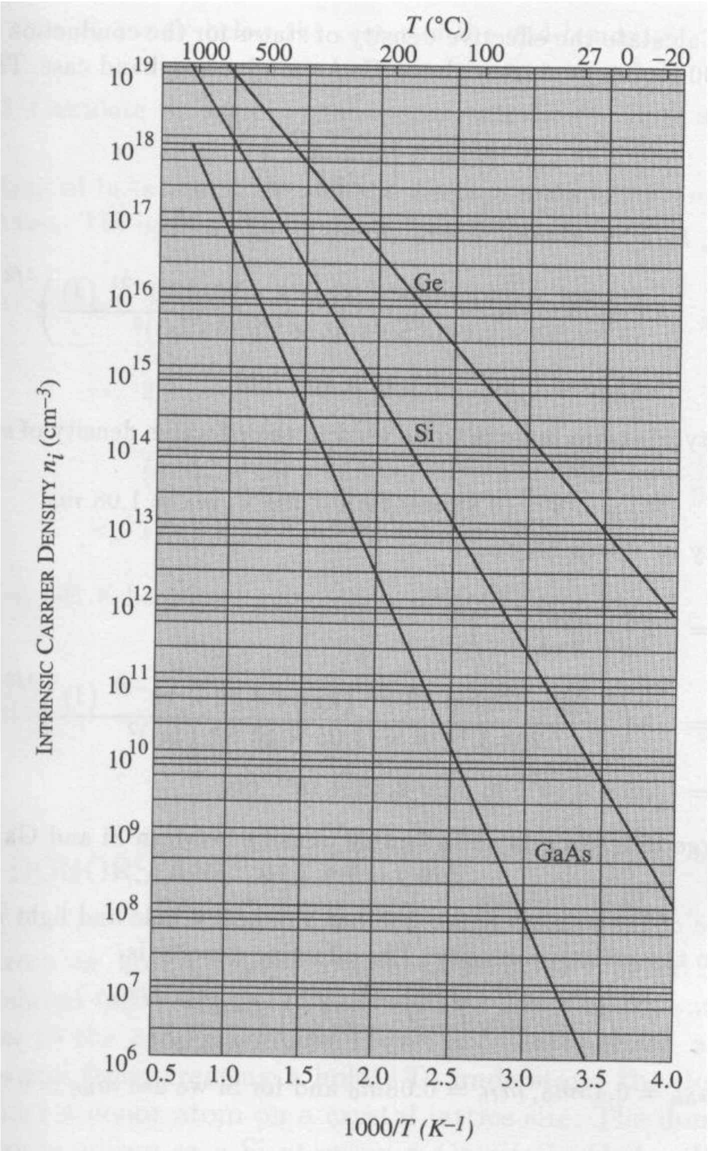
$$E_{Fi} = \frac{E_C + E_v}{2} + \frac{3}{4}k_B T\ln(m_h^*/m_e^*)$$

- The Fermi level of an intrinsic material lies close to the midgap.
- In calculating m_h^* and m_e^* the number of valleys and the sum of heavy and light hole states have to be included.

Material (300K)	Conduction band effective density (N_C)	Valence band effective density (N_v)	Intrinsic carrier concentration $(n_i=p_i)$
Si	$2.78\text{x}10^{19}$	$9.84\text{x}10^{18}$	$1.5\text{x}10^{10}$
Ge	$1.04\text{x}10^{19}$	$6.0\text{x}10^{18}$	$2.33\text{x}10^{13}$
GaAs	$4.45\text{x}10^{17}$	$7.72\text{x}10^{17}$	$1.84\text{x}10^{6}$

Carrier concentration: Remarks

- The carrier concentration increases exponentially as the bandgap decreases.
- Notice the strong dependence on temperature.
- In electronic devices where current has to be modulated, the concentration of intrinsic carriers is fixed by the temperature and this therefore detrimental to device performance.



Intrinsic carrier concentration

- Once the intrinsic carrier concentration increases to $\sim 10^{15}cm^{-3}$, the material becomes unsuitable for electronic devices due to the high leakage current arising from the intrinsic carriers.
- High bandgap semiconductors such as diamond (C), and SiC have attracted growing interest.
- They can be used in high temperature applications.
- Pure semiconductors have a low concentration of mobile carriers:
  - Typically $\sim 10^{11}cm^{-3}$ for intrinsic semiconductors
  - And $\sim 10^{21}cm^{-3}$ for metals.
- The addition of impurities – doping – can be used to change the conductivity of semiconductors.

Example: Intrinsic carriers

- Calculate the position of the intrinsic Fermi level in Si at 300K.
- The density of states effective mass of the combined six valleys of silicon is:

$$m^*_{dos} = (6)^{2/3}(m^*_l m^*_t)^{1/3} = 1.08m_0$$

- The density of states mass for the valence band is $0.55m_0$. Hence the intrinsic Fermi level is given by:

$$E_{Fi} = \frac{E_g}{2} + \frac{3}{4}k_B T \ln\left(\frac{m^*_h}{m^*_e}\right) = \frac{E_g}{2} + \frac{3}{4}(0.026)\ln\left(\frac{0.55}{1.08}\right)$$

$$= \frac{E_g}{2} - (0.0132eV)$$

- The Fermi level is then 13.2meV below the centre of the mid-bandgap.

Doping of semiconductors

- There are two kinds of dopants:
  - Donors, which donate an electron to the conduction band. (n-type)
  - Acceptors, which accept an electron from the valence band – and therefore create a hole. (p-type)
- A donor atom should have one or more electrons in its outer shell than the atom it replaces.
- In silicon (a four valent atom) the addition of a pentavalent atom means the remaining fifth electron now sees a positively charged ion to which it is attracted.
- The attractive potential is simply:

$$U(r) = \frac{-e^2}{4\pi\varepsilon r}$$

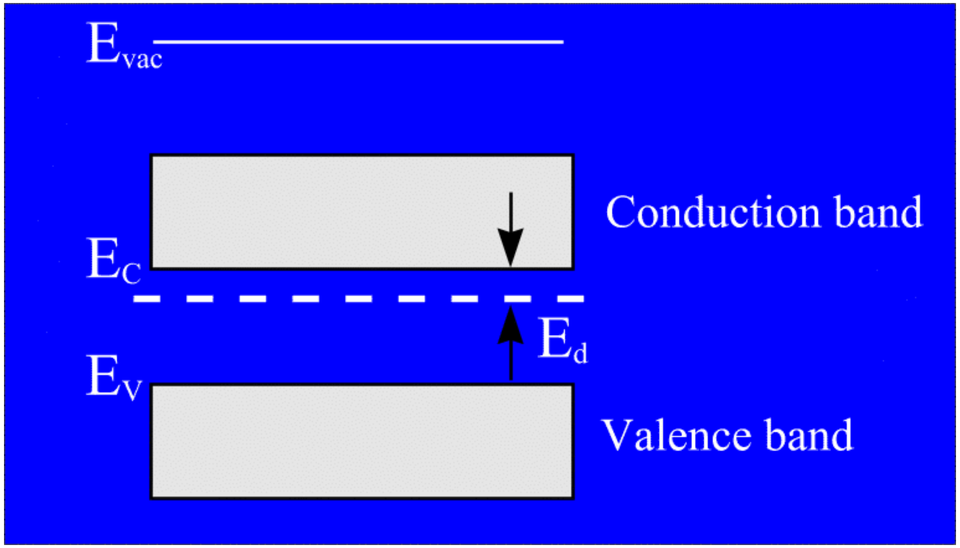
Donor impurities

- The lowest-energy solution for this problem is the **ionisation energy**:

$$E_d = E_C - \frac{e^4 m_e^*}{2(4\pi\varepsilon)^2 \hbar^2}$$

$$= E_C - 13.6\left(\frac{m^*}{m_0}\right)\left(\frac{\varepsilon_0}{\varepsilon}\right)^2 \text{eV}$$

- Notice the effective mass m* is used.
- The energy level is measured from the bandedge.



Limitations of hydrogen formalism

- The simple hydrogen model cannot account for the details of ionisation energy, particularly for the deep impurity levels in semiconductors (ionisation energies $>3kT$).
- Calculated values do predict the correct order of magnitude of the true ionisation energies for shallow impurity levels.
- For shallow impurities in Si and GaAs, there is usually enough thermal energy E_d to ionise all donor impurities, and thus provide an equal number of electrons in the conduction band.
- This condition is called **complete ionisation**.
- Under complete ionisation the electron density (n) equals the donor concentration (N_D).
- Carrier **freezeout** occurs if donor (acceptor) electrons (holes) are tied to there respective lattice sites.

Donor impurities

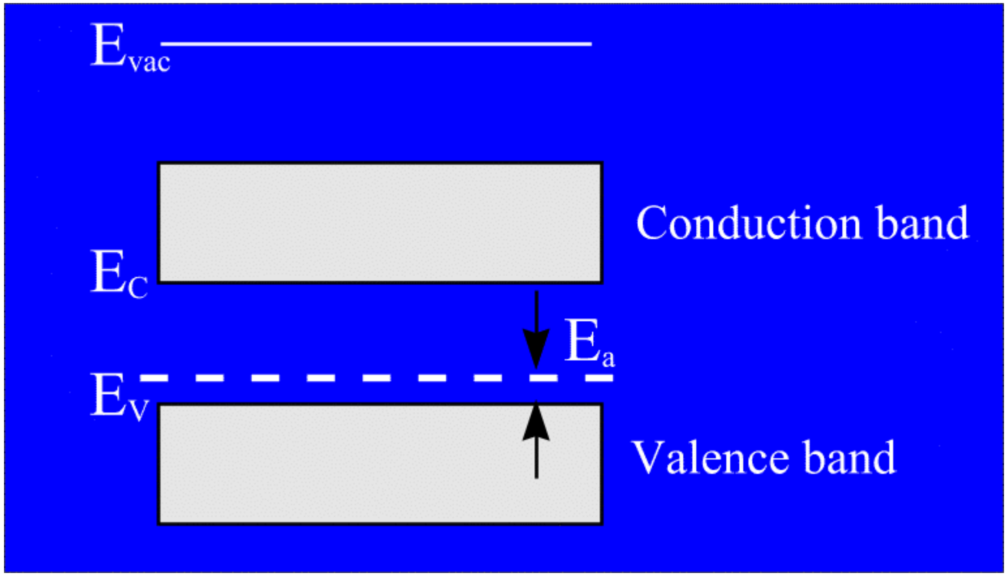
- Another effective mass m_σ*, the conductivity effective mass is introduced to describe how electrons respond to external potentials.
- This mass is used for donor energies as well as for charge transport in an electric field.
- For direct bandgap semiconductors, this is the effective mass.
- For indirect materials such as silicon the conductivity mass is:

$$m_\sigma^* = 3\left(\frac{2}{m_t^*} + \frac{1}{m_l^*}\right)^{-1}$$

- Reminder: The density of states mass represents the properties of the electrons at a constant-energy surface in the band structure.

Acceptor impurities

- The acceptor levels are produces when impurities that have a similar core potential as the atoms in the host lattice, but have one less electron in the outer shell, are introduced.
- Group III elements can act as acceptors in Si or Ge.
- Under complete ionisation the hole density (*p*) equals the acceptor concentration (N_A).



Summary of donors and acceptors

Semiconductor	Impurity (Donor)	Donor Energy (meV)	Impurity (Acceptor)	Acceptor Energy (meV)
GaAs	Si	5.8	C	26
	Ge	6	Be	28
	S	6	Mg	28
	Sn	6	Si	35
Si	Li	33	Be	45
	Sb	39	Al	67
	P	45	Ga	72
	As	54	In	160
Ge	Li	9.3	B	10
	Sb	9.6	Al	10
	P	12	Ga	11
	As	13	In	11

Doped semiconductors are referred to as **extrinsic** semiconductors.

Extrinsic Fermi Level

- We can now write the Fermi level in terms of the effective density of states N_C and the donor concentration N_d:
- Recall: $n = N_C \exp\left[\frac{(E_F - E_C)}{k_B T}\right]$ and $n = N_d$
- Therefore: $E_F - E_C = k_B T \ln\left[\frac{N_d}{N_C}\right]$
- Similarly for shallow acceptors $p=N_A$ and:

$$E_v - E_F = k_B T \ln\left[\frac{N_a}{N_v}\right]$$

- For higher donor concentration, the smaller the energy difference (E_F-E_C), the Fermi level moves closer to the bottom of the conduction band.

Extrinsic Carrier Density

- Considering material that have a dominance of donors (n-type) or acceptors (p-type).
- There is no longer equality between the electrons and holes: $n - p \neq 0$
- The law of mass action still holds – and only changes at high doping levels:

$$np = \text{constant} = n_i^2$$

- As indicated, when the semiconductor is doped n-type (p-type), the Fermi level moves towards the conduction (valence) bandedge.
- When the Fermi level approaches the bandedge, the Boltzmann approximation is not very good and the simple expressions relating the carrier concentration and Fermi level are not very accurate.

Summary of lecture 3

- Mobile charge carriers
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  - Law of mass action
  - Intrinsic Fermi level
  - Intrinsic carrier concentration
- Doping of semiconductors