

# PHYS389: Semiconductor Applications

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- **Semiconductor Physics**

- Lattice structure
- Electrons in semiconductors
- Doping



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- **Semiconductor Applications**

- P-N Junctions
- Field Effect Transistors
- Integrated circuits

- **Applications in Nuclear and Particle Physics**

- Accelerators and Nuclear Reactions
- Nuclear radiation detection
  - Range of charged particles
  - Silicon and Germanium radiation detectors
  - Tracking

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# Semiconductor Research at Liverpool

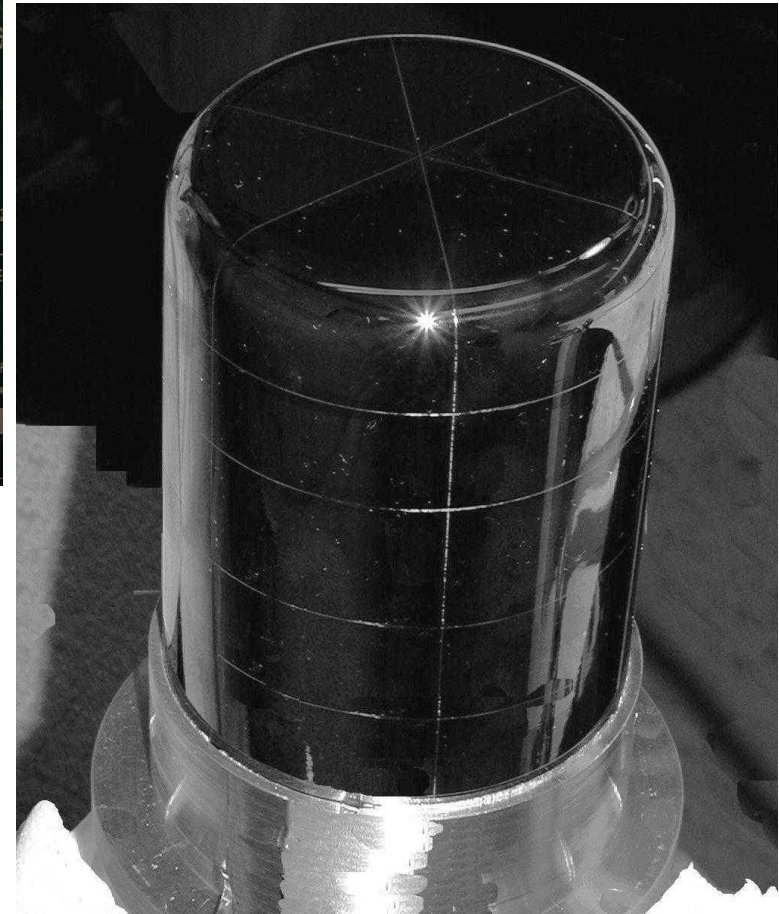
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- LHCb Silicon tracker

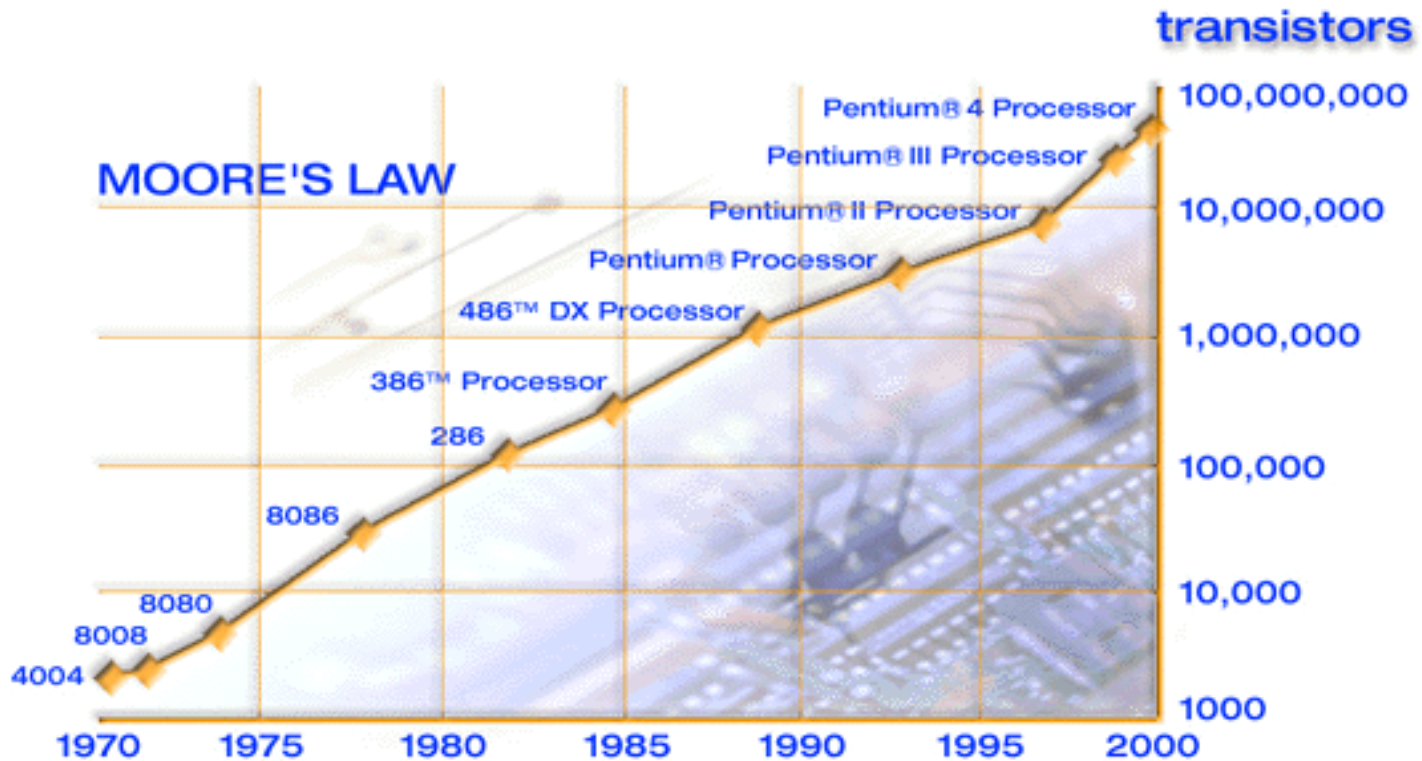


- Germanium imaging detector



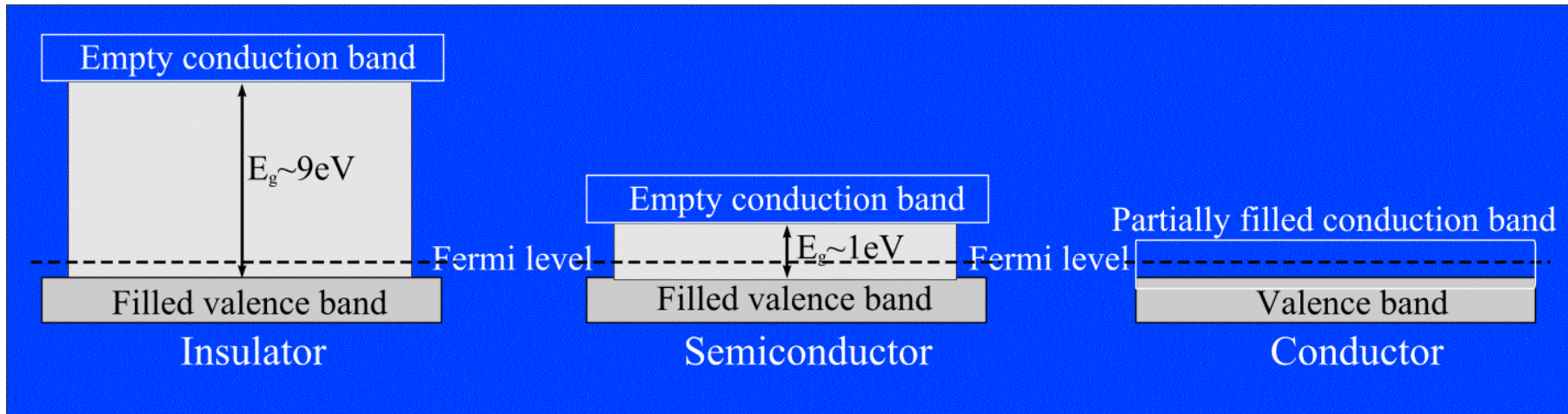
# Lecture 1: What are Semiconductors?

- History
- Why all the fuss?
- Crystal structure
- Energy Bands
- Density of states
- Fermi Level
- The Maxwell-Boltzmann approximation



- “The number of transistors per integrated circuit will double every 18 months”, Gordon Moore, 1965.

# What is a semiconductor?



- When an allowed band is completely filled with electrons, the electrons in the band cannot conduct any current.
- Metals have a high conductivity because of the large number of electrons that can participate in current transport
- Semiconductors have zero conductivity at 0K.

# Semiconductors: Classification

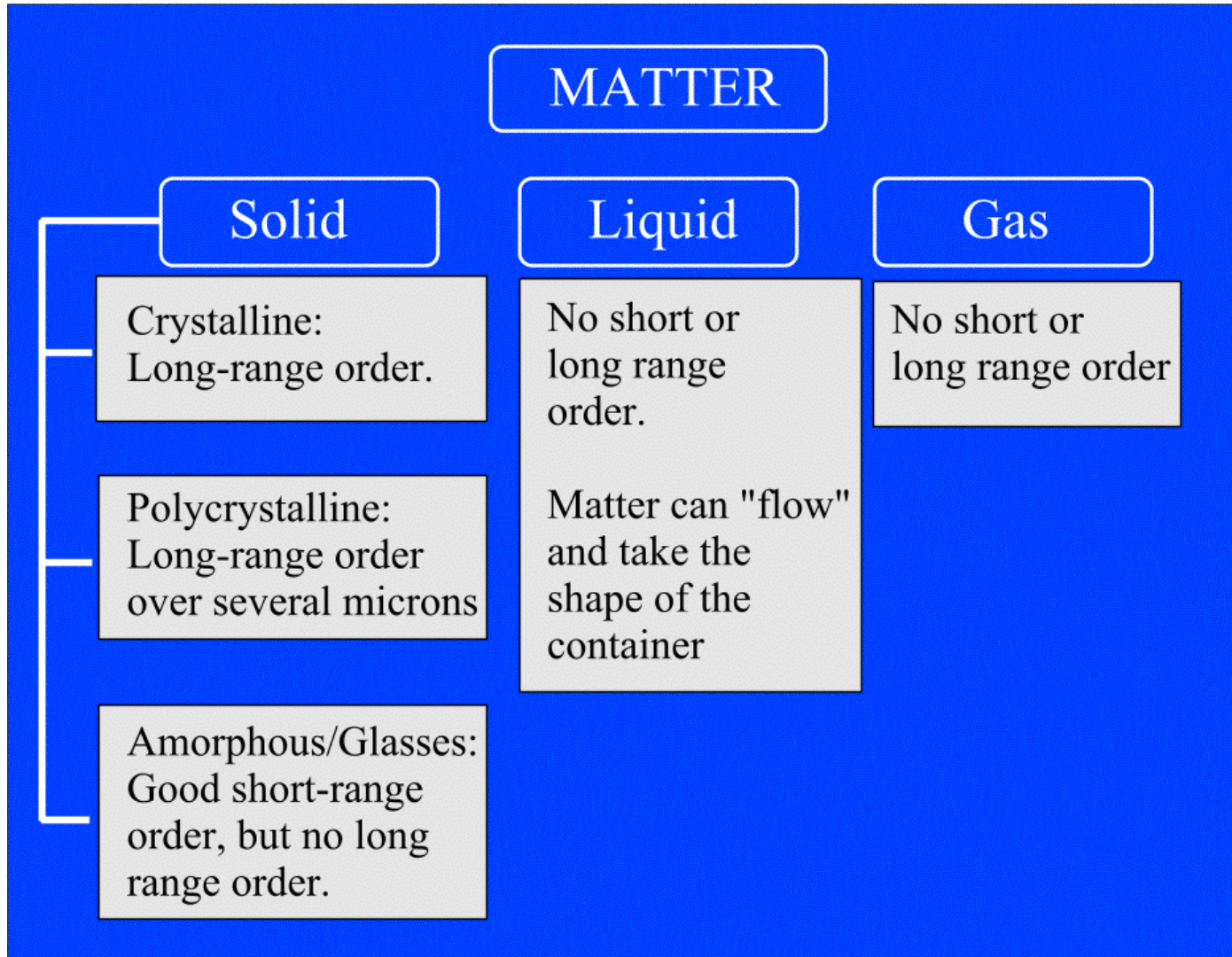
Period	Column II	III	IV	V	VI
2		B	C	N	
		Boron	Carbon	Nitrogen	
3	Mg	Al	Si	P	S
	Magnesium	Aluminium	Silicon	Phosphorus	Sulphur
4	Zn	Ga	Ge	As	Se
	Zinc	Gallium	Germanium	Arsenic	Selenium
5	Cd	In	Sn	Sb	Te
	Cadmium	Indium	Tin	Antimony	Tellurium
6	Hg		Pb		
	Mercury		Lead		

Semiconductors composed of a single species of atoms, such as silicon and germanium are found in column IV of the periodic table. They are often termed **elemental** semiconductors.

**Compound** semiconductors are composed of two or more elements. For example, GaAs, AlSb and InSb are all III-V semiconductors. CdS, CdTe and ZnTe are all II-VI.

- Semiconductors have special properties that allow you to alter their **conductivities** from very low to very high values.
- Charge transport in semiconductors can occur by two different kinds of particles – **electrons** and **holes**.
- Semiconductor devices can be designed that have input-output relations to produce **rectifying** properties; inverters and amplifiers.
- Semiconductor devices can be combined with other elements (resistors, capacitors etc) to produce circuits on which modern information-processing chips are based.

# The different states of matter





- The periodic arrangement of atoms in a crystal is called a **lattice**.
- The lattice by itself is a mathematical abstraction.
- A building block of atoms called the **basis** is then attached to each lattice point, yielding a crystal structure.
- For a given semiconductor there is a basis that is representative of the entire lattice.
- In a crystal an atom never strays far from a single fixed position.
- The thermal vibrations associated with the atom are centred about this position.

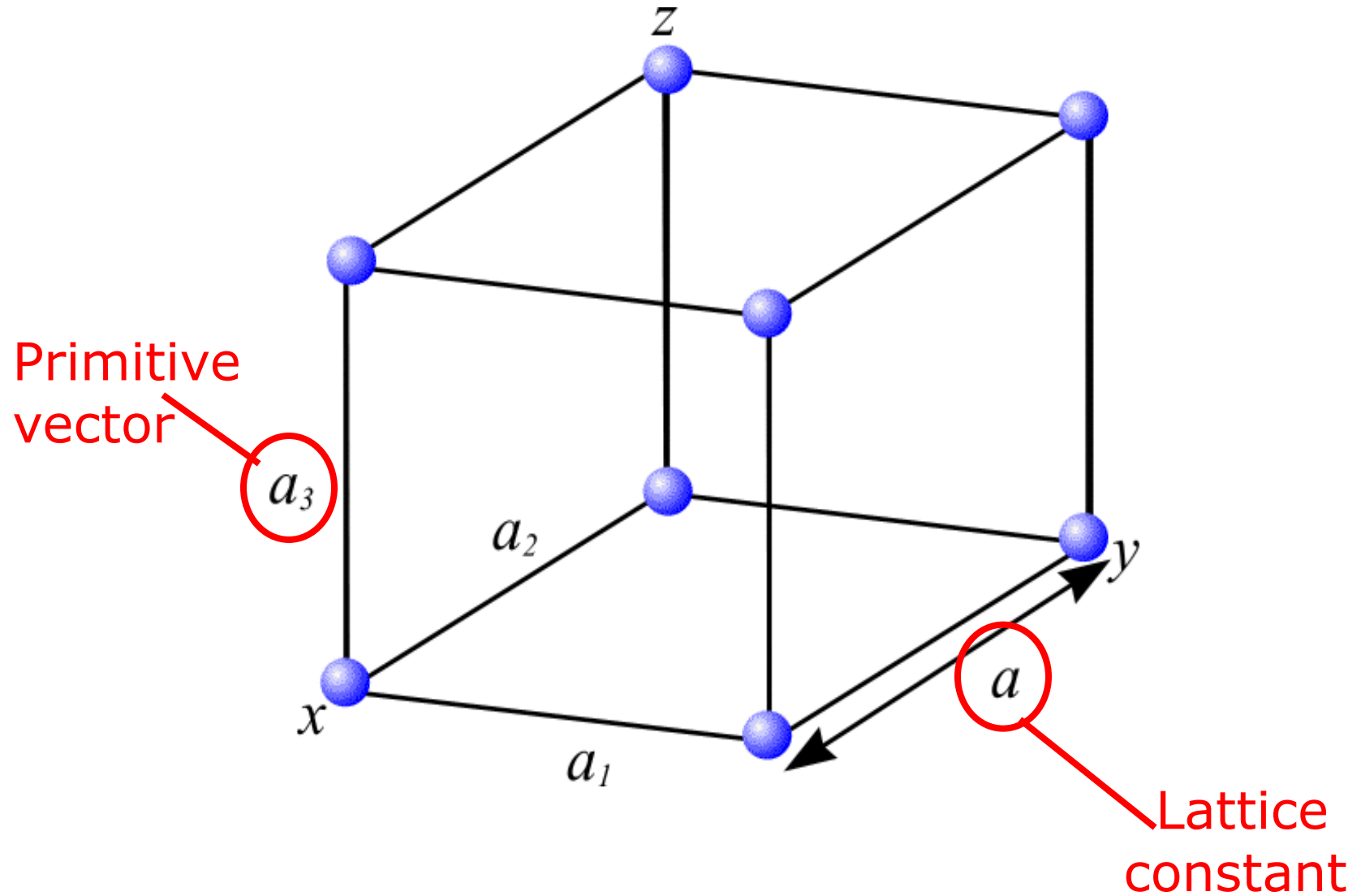
**Lattice + Basis = Crystal Structure**

- An important property of a lattice is the ability to define three vectors  $\mathbf{a}_1$ ,  $\mathbf{a}_2$  and  $\mathbf{a}_3$  such that any lattice point  $\mathbf{R}'$  can be obtained from any other lattice point  $\mathbf{R}$  by a translation:

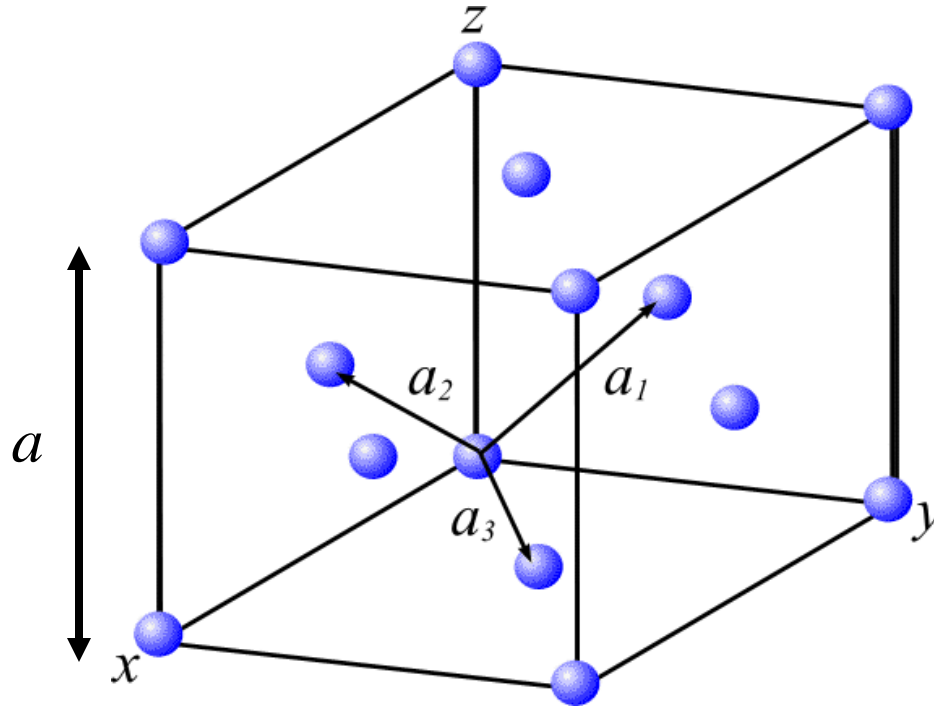
$$\mathbf{R}' = \mathbf{R} + m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + m_3\mathbf{a}_3$$

- $m_1$ ,  $m_2$  and  $m_3$  are integers. Such a lattice is called a Bravais lattice.
- $\mathbf{a}_1$ ,  $\mathbf{a}_2$  and  $\mathbf{a}_3$  are termed the **primitive** if the volume of the cell formed by them is the smallest possible.
- Various kinds of lattice structures are possible in nature.
- We will concentrate on the cubic lattice.

# The Cubic Lattice Structure



# Face Centred Cubic lattice structure



- The Face Centred Cubic (FCC) lattice is the most important for semiconductors.
- A symmetric set of primitive vectors:

$$\mathbf{a}_1 = \frac{a}{2}(\hat{y} + \hat{z}), \quad \mathbf{a}_2 = \frac{a}{2}(\hat{z} + \hat{x}), \quad \mathbf{a}_3 = \frac{a}{2}(\hat{x} + \hat{y})$$

The Lattice Constant

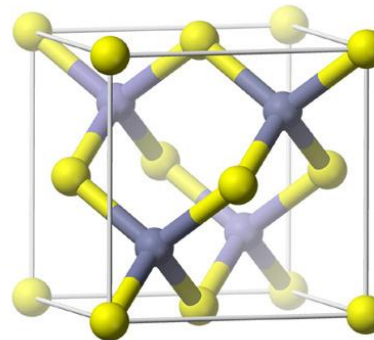
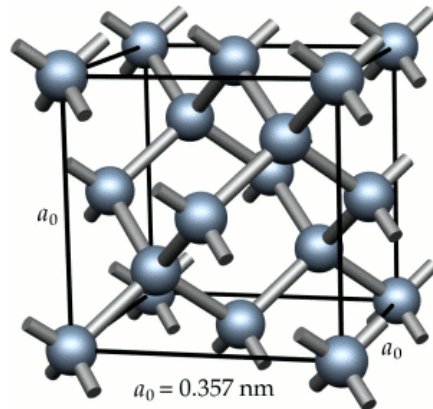
# Semiconductor lattices

- Essentially all semiconductors of interest for electronics and opto-electronics have an underlying FCC lattice structure.
- However, they have two atoms per basis:

$$(000) \text{ and } \left( \frac{a}{4}, \frac{a}{4}, \frac{a}{4} \right)$$

- This can be seen as two interpenetrating FCC sub-lattices with one sub-lattice displaced from the other by one quarter of the distance along a diagonal of the cube.
- The separation between the atoms is  $\sqrt{3}a/4$ .
- If the two atoms of the basis are the same, the structure is called **diamond**, semiconductors such as silicon and germanium fall into this category.
- If the two atoms are different, the structure is called **zinc blende**, example III-V semiconductors include GaAs and AlAs.

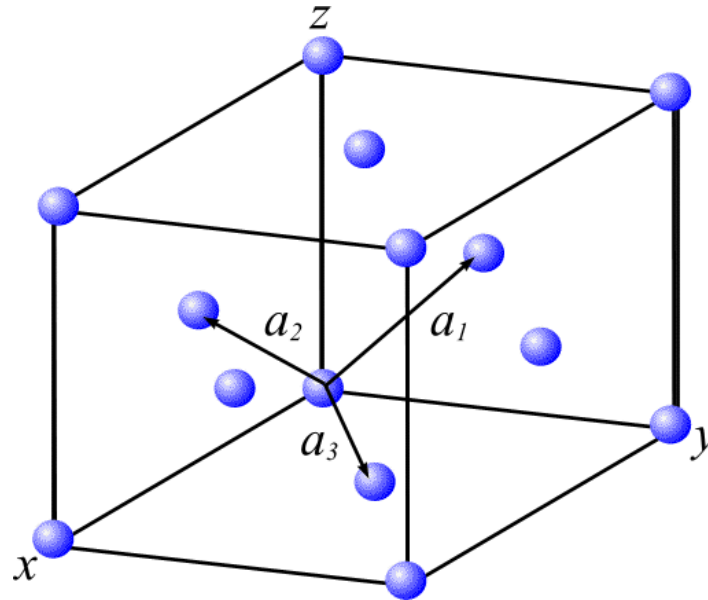
Diamond



Zinc  
blende

# Semiconductor example

- At 300K the lattice constant for silicon is 0.543nm. Suppose we want to calculate the number of silicon atoms per cubic centimetre.

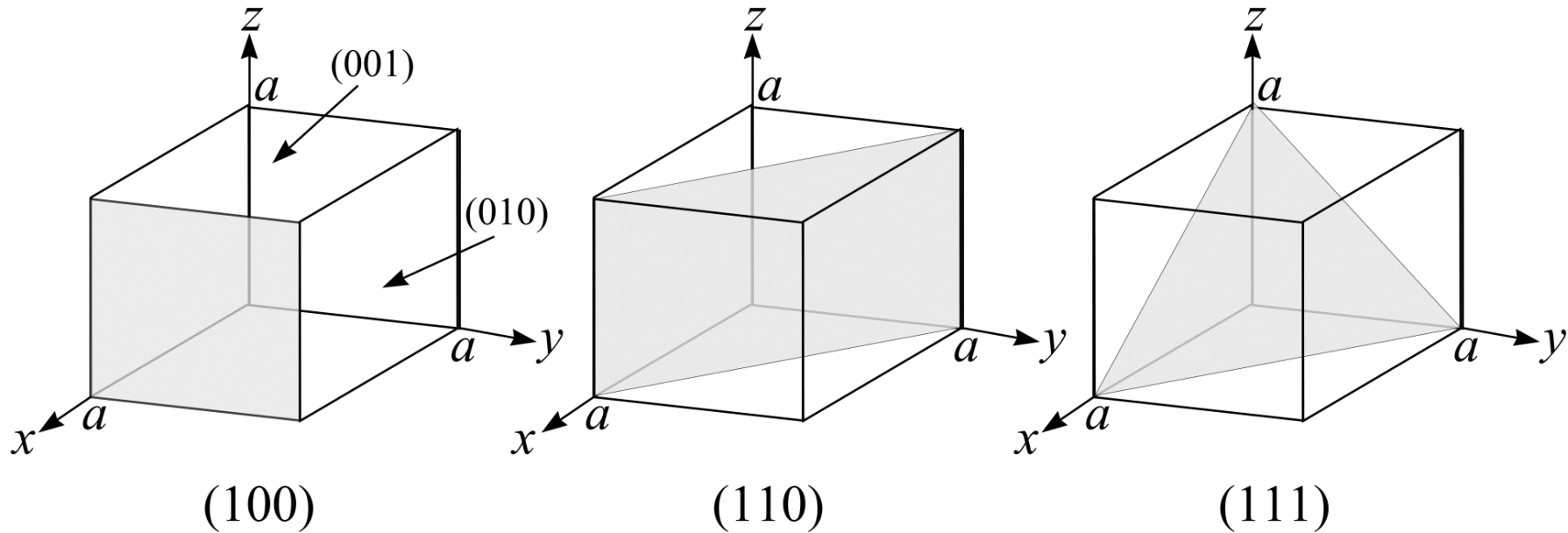


- There are 8 atoms per unit cell, therefore:

$$N_{\text{si}} = \frac{8}{a^3} = \frac{8}{(5.43 \times 10^{-8})^3} = 5 \times 10^{22} \text{ atoms/cm}^3$$

# Semiconductor lattice properties

- We need a convenient method of defining the various planes – **Miller Indices**.
  - Define the  $x, y, z$  axes.
  - Take intercepts of the plane along the axes in units of lattice constants
  - Take the reciprocal of the intercepts and reduce them to the smallest integers  $h, k$  and  $l$ .
- $(hkl)$  denotes a family of parallel planes.
- $\{hkl\}$  denotes a family of equivalent planes. For example  $\{100\}$ ,  $\{010\}$  and  $\{001\}$  are all equivalent in the cubic structure.
- $[hkl]$  denotes a crystal direction e.g.  $[100]$  x-axis
- $\langle hkl \rangle$  denotes a full set of equivalent directions.



So what does this mean?

The crystal properties along different planes are different  
 - there are differences in the atomic spacings.

This means **electrical** and other device properties are dependent on the crystal orientation.



- Electrons inside semiconductors can be regarded as “free” under proper conditions – allowing rules for free electrons to be easily adapted for semiconductors.
- Solving Schrödinger equation:

$$\left[ -\frac{\hbar^2}{2m_0} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) + V_0 \right] \psi(\mathbf{r}) = E\psi(\mathbf{r})$$

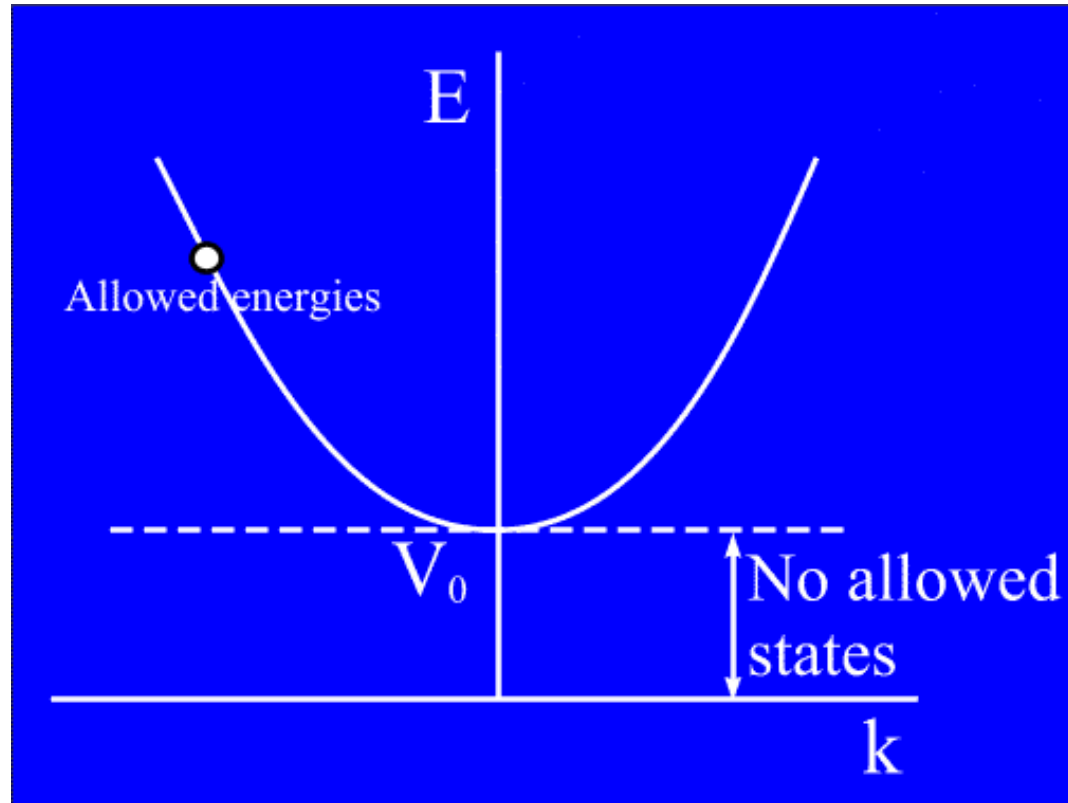
- The **energy** of the electron is obtained as:

$$E = \frac{\hbar^2 \mathbf{k}^2}{2m_0} + V_0 \quad \text{Classically: } E = \frac{p^2}{2m_0}$$

- And the **momentum** is obtained as:  $p = \hbar k$
- Equation of motion:  $\hbar \frac{d\mathbf{k}}{dt} = \mathbf{F}_{\text{ext}}$
- Where  $k$  is the wavevector:  $k = \frac{2\pi}{\lambda}$
- The energy-momentum (E-k) relation for free electrons now be obtained.

# Electrons in free space E-k relation

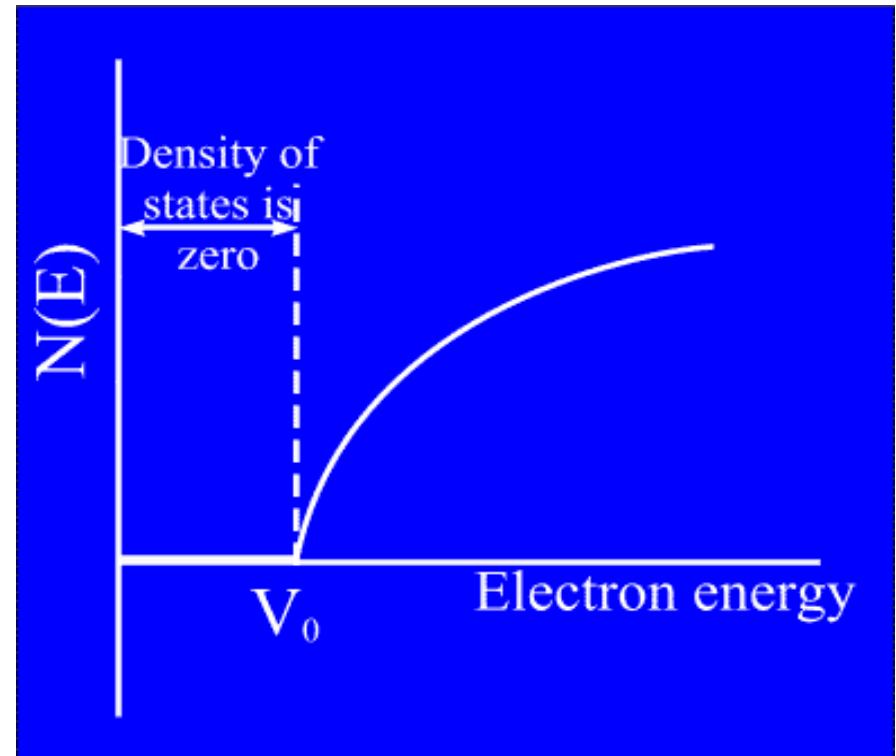
- The energy-momentum relationship:



- The allowed energies form a **continuous** band.

- The density of states is the number of available electronic states per unit volume per unit energy around an energy  $E$ .
- Is a very important, and important physical phenomena such as optical absorption and transport are intimately dependent on this concept.
- The density of states  $N(E)$  can be written as:

$$N(E) = \frac{\sqrt{2}m_0^{3/2} (E - V_0)^{1/2}}{\pi^2 \hbar^3}$$



- The density of states of electrons moving in zero potential at an energy of 0.1eV:

$$\begin{aligned}
 N(E) &= \frac{\sqrt{2}m_0^{3/2}E^{1/2}}{\pi^2\hbar^3} \\
 &= \frac{\sqrt{2}(0.91 \times 10^{-30}\text{kg})^{3/2}(E^{1/2})}{\pi^2(1.05 \times 10^{-34}\text{Js})^3} \\
 &= 1.07 \times 10^{56}E^{1/2}\text{J}^{-1}\text{m}^{-1}
 \end{aligned}$$

- Expressed in the more commonly used units of  $\text{eV}^{-1}\text{cm}^{-3}$  gives,

$$N(E) = 2.15 \times 10^{21} \text{eV}^{-1} \text{cm}^{-3}$$

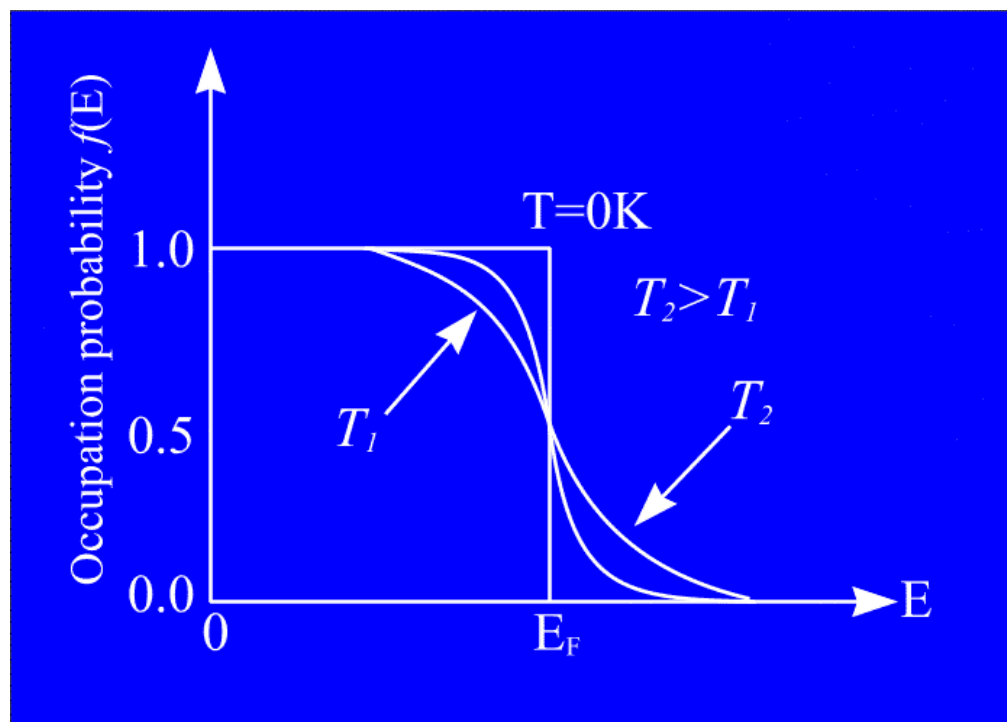
- When the electron wavefunction is confined to  $\sim 10\text{nm}$  around the nucleus, only discrete or bound state energies are allowed.
- When the atomic spacing becomes 10-20nm, electrons will sense the neighbouring nuclei, and will be influenced by them.
- The result of these interactions is:
  - Lower-energy core levels remain relatively unaffected
  - Electronic levels with higher energies and whose wavefunctions are not confined to the nucleus, broaden into bands of allowed energies.
  - These allowed bands are separated by **bandgaps**.
- Within each band the electron is described by a k-vector (as before), only the relation is more complicated.
- The electron behaves as if it were in free space, except it responds as if it had a different or **effective** mass.

# Filling of electronic states

- How do the electrons distribute themselves among the various allowed electronic states?
- The distribution function  $f(E)$  tells us the probability that an allowed level at energy  $E$  is occupied.
- Is the Maxwell-Boltzmann distribution function :

$$F(E) = \exp\left(-\frac{E - E_F}{k_B T}\right)$$

- $E_F$  is the Fermi level representing the energy where  $F(E_F) = 1/2$ .



# The Fermi level in semiconductors

- If the electron density is small, so that  $F(E)$  is always small. The Fermi function can be presented by the Boltzmann function.
- The electron density can now be analytically evaluated as:

$$n = \int_{E_0}^{\infty} N(E)F(E)dE$$

$$= N_C \exp\left(\frac{E_F - E_0}{k_B T}\right)$$

- Where  $N_C$  is called the **effective density of states** and is defined as:

$$N_C = 2 \left( \frac{m_0}{2\pi\hbar^2} \right)^{3/2} (k_B T)^{3/2}$$

# Fermi level: Example calculation

- Calculate the Fermi level at 77K for a case where the electron density is  $10^{19}\text{cm}^{-3}$ . Assume the energy band starts at  $E=0$ .
- In the Boltzmann approximation, the Fermi level is:

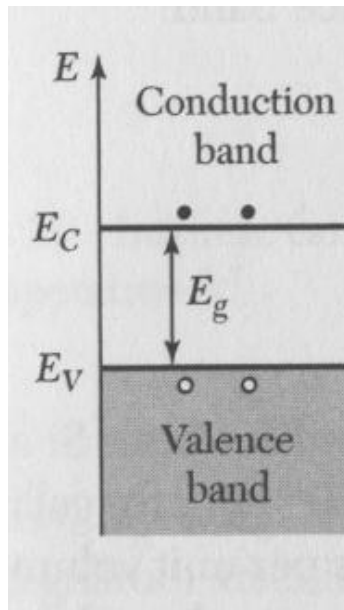
$$E_F = k_B T \ln\left(\frac{n}{N_C}\right)$$

$$N_C = 3.34 \times 10^{18} \text{cm}^{-3}$$

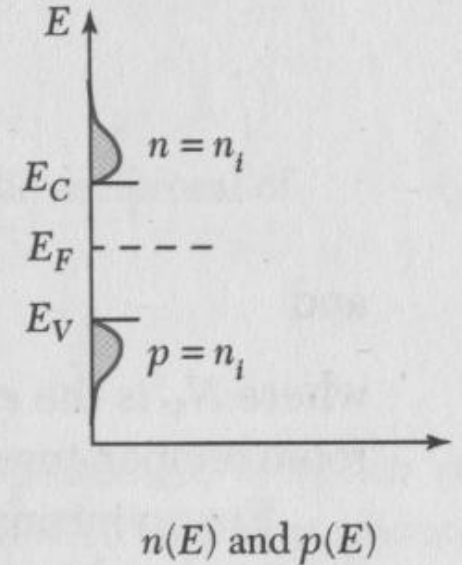
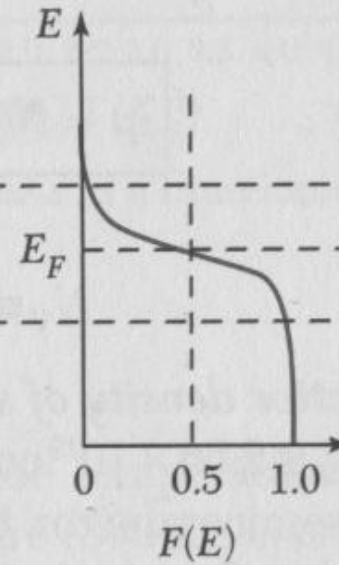
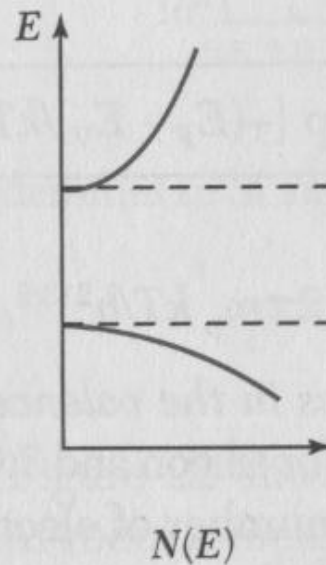
$$E_F = 0.0067 \left[ \ln\left(\frac{10^{19}}{3.34 \times 10^{18}}\right) \right] = 7.35 \text{meV}$$



## Semiconductor band diagram



## Fermi distribution function



**Density of states**

**Carrier concentration**

$$n = \int_{E_C}^{\infty} N(E)F(E)dE$$