Notes on Cross-Section Measurements, Bayesian Unfolding and Model Dependence.

by
Morgan WASCKO

NuSTEC Generator School
Liverpool

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- For an interaction,

$$\sigma = \frac{Rate}{Flux}.$$

- Fermi's Golden Rule:

$$Rate = \underbrace{|\mathcal{M}|^2}_{\text{dynamics}} \underbrace{\rho_f \prod_{in} \frac{1}{2E_{in}}}_{\text{kinematics}}$$

- Use Feynman diagrams to calculate $\mathcal{M}$.*

$$\sigma = \frac{1}{Flux}\left(|\mathcal{M}|^2 \rho_f \prod_{in} \frac{1}{2E_{in}}\right)$$

- Need to compare this to experiments
  → not science otherwise!

- σ is total cross section → also interesting to know dependence on : E, Q^2, W etc

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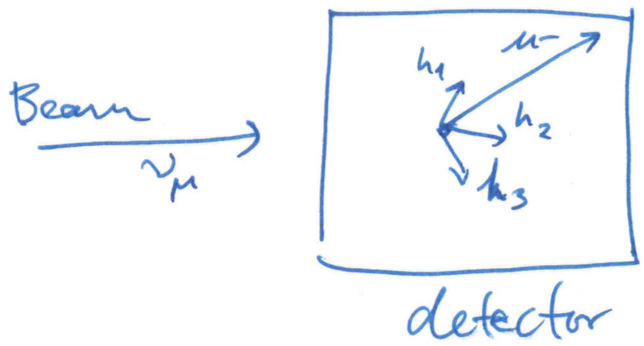
* Feynman diagrams usually used in a perturbative approach, which works for QED, GWS, and high Q^2 QCD (asymptotic freedom limit). We live in non-perturbative QCD land! ouch!

E, Q^2, W etc., "initial" variables,

$p_\mu, \theta_\mu, p_\pi, N_\pi$ etc., "final" variables.

- What do we measure?

We are selecting specific event topologies in detect[or] and counting events.



- Observe some event sample, N.
- Can convert to a rate by POT normalising, but will neglect until end when we handle flux term.

- Bin up data in interesting variables, X.

Examples: $p_\mu, \theta_\mu, p_\pi, N_\pi, \ldots$

$$N \to \sum_j^n N_j, \quad j = \text{bins of reconstructed variable } X$$ (n

- Now a differential rate, well almost — need to divide by bin width detected events = N_j

EFFICIENCY

- Detector does not actually find all interactions.
- Need to estimate efficiency with MC

$$\varepsilon_j = \frac{N_j^{\text{detected}}}{N_j^{\text{occured}}} \Big\} \text{ in FV}$$

$$\text{True events} = \frac{N_j}{\varepsilon_j}.$$

BACKGROUNDS

- Sometimes select the "wrong" event.
- Must estimate fraction of events that are not desired signal (with MC)
- Can estimate PURITY or BACKGROUND
- $\eta_j = \frac{N_j^{sig}}{N_j^{\text{detected}}} \Big\}$ in FV $\qquad B_j = N_j^{\text{NOT SIG}}$

→ Multiplicative and additive (background) corrections have different issues in statistical analysis.

Now, true events $= \frac{N_j \eta_j}{\varepsilon_j}$ OR $\frac{(N_j - B_j)}{\varepsilon_j}$

- UNFOLDING

- Detectors do not make perfect measurements.
  → Must correct observed variables for known smearing effects

— Many experiments use "Bayesian Unfolding"

- The technique essentially asks the question:

"If we observe N_j events in bin j, in which bins did they really happen?"

- Now, let j = events ~~of~~ observed in recon. bin j
  k = events occuring in true bin k.

- Start with identity of conditional probability:

$$P(k|j)\,P(j) = P(j|k)\,P(k);$$

then can write

$$P(k|j) = \frac{P(j|k)\,P(k)}{P(j)}$$ } Bayes's Theorem

$$P(k|j) = \frac{P(j|k)\,P(k)}{\sum_\alpha P(j|\alpha)\,P(\alpha)}$$ ← expand over all true states α

Use MC: $P(k) = \dfrac{N_k}{N_{TOT}}$ ← bin k of true variable x; N_{TOT} ← all bins

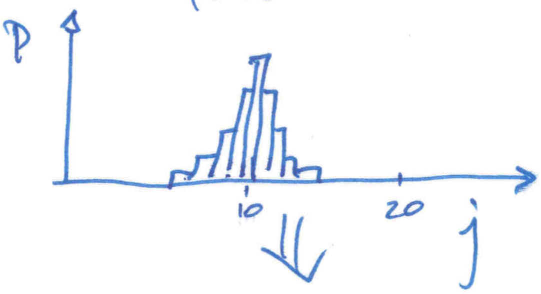
$P(j|k) = \dfrac{N_{jk}}{N_k}$ ← events in bin j given it started in bin k; N_k ← events in bin k

$$P(k|j) = \frac{N_{jk}}{\sum_\alpha N_{j\alpha}}$$ ~~unsmearing matrix~~

← $N_{j\alpha}$ = events in bin j given it started in true bin α.

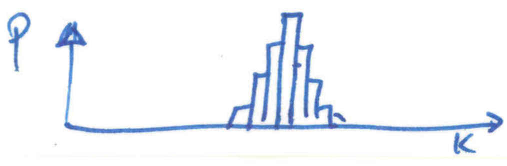
$\equiv U_{jk}$ = unsmearing matrix

idea: true $k = 10$



← smearing of detector

⇓



← unsmearing

Notes on unfolding

~~[illegible]~~

- Spreads 1 event into several bins
- Jumbles up statistical error between bins

$$N_k^{True} = \sum_j \frac{U_{jk}(N_j - B_j)}{\varepsilon_k}$$

Note: ε_k is now function of k:

$$\varepsilon_k = \frac{N_k^{\text{sig detected}}}{N_k^{\text{sig occurred}}} \Bigg\} \text{ in true bins } k.$$

- To get an xsec, need to normalise by flux, number of Targets, and bin width:

$$\frac{d\sigma}{dx_k} = \frac{1}{T\Phi} \frac{N_k^{true}}{\Delta x_k}$$

$$\frac{d\sigma}{dx_k} = \frac{1}{T\Phi_\nu} \frac{\sum_j U_{jk}(N_j - B_j)}{\varepsilon_k \, \Delta x_k}.$$

- Can iterate this process - led to problems in my experience but seems to work for MINERvA.

- ~~TARGET~~ FLUX NORMALISATION

- We have integrated over the entire neutrino flux:

$$\Phi_\nu = \int \Phi_\nu(E_\nu)\, dE_\nu = \sum_i \Phi_i$$

i = true neutrino energy bin

- Usually provided by experiment's flux group, along with covariance matrix.
- For an ~~a~~ absolute cross section, this error is absorbed fully into the total error on the measurement

- TARGET NORMALISATION

- Need to make a choice about what you are measuring.

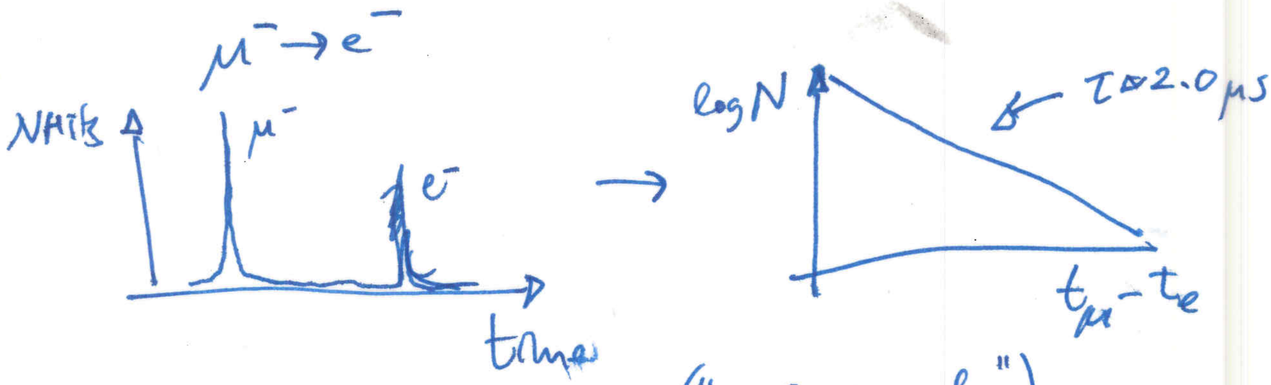
example: plastic scintillator → are you measuring on

- molecules? → CH
- nuclei? → C, H
- nucleons? → 6 n, 7 p

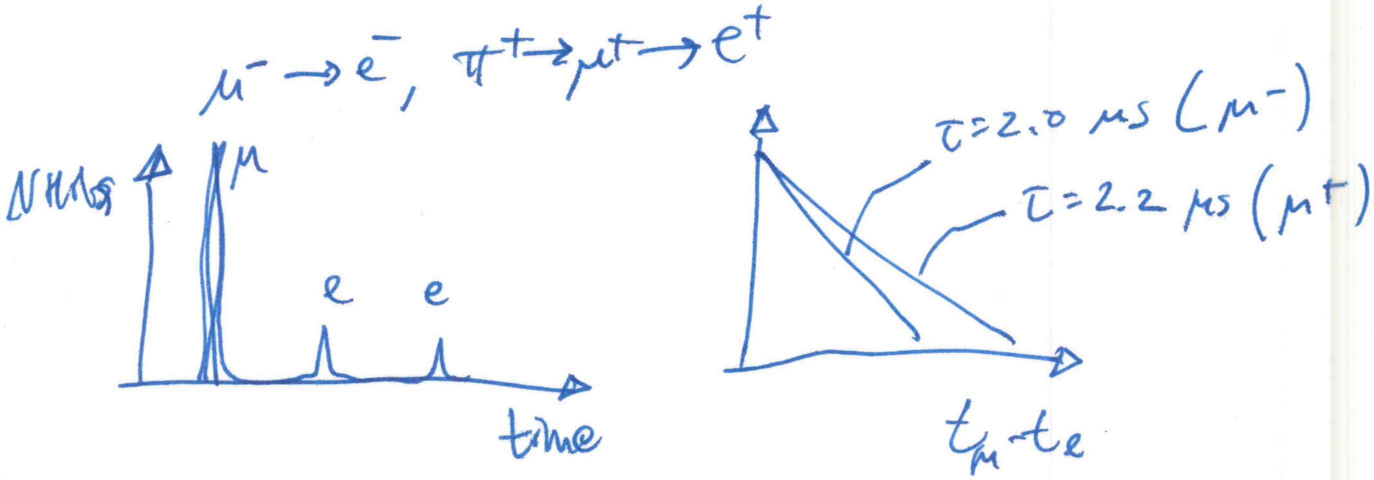
- The choice of signal definition can make the result more or less model dependent.
- "True" process (like CCQE) or final state topology (like $1\mu^- 0\pi^+$)
  - ↳ "True" definition depends on xsec theory for the result
  - ↳ "Final state" definition depends only on knowledge of detector response
- But, what does a theorist do with a single-μ cross-section result?
- Background estimates also can lead to model dependence
  - Usually must use MC to define BG estimates, and that relies on models via generators.
  - Can mitigate using data constraints or sideband data samples.

- example CCQE / CC 1π from MiniBooNE

MB: 2 subevents = single μ ("2SE sample")



3 subevents = $\mu^-\pi^+$ ("3SE sample")



MC says 2 subevent sample has 80% QE, 20% 1π

3 subevent has 80% 1π, 20% other

if MC gets 3 subevent wrong, then 2SE BG is wrong! So ~~apply~~ use 3 subevent sample to ~~redo~~ scale 2SE BG correction → more reliable QE result!

- Even more reliable: report 1μ xsec!

- For initial variables (E_ν, Q^2 etc),
  - could follow similar procedure :

$$x_\mu \to Q^2_k \text{ etc.}$$

  - More common to use MC to directly compare against data, and use scale factors to extract data.

- Use, for example, χ^2 minimisation to find values of internal parameters (e.g. M_A, p_F) that give best agreement with data.
- Or, can simply scale the MC bin by bin to agree with data.

$$\text{scale factors} \quad f_i = \frac{\sigma_i^{\text{scaled MC}}}{\sigma_i^{MC}}$$

$$\text{cross section} \quad \sigma_i = \frac{f_i N_i^{pred} \eta_i}{\varepsilon_i T \Phi_i}$$

see note on next page

$$\sigma_i = \frac{f_i N_i^{pred} \eta_i}{\varepsilon_i T \Phi}$$

this flux term is integrated in the case of Q^2, W ~~and is the differential~~ but is the flux per bin for σ vs. E_ν

- This method uses the MC to predict the number of events per bin, which automatically takes into account the detector smearing and inefficiencies, as well as BG.
  - → The "unfolding" is already done by the MC. Actually, the MC is comparing smeared simulation to data, which means we can extract the "true" xsec that would yield that number of events.
- The first method makes MC-based corrections to data.

→ What's best???

- Depends on your purpose.
- Want to measure an unambiguous xsec that can be used by others?
  - → Do unfolded flux int. differential xsecs

SYSTEMATICS

- change ~~one~~ parameters in MC, run analysis forward and check change on σ
- add all variations in quadrature to get total sys.
- need to know (or assume) covariance of parameters.

$$E_\nu = E(\theta_\mu, p_\mu ; h_1 \ldots h_m)$$

h = underlying params

i, j = energy bins

n, m = parameter bins (M_A or Φ_ν)

$$\frac{1}{N_{\alpha}} \sum_\alpha (N_i^\alpha - N_i^0)(N_j^\alpha - N_j^0) = \langle \sigma_i \sigma_j \rangle = M_{ij}$$

$$M_{ij}^{TOT} = \sum_k^{N_{SYS}} M_{ij}^k \; ;$$

DATA RELEASE

- vectors (text file) of xsec
- ~~matrix of cov~~
- covariance matrix

use it in χ^2 fit as: for example.

$$\chi^2 = \sum_i \sum_j (d_i - m_i) M^{-1}_{ij} (d_j - m_j)$$

PUBLISH A TOOL BOX

- EFFICIENCY FUNCTIONS ?
- EVENT LIBRARY ?
- SINGLE EVENT XSEC?