NuSTEC Neutrino Generator School

UNIVERSITY OF LIVERPOOL

Lecture T3

Basics of electroweak interactions

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Outline

- Electroweak interactions in the Standard Model
- Strong interactions in the Standard Model
  - Example: charged pion decay
- Inclusive neutrino-nucleon(nucleus) cross section
  - Example: EM scattering on a pointlike particle

Electroweak interactions in the SM

- Spontaneously broken SU(2) x U(1) gauge symmetry

$$\mathcal{L}_{\rm EW} = -e J^\mu_{em} A_\mu - \frac{g}{2\cos\theta_W} J^\mu_{nc} Z_\mu - \frac{g}{2\sqrt{2}} J^\mu_{cc} W^+_\mu + h.c.$$

$$\sin\theta_W = \frac{e}{g} \qquad \cos\theta_W = \frac{M_W}{M_Z} \qquad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

in the leptonic sector:

$$\begin{aligned} J^\mu_{em} &= \bar{l}_i \gamma^\mu l_i \qquad i = e, \mu, \tau \\ J^\mu_{cc} &= \bar{\nu}_i \gamma^\mu (1-\gamma_5) l_i \\ J^\mu_{nc} &= \frac{1}{2} \bar{l}_i \gamma^\mu (g_V - g_A \gamma_5) l_i + \frac{1}{2} \bar{\nu}_i \gamma^\mu (1-\gamma_5) \nu_i \end{aligned}$$

$$g_V = -1 + 4\sin^2\theta_W, \; g_A = -1$$

$$|g_V| \approx 0.04 \ll |g_A|$$

Electroweak interactions in the SM

- Spontaneously broken SU(2) x U(1) gauge symmetry

$$\mathcal{L}_{\mathrm{EW}} = -e J^{\mu}_{em} A_{\mu} - \frac{g}{2\cos\theta_W} J^{\mu}_{nc} Z_{\mu} - \frac{g}{2\sqrt{2}} J^{\mu}_{cc} W^{+}_{\mu} + h.c.$$

$$\sin\theta_W = \frac{e}{g} \qquad \cos\theta_W = \frac{M_W}{M_Z} \qquad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$

in the quark sector:

$$J^{\mu}_{em} = Q_i \bar{q}_i \gamma^{\mu} q_i = \frac{2}{3} \bar{q}_u \gamma^{\mu} q_u - \frac{1}{3} (\bar{q}_d \gamma^{\mu} q_d + \bar{q}_s \gamma^{\mu} q_s) + \ldots$$

$$\begin{aligned} J^{\mu}_{nc} &= \bar{q}_u \gamma^{\mu} \left[\frac{1}{2} - \left(\frac{2}{3} \right) 2 \sin^2\theta_W - \frac{1}{2} \gamma_5 \right] q_u + (u \to c) + (u \to t) \\ &+ \bar{q}_d \gamma^{\mu} \left[-\frac{1}{2} - \left(-\frac{1}{3} \right) 2 \sin^2\theta_W + \frac{1}{2} \gamma_5 \right] q_d + (d \to s) + (d \to b) \end{aligned}$$

Electroweak interactions in the SM

- Spontaneously broken SU(2) x U(1) gauge symmetry

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$$\sin\theta_W = \frac{e}{g} \qquad \cos\theta_W = \frac{M_W}{M_Z} \qquad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$

in the quark sector:

$$J^\mu_{cc} = (\bar{q}_u \bar{q}_c \bar{q}_t)\, \gamma^\mu\, (1-\gamma_5)\, U \begin{pmatrix} q_d \\ q_s \\ q_b \end{pmatrix}$$

$U \leftarrow$ CKM matrix

$$U \approx \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

$\theta_C \leftarrow$ Cabibbo angle

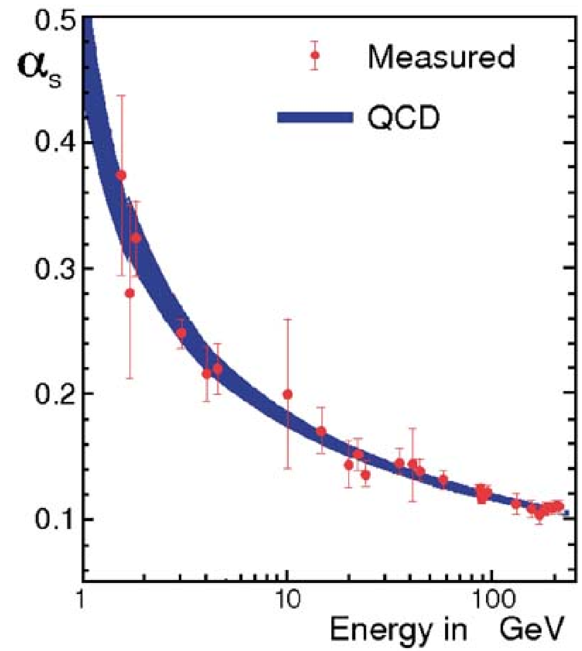
Strong interactions in the SM

- SU(3) (color) gauge symmetry: QCD

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_q \left(i\gamma^\mu D_\mu - m_q \right) \psi_q - \frac{1}{4} G_a^{\mu\nu} G_{a\mu\nu} \qquad q = u, d, s, \ldots \quad a = 1-8$$

$$D_\mu \psi = \left(\partial_\mu - i g \frac{\lambda_a}{2} A_\mu^a \right) \psi \qquad G_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_{abc} A_b^\mu A_c^\nu$$

- Asymptotically free $\Rightarrow$ perturbative at high energies
- Nonperturbative at low energies
- Confining



$$\alpha_s = \frac{g^2}{4\pi}$$

Strong interactions in the SM

- Approximate symmetries of N_f =3 QCD
  - $m_u = m_d = m_s$ ⇔ Global $SU(3)_{flavor}$ symmetry

$$V_a^\mu = \bar{q}\gamma^\mu \frac{\lambda_a}{2} q \Leftrightarrow \partial_\mu V_a^\mu = 0 \qquad a = 1-8 \qquad q = \begin{pmatrix} q_u \\ q_d \\ q_s \end{pmatrix}$$

m_u (1 GeV) = 4 ± 2 MeV
m_d (1 GeV) = 8 ± 4 MeV
m_s (1 GeV) = 164 ± 33 MeV

$$\partial_\mu V_a^\mu = \bar{q}\left[m, \frac{\lambda_a}{2}\right] q \qquad m = \mathrm{diag}(m_u, m_d, m_s)$$

  - $m_u = m_d$ ⇔ Global $SU(2)_{flavor}$ isospin symmetry

$$V_a^\mu = \bar{q}\gamma^\mu \frac{\lambda_a}{2} q = \bar{q}'\gamma^\mu \frac{\tau_a}{2} q' \Leftrightarrow \partial_\mu V_a^\mu = 0 \qquad a = 1-3 \qquad q' = \begin{pmatrix} q_u \\ q_d \end{pmatrix}$$

Strong interactions in the SM

Gell-Mann matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ +i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ +i & 0 & 0 \end{pmatrix},$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & +i & 0 \end{pmatrix}, \lambda_8 = \tfrac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Strong interactions in the SM

- Approximate symmetries of N_f =3 QCD
  - $m_u = m_d = m_s \Leftrightarrow$ Global $SU(3)_{flavor}$ symmetry

$$V_a^\mu = \bar{q}\gamma^\mu \frac{\lambda_a}{2} q \Leftrightarrow \partial_\mu V_a^\mu = 0 \qquad a = 1-8 \qquad q = \begin{pmatrix} q_u \\ q_d \\ q_s \end{pmatrix}$$

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$$V_a^\mu = \bar{q}\gamma^\mu \frac{\lambda_a}{2} q = \bar{q}'\gamma^\mu \frac{\tau_a}{2} q' \Leftrightarrow \partial_\mu V_a^\mu = 0 \qquad a = 1-3 \qquad q' = \begin{pmatrix} q_u \\ q_d \end{pmatrix}$$

Strong interactions in the SM

- Flavor structure of the EW quark currents:

$$\mathcal{L}_{\mathrm{EW}} = -eJ^\mu_{em}A_\mu - \frac{g}{2\cos\theta_W}J^\mu_{nc}Z_\mu - \frac{g}{2\sqrt{2}}J^\mu_{cc}W^+_\mu + h.c.$$

$$\begin{aligned} J^\mu_{em} &= \frac{2}{3}\bar{q}_u\gamma^\mu q_u - \frac{1}{3}(\bar{q}_d\gamma^\mu q_d + \bar{q}_s\gamma^\mu q_s) \\ &= \frac{1}{2}V^\mu_Y + V^\mu_3 \end{aligned}$$

$$J^\mu_{cc} = \bar{q}_u\gamma^\mu(1-\gamma_5)(q_d\cos\theta_C + q_s\sin\theta_C)$$

$$V^\mu_+ = \bar{q}_u\gamma^\mu q_d = \bar{q}_u\gamma^\mu\frac{\lambda_1 + i\lambda_2}{2}q_d = V^\mu_1 + iV^\mu_2$$

$V_{1,2,3}$: components of the same conserved flavor vector current

$$J^\mu_{nc} = \bar{q}_u\gamma^\mu\left[\frac{1}{2} - \left(\frac{2}{3}\right)2\sin^2\theta_W - \frac{1}{2}\gamma_5\right]q_u + \bar{q}_d\gamma^\mu\left[-\frac{1}{2} - \left(-\frac{1}{3}\right)2\sin^2\theta_W + \frac{1}{2}\gamma_5\right]q_d + (d\to s)$$

$$V^\mu_{nc} = (1-2\sin^2\theta_W)V^\mu_3 - 2\sin^2\theta_W\frac{1}{2}V^\mu_Y - \frac{1}{2}\bar{q}_s\gamma^\mu q_s$$

Strong interactions in the SM

- Approximate symmetries of $N_f = 3$ QCD
  - $m_u = m_d = m_s = 0 \Leftrightarrow$ Chiral $SU(3)_L \times SU(3)_R$ symmetry

m_u (1 GeV) = 4 ± 2 MeV
m_d (1 GeV) = 8 ± 4 MeV
m_s (1 GeV) = 164 ± 33 MeV

$\ll$ nucleon mass

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_{qL} i\gamma^\mu D_\mu \psi_{qL} + \bar{\psi}_{qR} i\gamma^\mu D_\mu \psi_{qR} - m_q(\bar{\psi}_{qL}\psi_{qR} + \bar{\psi}_{qL}\psi_{qR}) + \ldots$$

- Conserved currents:

$$\begin{aligned} R_a^\mu &= \bar{q}_R \gamma^\mu \frac{\lambda_a}{2} q_R \\ L_a^\mu &= \bar{q}_L \gamma^\mu \frac{\lambda_a}{2} q_L \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} V_a^\mu &= R_a^\mu + L_a^\mu = \bar{q}\gamma^\mu \frac{\lambda_a}{2} q \\ A_a^\mu &= R_a^\mu - L_a^\mu = \bar{q}\gamma^\mu \gamma_5 \frac{\lambda_a}{2} q \end{aligned} \qquad q = \begin{pmatrix} q_u \\ q_d \\ q_s \end{pmatrix}$$

- Explicit chiral symmetry breaking:

$$\partial_\mu V_a^\mu = \bar{q}\left[m, \frac{\lambda_a}{2}\right] q \qquad \partial_\mu A_a^\mu = i\bar{q}\left\{m, \frac{\lambda_a}{2}\right\}\gamma_5 q \quad \leftarrow \text{PCAC}$$

Electroweak nucleon current

- Flavor structure of the quark currents:

$$\mathcal{L}_{\rm EW} = -e J^\mu_{em} A_\mu - \frac{g}{2\cos\theta_W} J^\mu_{nc} Z_\mu - \frac{g}{2\sqrt{2}} J^\mu_{cc} W^+_\mu + h.c.$$

$$J^\mu_{cc} = \bar{q}_u \gamma^\mu (1-\gamma_5)(q_d \cos\theta_C + q_s \sin\theta_C)$$

$$A^\mu_+ = \bar{q}_u \gamma^\mu \gamma_5 q_d = \bar{q}_u \gamma^\mu \gamma_5 \frac{\lambda_1 + i\lambda_2}{2} q_d = A^\mu_1 + i A^\mu_2$$

$$J^\mu_{nc} = \bar{q}_u \gamma^\mu \left[\frac{1}{2} - \left(\frac{2}{3}\right) 2\sin^2\theta_W - \frac{1}{2}\gamma_5\right] q_u + \bar{q}_d \gamma^\mu \left[-\frac{1}{2} - \left(-\frac{1}{3}\right) 2\sin^2\theta_W + \frac{1}{2}\gamma_5\right] q_d + (d \to s)$$

$$A^\mu_{nc} = A^\mu_3 + \frac{1}{2}\bar{q}_s \gamma^\mu \gamma_5 q_s$$

$A_{1,2,3}$: components of the same partially conserved flavor axial current

Strong interactions in the SM

- Approximate symmetries of N_f =3 QCD
  - $m_u = m_d = m_s = 0 \Leftrightarrow$ Chiral $SU(3)_L \times SU(3)_R$ symmetry
  - Explicit chiral symmetry breaking:

$$\partial_\mu V_a^\mu = \bar{q}\left[m, \frac{\lambda_a}{2}\right] q \qquad \partial_\mu A_a^\mu = i\bar{q}\left\{m, \frac{\lambda_a}{2}\right\}\gamma_5 q \qquad \leftarrow \text{PCAC}$$

  - Spontaneous chiral symmetry breaking:
    - the ground state does not have the chiral symmetry of the Lagrangian
    - m_ρ = 770 MeV (1^-) $\neq$ m_{a_1} = 1230 MeV (1^+)
    - $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$
    - Goldstone bosons: π, K, η

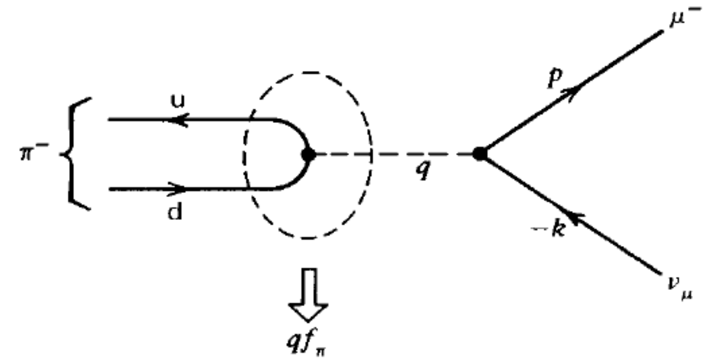
$$\mathcal{L}_{\mathrm{QCD}} \rightarrow \mathcal{L}_{\mathrm{effective}}(\pi, K, \eta \ldots)$$

    - In terms of hadronic degrees of freedom: $A_a^\mu = -f_\pi \partial^\mu \pi_a + \ldots$

$$\langle 0 | A_a^\mu | \pi^- \rangle = \sqrt{2} f_\pi q^\mu$$

Electroweak interactions in the SM

- Example: $\pi^-(q) \to \mu^-(p) + \bar{\nu}_\mu(k)$



$$\mathcal{L}_{\rm EW} = -\frac{g}{2\sqrt{2}} J^\mu_{cc} W^+_\mu + h.c.$$

$$(-i)\mathcal{M} = (-i)\left\langle \mu^- \bar{\nu}_\mu \right| \left(-\frac{g}{2\sqrt{2}}\right) J^\mu_{cc} \left|0\right\rangle (-i) D_{\mu\nu}(q) (-i) \left\langle 0\right| \left(-\frac{g}{2\sqrt{2}}\right) J^\nu_{cc} \left|\pi^-\right\rangle$$

$$D_{\mu\nu} = \frac{1}{q^2 - M_W^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right) \approx -\frac{g_{\mu\nu}}{M_W^2} \qquad q^2 = m_\pi^2 \ll M_W^2$$

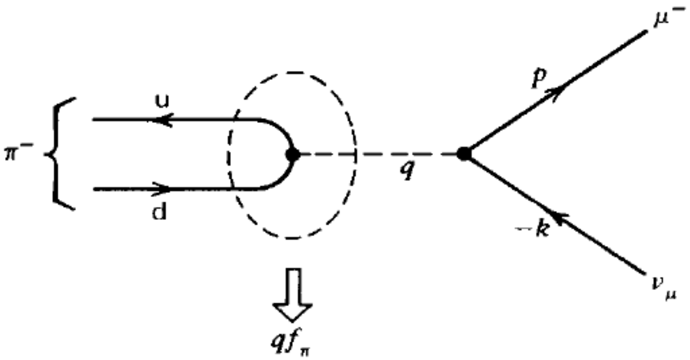
$$\left\langle \mu^- \bar{\nu}_\mu \right| J^\mu_{cc} \left|0\right\rangle = \bar{u}(p)\gamma_\mu (1-\gamma_5) v(k)$$

$$\left\langle 0\right| J^\nu_{cc} \left|\pi^-\right\rangle = \bar{v}_u \gamma^\nu (1-\gamma_5) u_{d'} = \sqrt{2} f_\pi q^\nu$$

$$\left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{M_W^2} = \frac{G_F}{\sqrt{2}}$$

Electroweak interactions in the SM

- Example: $\pi^-(q) \rightarrow \mu^-(p) + \bar{\nu}_\mu(k)$



$$\overline{|\mathcal{M}|^2} = \overline{\sum_{\text{polar.}}} |\mathcal{M}|^2 = 4G_F^2 L_{\mu\nu} H^{\mu\nu}$$

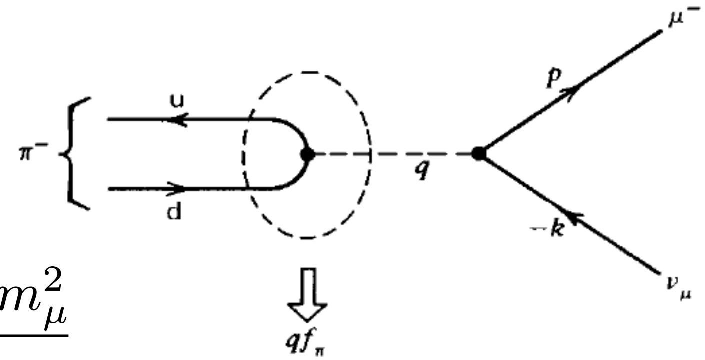
$$\mathrm{Tr}\left[(\not{p} + m_\mu)\gamma_\mu(1-\gamma_5)\not{k}\gamma_\nu(1-\gamma_5)\right] = 8L_{\mu\nu}$$

$$L_{\mu\nu} = p_\mu k_\nu + p_\nu k_\mu - g_{\mu\nu} k \cdot p + i\epsilon_{\mu\nu\alpha\beta} p^\alpha k^\beta$$

$$H^{\mu\nu} = 2f_\pi^2 q^\mu q^\nu$$

Electroweak interactions in the SM

- Example: $\pi^-(q) \to \mu^-(p) + \bar{\nu}_\mu(k)$



$$\overline{|\mathcal{M}|^2} = 8 G_F^2 f_\pi^2 m_\mu^2 (p \cdot k)$$

$$q^2 = (p+k)^2 = m_\mu^2 + 2(p \cdot k) \Rightarrow (p \cdot k) = \frac{m_\pi^2 - m_\mu^2}{2}$$

Decay width, in the π rest frame:

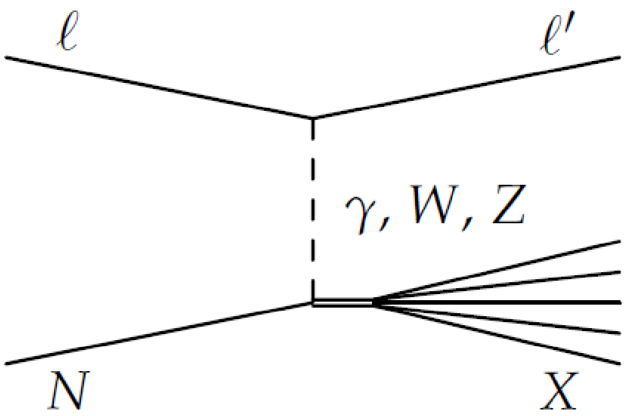
$$\Gamma = \frac{1}{2 m_\pi} \int \frac{d^3 p}{2p^0 (2\pi)^3} \frac{d^3 k}{2k^0 (2\pi)^3} (2\pi)^4 \delta^4 (k + p - q) \overline{|\mathcal{M}|^2}$$

$$\Gamma = \frac{G_F^2}{4\pi} f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2} \right)^2$$

$$\tau = \frac{1}{\Gamma} = 2.6\, 10^{-8}\, s \Rightarrow f_\pi = 92.4\, \mathrm{MeV}$$

Inclusive cross section

$$l(k) + N(p) \to l'(k') + X(p')$$



$$k = (k_0, \vec{k}) \qquad p = (E, \vec{p})$$

$$k' = (k'_0, \vec{k}') \qquad p' = (E', \vec{p}')$$

$$q = k - k' = p' - p = (\omega, \vec{q}) \qquad q^2 = -Q^2 < 0$$

In Lab: $p = (M, \vec{0})$

For CC: $$\frac{d\sigma}{dk'_0 d\Omega(\vec{k}')} = \frac{G_F^2}{(2\pi)^2} \frac{|\vec{k}'|}{k_0} L_{\mu\nu} W^{\mu\nu}$$

$$L_{\mu\nu} = k'_\mu k_\nu + k'_\nu k_\mu - g_{\mu\nu} k \cdot k' + i\epsilon_{\mu\nu\alpha\beta} k'^\alpha k^\beta$$

$$W^{\mu\nu} = \frac{1}{2M} \overline{\sum_{\text{polar.}}} \sum_i \left(\int \frac{d^3 p_i}{2E'_i (2\pi)^3} \right) (2\pi)^3 \delta^4(k' + p' - k - p) \langle X | J^\mu | N \rangle \langle X | J^\nu | N \rangle^*$$

For EM: $L_{\mu\nu} \to L_{\mu\nu}^{(\text{sym})} \qquad \frac{G_F^2}{(2\pi)^2} \to \frac{\alpha^2}{q^4}$

Inclusive cross section

$$l(k) + N(p) \rightarrow l'(k') + X(p')$$

$$k = (k_0, \vec{k}) \qquad p = (E, \vec{p})$$

$$k' = (k'_0, \vec{k}) \qquad p' = (E', \vec{p'})$$

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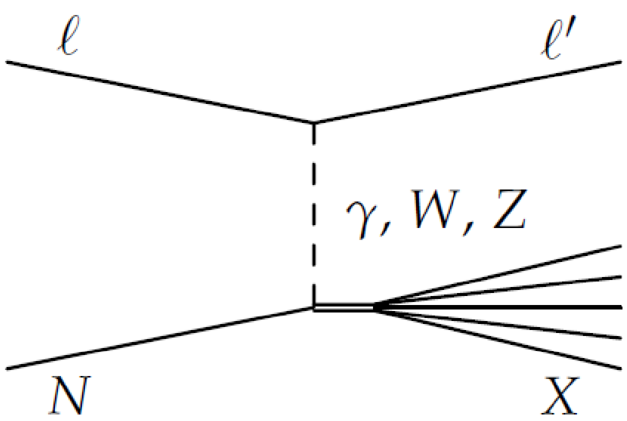


General structure of the hadronic tensor

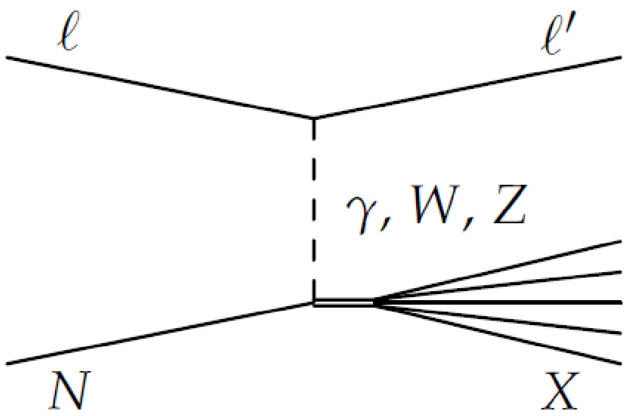
Ingredients: $g^{\mu\nu}$, q^{μ}, p^{μ}, $\epsilon^{\alpha\beta\mu\nu}$ $\quad p' = p + q$ $\leftarrow$ not independent

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + W_2 \frac{p^\mu p^\nu}{M^2} + W_4 \frac{q^\mu q^\nu}{M^2} + W_5 \frac{p^\mu q^\nu + q^\mu p^\nu}{M^2} + W_3 i\epsilon^{\mu\nu\alpha\beta} \frac{p_\alpha q_\beta}{2M^2} + W_6 \frac{p^\mu q^\nu - q^\mu p^\nu}{M^2}$$

Structure functions: $W_i = W_i(p^2 = M^2, q \cdot p = \omega M, q^2) = W_i(\omega, q^2)$

Inclusive cross section

$$l(k) + N(p) \to l'(k') + X(p')$$



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General structure of the hadronic tensor:

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + W_2 \frac{p^\mu p^\nu}{M^2} + W_4 \frac{q^\mu q^\nu}{M^2} + W_5 \frac{p^\mu q^\nu + q^\mu p^\nu}{M^2} + W_3 i\epsilon^{\mu\nu\alpha\beta} \frac{p_\alpha q_\beta}{2M^2} + W_6 \frac{p^\mu q^\nu - q^\mu p^\nu}{M^2}$$

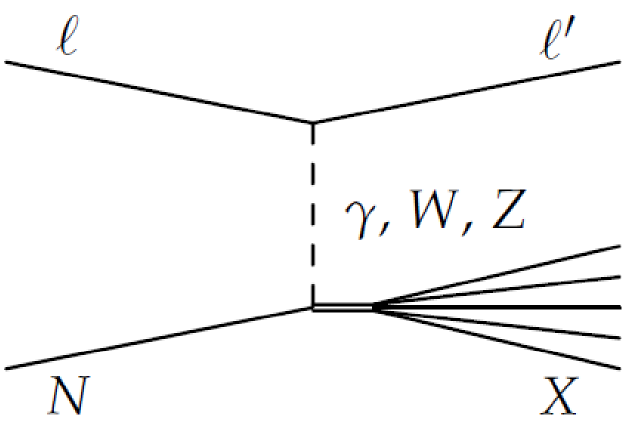
Structure functions: $W_i = W_i(\omega, q^2)$

For EM interactions: $q_\mu J^\mu = 0 \Rightarrow q_\mu W^{\mu\nu}_{em} = W^{\mu\nu}_{em} q_\nu = 0$

$$W^{\mu\nu}_{em} = W_1 \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right) + \frac{W_2}{M^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right)$$

Inclusive cross section

$$l(k) + N(p) \to l'(k') + X(p')$$



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$$q = k - k' = p' - p = (\omega, \vec{q}) \qquad q^2 = -Q^2 < 0$$

In Lab: $p = (M, \vec{0})$

$$\frac{d\sigma}{dk_0' d\Omega(\vec{k}')} = \frac{G_F^2}{(2\pi)^2} \frac{|\vec{k}'|}{k_0} \{W_1 2k \cdot k' + W_2(2k_0' k_0 - k \cdot k') + 2\frac{m_l^2}{M^2}[W_4 k \cdot k' - W_5 M k_0] + \frac{W_3}{M}[(k_0 + k_0')k \cdot k' - k_0 m_l^2]\}$$

$m_l \to 0$

$$\frac{d\sigma}{dk_0' d\Omega(\vec{k}')} = \frac{G_F^2}{2\pi^2}(k_0')^2 \left[W_1 2\sin^2\frac{\theta'}{2} + W_2\cos^2\frac{\theta'}{2} \pm W_3 \frac{(k_0 + k_0')}{M}\sin^2\frac{\theta'}{2}\right]$$

Inclusive cross section

- Example: EM scattering on a point-like particle

$$\langle X | \, J^\mu \, | N \rangle \rightarrow \bar{u}(p')\gamma^\mu u(p)$$

$$W^{\mu\nu} = \frac{1}{2M} \int \frac{d^3p'}{2E'} \delta^4(k' + p' - k - p) 4H^{\mu\nu}$$

$$H^{\mu\nu} = p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu}(p \cdot p' - M^2)$$

Using that:

$$\delta(E' + k_0' - M - k_0) = \frac{E'}{M} \delta \left(k_0' - k_0 - \frac{q^2}{2M} \right)$$

$$q^2 = (p' - p)^2 = 2M^2 - 2p \cdot p' \;\Rightarrow\; p \cdot p' - M^2 = -\frac{q^2}{2}$$

$$\frac{p \cdot q}{q^2} = \frac{M\omega}{q^2} = -\frac{1}{2}$$

Inclusive cross section

- Example: EM scattering on a point-like particle

$$\langle X|\, J^\mu \,|N\rangle \rightarrow \bar{u}(p')\gamma^\mu u(p)$$

$$W^{\mu\nu} = \frac{1}{2M}\int \frac{d^3p'}{2E'}\delta^4(k' + p' - k - p)4H^{\mu\nu}$$

$$H^{\mu\nu} = p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu}(p\cdot p' - M^2)$$

one finds:

$$\begin{aligned} W_1 &= -\frac{q^2}{4M^2\omega}\delta\left(1 + \frac{q^2}{2M\omega}\right) \\ W_2 &= \frac{1}{\omega}\delta\left(1 + \frac{q^2}{2M\omega}\right) \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} 2MW_1 &\equiv F_1 = x\delta(1-x) = F_1(x) \\ \omega W_2 &\equiv \delta(1-x) = F_2(x) \end{aligned}$$

$$x = -\frac{q^2}{2M\omega}$$

Inclusive cross section

- Example: EM scattering on a point-like particle

$$\langle X|\, J^\mu \,|N\rangle \to \bar{u}(p')\gamma^\mu u(p)$$

$$W^{\mu\nu} = \frac{1}{2M}\int \frac{d^3p'}{2E'}\delta^4(k'+p'-k-p)4H^{\mu\nu}$$

$$H^{\mu\nu} = p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu}(p\cdot p' - M^2)$$

for real nucleons, at low $\vec{q}^{\,2}$

$$W_1 = -\frac{q^2}{4M^2\omega}\delta\left(1+\frac{q^2}{2M\omega}\right) \to -\frac{q^2}{4M^2\omega}G^2(q^2)\delta\left(1+\frac{q^2}{2M\omega}\right)$$

$$W_2 = \frac{1}{\omega}\delta\left(1+\frac{q^2}{2M\omega}\right) \to \frac{1}{\omega}G^2(q^2)\delta\left(1+\frac{q^2}{2M\omega}\right)$$

Form factors

$\rho(r)$	$\lvert F(\boldsymbol{q}^2) \rvert$	Example
pointlike	constant	Electron
exponential	dipole	Proton
gauss	gauss	^{6}Li
homogeneous sphere	oscillating	–
sphere with a diffuse surface	oscillating	^{40}Ca

$r \longrightarrow$ $\quad |q| \longrightarrow$

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