Update on Coupled Bunch Instabilities

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Since the last update in April ...

1. The finite wall wake function (which is tedious to calculate) has been used in the jitter calculation.

2. The resulting jitter is only a few percent higher than the thick wall case (which is easy).

3. Using the injection offset specified for the ILC design, the rms jitter is 50% of the specified 2 pm emittance.

4. The jitter shows clear oscillations that can be explained in terms of the Fourier modes due to wake field coupling.

5. An analytic solution for the jitter is derived and agrees well with the simulation results.

Overview

1. The ILC damping ring and transient effects arising from the injection of fresh bunches.

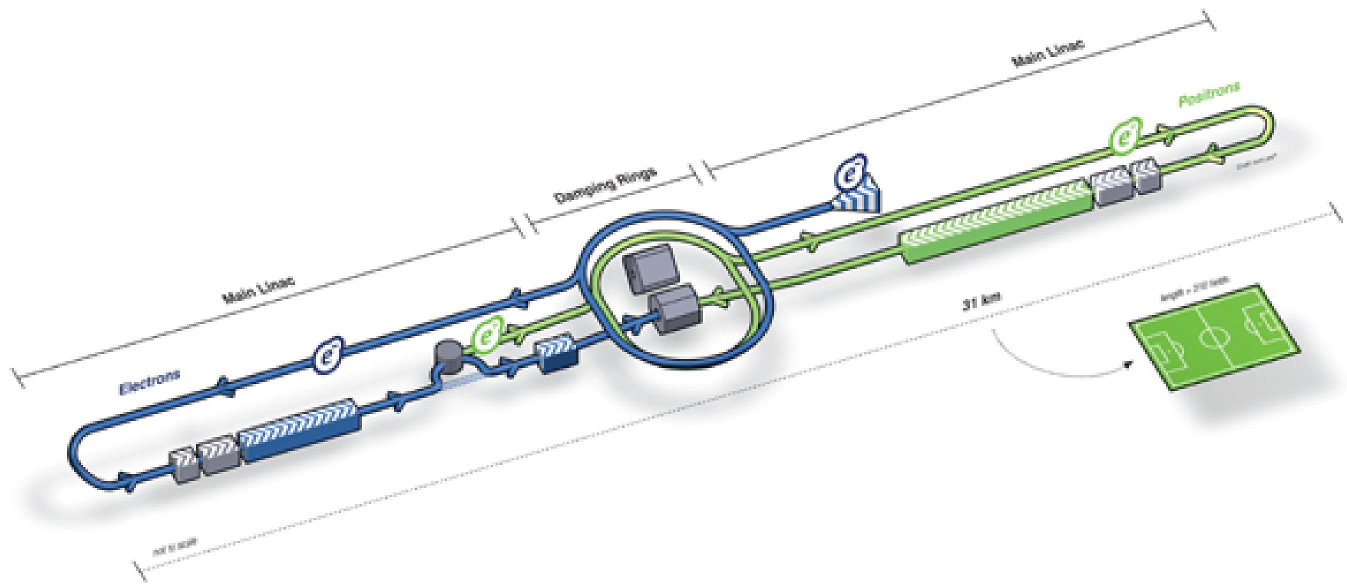
2. Calculation of the finite wall wake function, and effect of finite wall thickness on the extraction jitter.

3. Using FFT convolution to make the simulation possible, and determining the effect of the lattice on the jitter.

4. Effect of extraction jitter on the vertical emittance, and the presence of oscillation modes in the jitter.

5. An analytic solution for the extraction jitter, and how it compares with the simulation results.

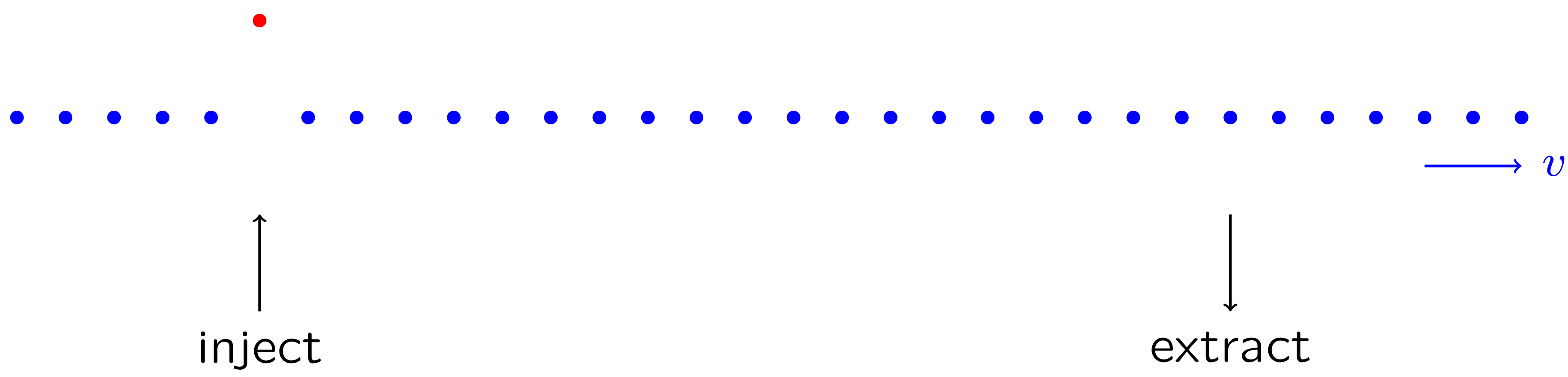
The jitter in the ILC damping ring arises because of the need to produce positrons from the electron bunches.



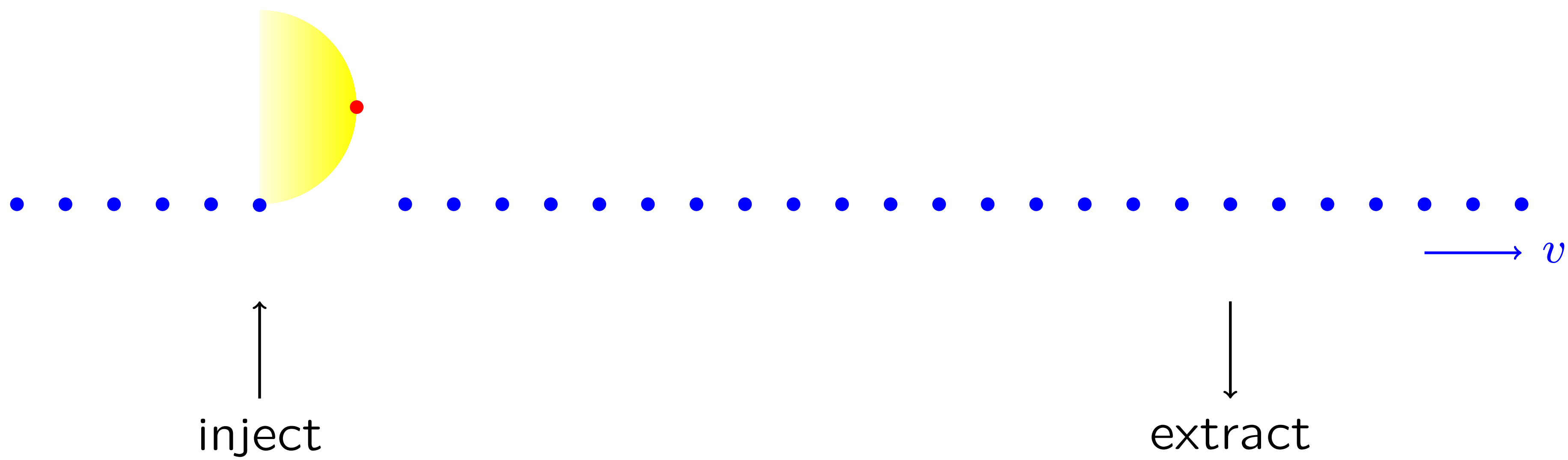
Positron bunches are required at the same time as the electron bunches. The timing means that fresh electron bunches have to be injected while the damped bunches are being extracted.

This induces jitter in the damped bunches.

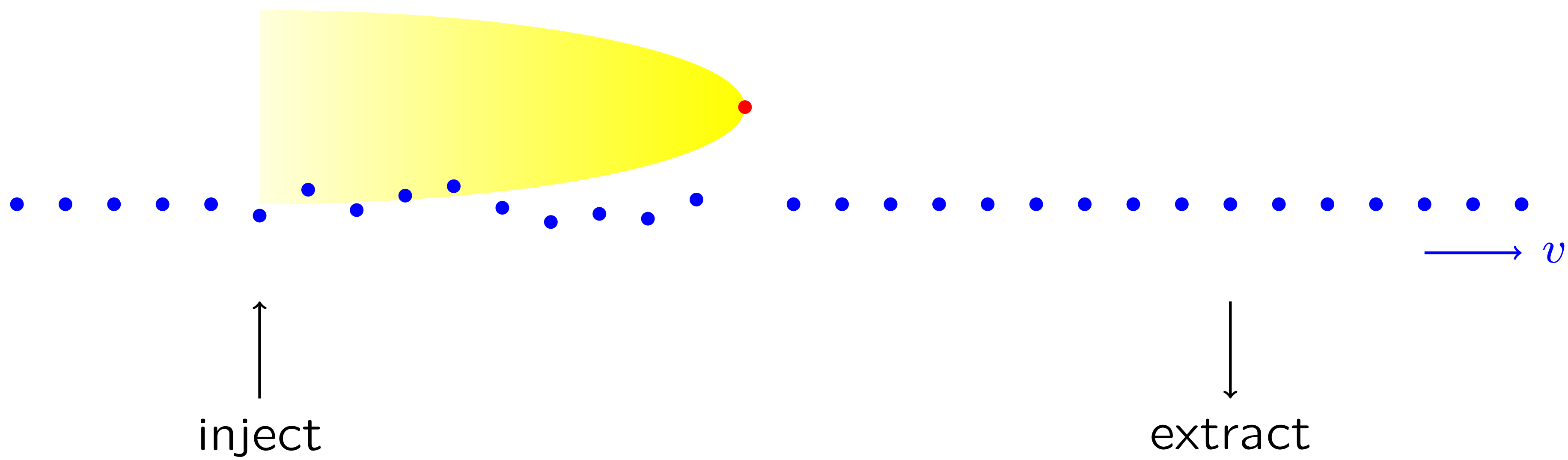
In the ring, a fresh bunch is injected among damped bunches.

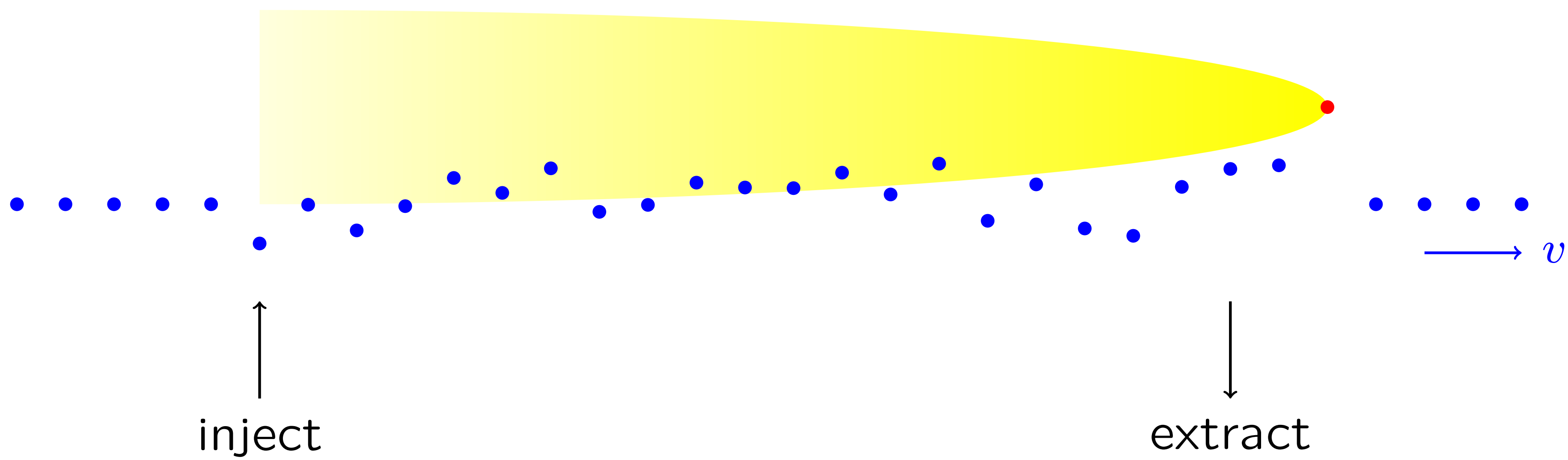


A wake field is generated because of the resistive wall.



This perturbs and induces jitter in the damped bunches ...




$\longrightarrow v$
inject
extract

The resistive wall wake field.

1. The wake force on one bunch due to a leading bunch can be calculated by multiplying the charges of the two bunches and a wake function.

2. A simple formula for the wake function is available in the thick wall, high frequency limit.

3. To obtain a reliable estimate of the jitter, we study the effect of a finite wall, which is more realistic.

4. This requires a longer calculation, using a full wave solution of Maxwell's equations. For cylindrical symmetry, the solutions are modified Bessel's functions.

5. By matching these at the interfaces of the beam pipe and vacuum or air, the electric field at the axis - i.e. the wake field - can be obtained. (A variety of techniques are available from the literature.)

A snapshot of the required equations:

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{l^2}{r^2} - \frac{k_z^2}{\gamma^2}\right] E_{1,z}(r,\omega) = i\frac{Pk_z \cos\theta}{\epsilon_0 \gamma_0^2 \beta c \pi a^2}\delta(a-r)$$

where

$$\gamma^{-2} = \gamma_0^{-2} - \frac{i\mu\omega S}{k_z^2}$$

and r is the distance from the axis, θ the azimuthal angle, γ_0 the electron energy, S the medium conductivity, and k_z the spatial frequency.

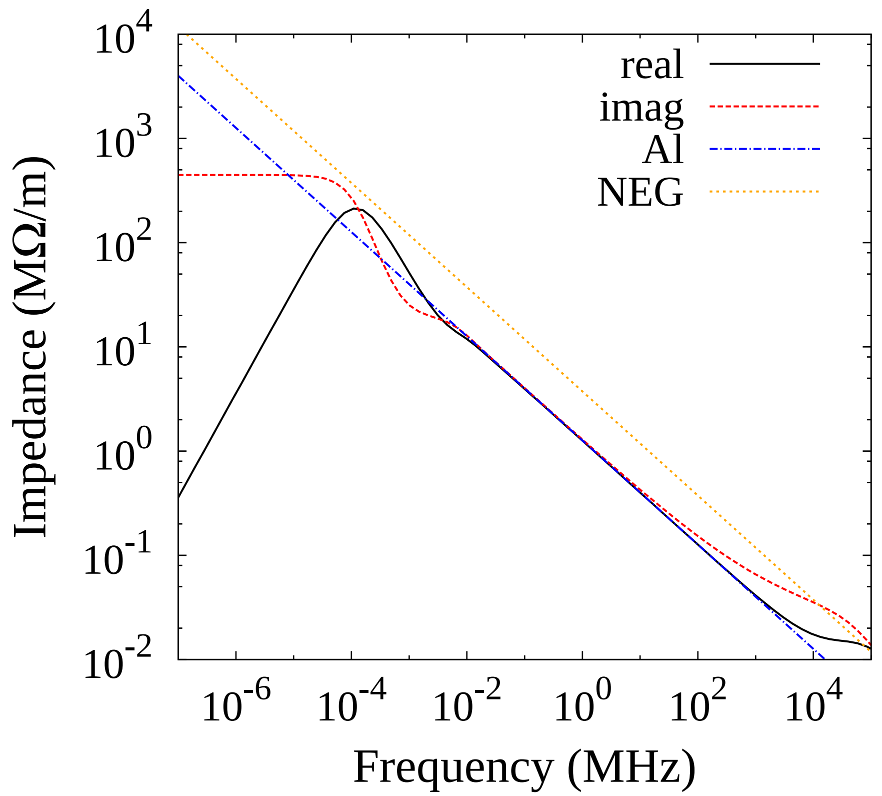
The solutions are modified Bessel's functions:

$$E_{1,z}(r,\omega) = A_n I_1(\sigma r) + B_n K_1(\sigma r)$$

where $\sigma = k_z/\gamma$.

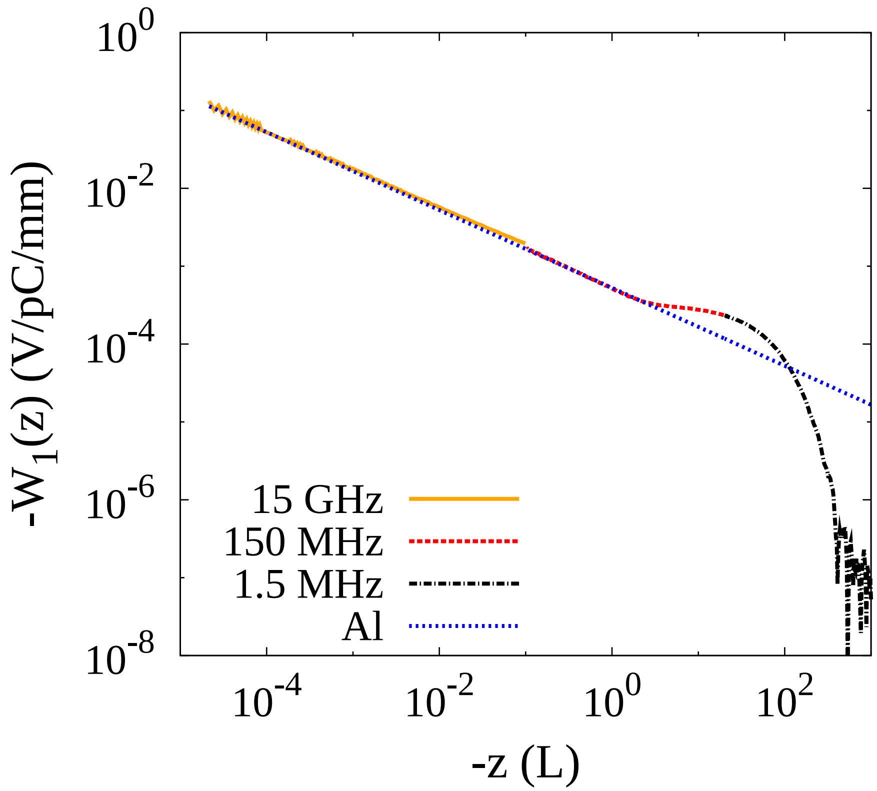
Other components of the electric and magnetic fields can be obtained using Maxwell's equations. The coefficients A_n and B_n are then obtained by matching these components at the interfaces between wall and vacuum.

In the frequency domain, the wake function is known as the impedance.



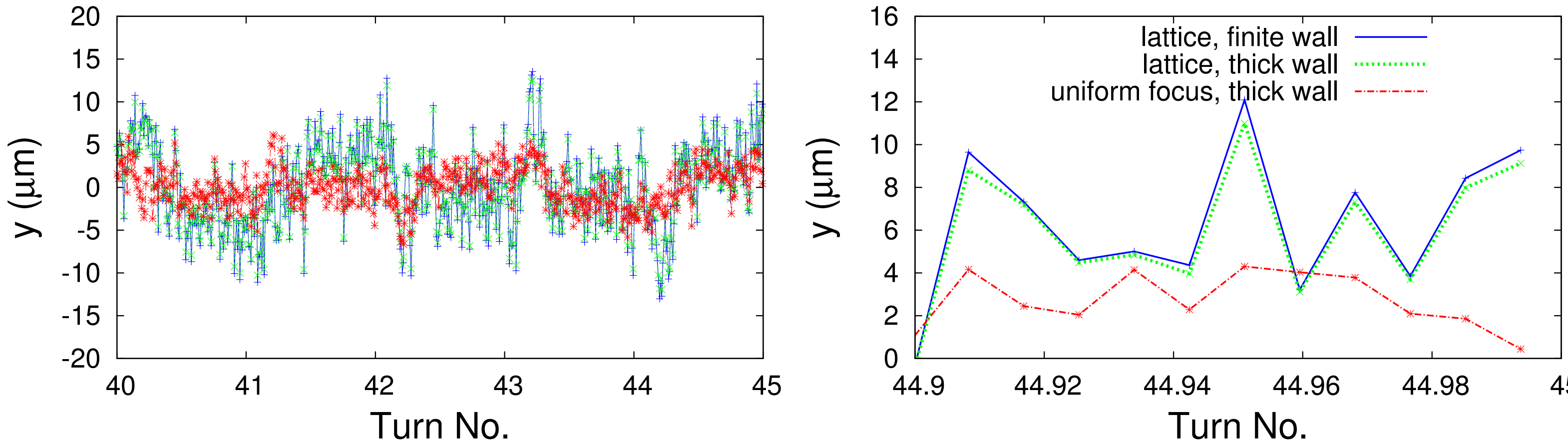
The impedance peaks at a frequency where the half wavelength in the beam pipe is equal to its thickness. This may be attributed to constructive interference in the wall.

The wake function is obtained from the impedance by Fourier transform.



The resulting peak between 10 and 100 turns corresponds to the "2 mm" peak in the impedance. Therefore, a thinner wall would give a stronger wake force at a smaller distance.

Compared with the thick wall case, the jitter is increased by a few percent. This suggests that the effect of the finite wall may be neglected if a quick jitter estimate is needed.



However, it should be noted that a thinner wall is expected to give a higher jitter, since it gives a stronger wake force at a smaller distance.

A bunch of electrons in the ring experiences forces from the focusing magnets, wake fields from leading bunches, and the feedback control system.

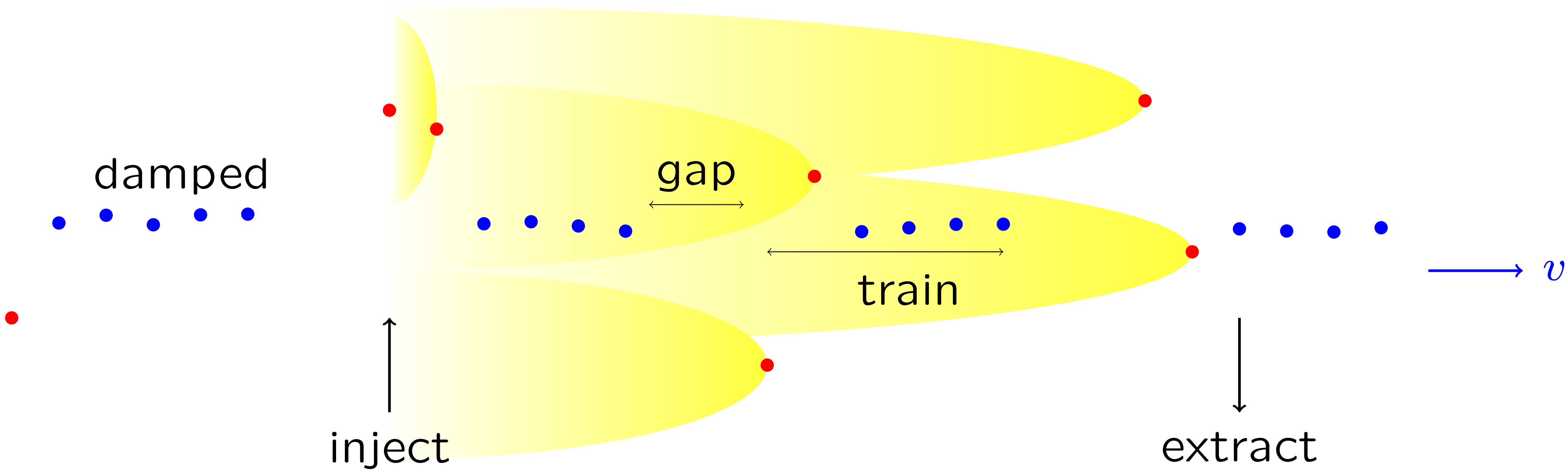
The equation of motion for the displacement of the m^{th} bunch is therefore given by

$$\frac{d^2 y_m}{dt^2} + 2\zeta \frac{dy_m}{dt} + K(t) y_m = -\frac{N r_0 c}{\gamma T_0} \sum_{n=1}^{\infty} W_1(-cn\tau) y_{m+n}(t - n\tau)$$

where ζ is the damping factor from feedback control, $K(t)$ is the focusing strength, $W_1(z)$ is the wake function, τ is the for light to traverse the bunch spacing, N the bunch population, T_0 the revolution time, and γ the electron energy.

The simulation can be carried out by numerical integration for any fill pattern of the bunches, as well as injection and extraction procedure.

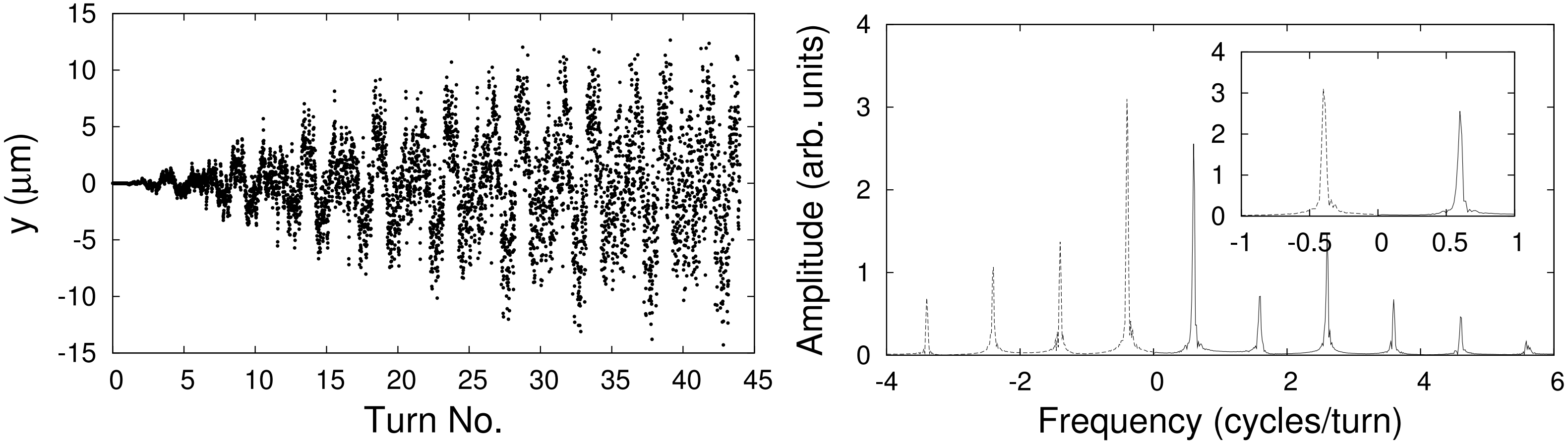
A typical way of filling the ring consists of trains of equally spaced bunches, separated by gaps for ion clearing. In the ILC damping ring, injection and extraction starts from the end of each train in order to minimise the disturbance due to the wake fields of injected bunches.



For the simulation, a typical fill pattern would be 45 bunches per train with gaps of 15 bunch spacing each, and a total of 116 trains in the ring.

The simulation is carried out using the lattice for the DCO2 damping ring, and a bunch population of 1.04×10^{10}. The rms of the injection offsets is assumed to correspond to 10% of the recommended betatron action for injected positrons.

Displacement of each bunch just before extraction is plotted.

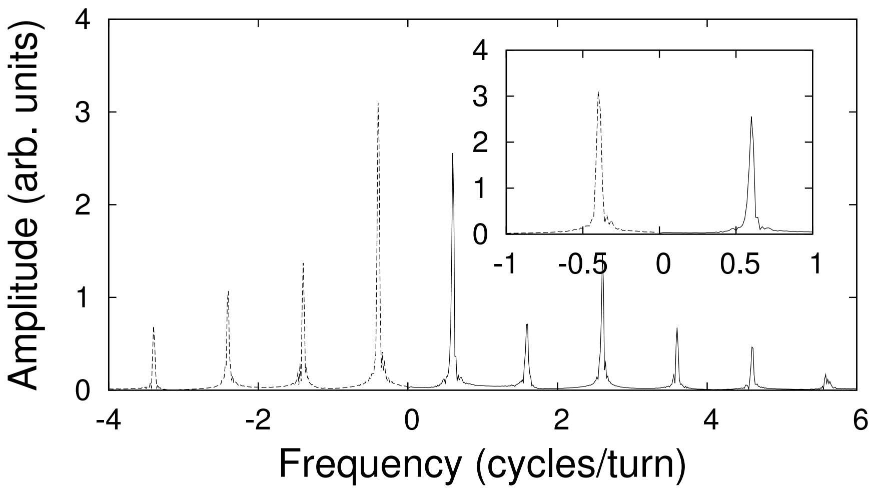


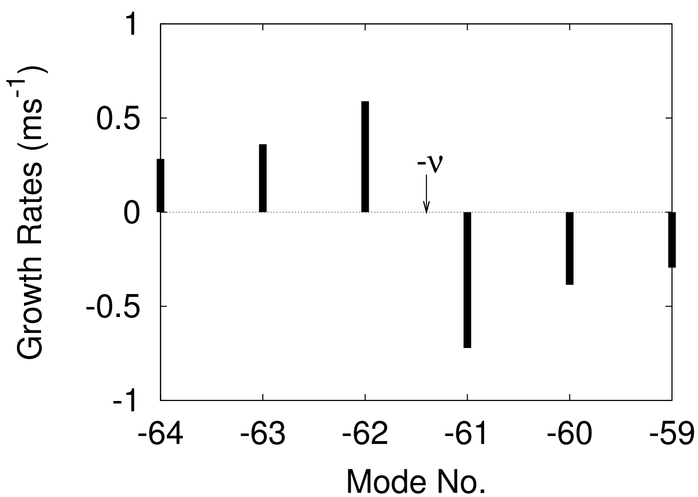
Comparison with the specified vertical emittance of 2 pm gives a jitter of 50%. If a tolerance of 10% is reasonable, this is too high and the positron action should be reduced.

Unexpected modulation in the jitter is also observed.

The peaks in jitter spectrum arise because the bunch displacements at the extraction location approximately repeat themselves every turn.

A Fourier transform of the displacements decompose them into components known as Fourier modes. An approximate formula is available for the growth rates of these modes.

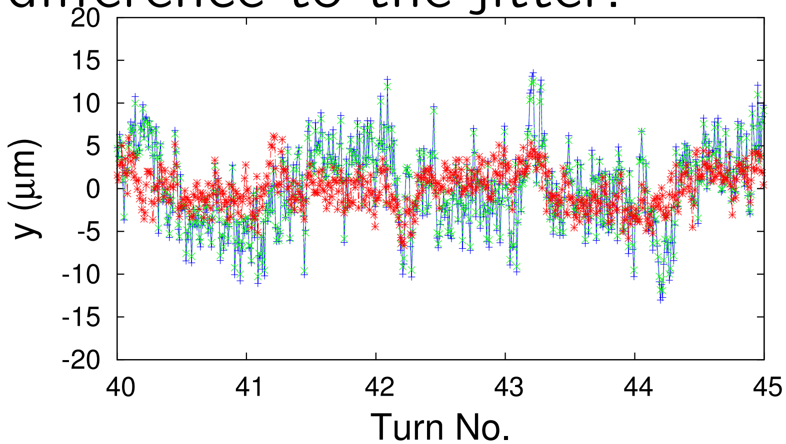


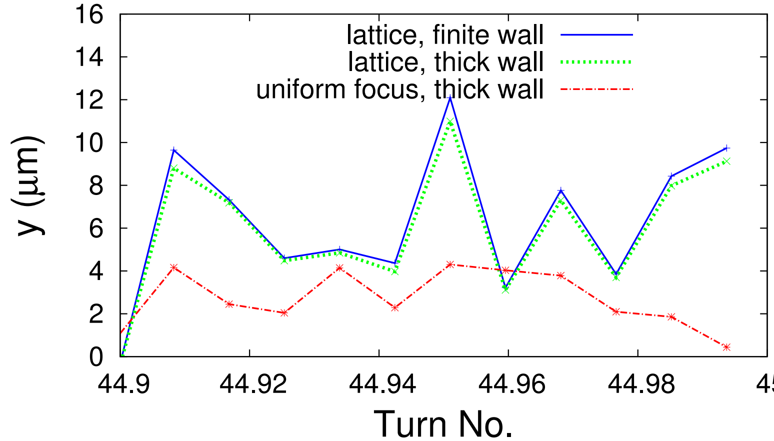


It can be shown analytically that the dominant jitter peaks correspond to Fourier modes with the highest growth rates.

An analytical solution for the growth rates of the unstable oscillations is available, if we assume that the focusing strengths around the lattice are completely smeared out and become uniform.

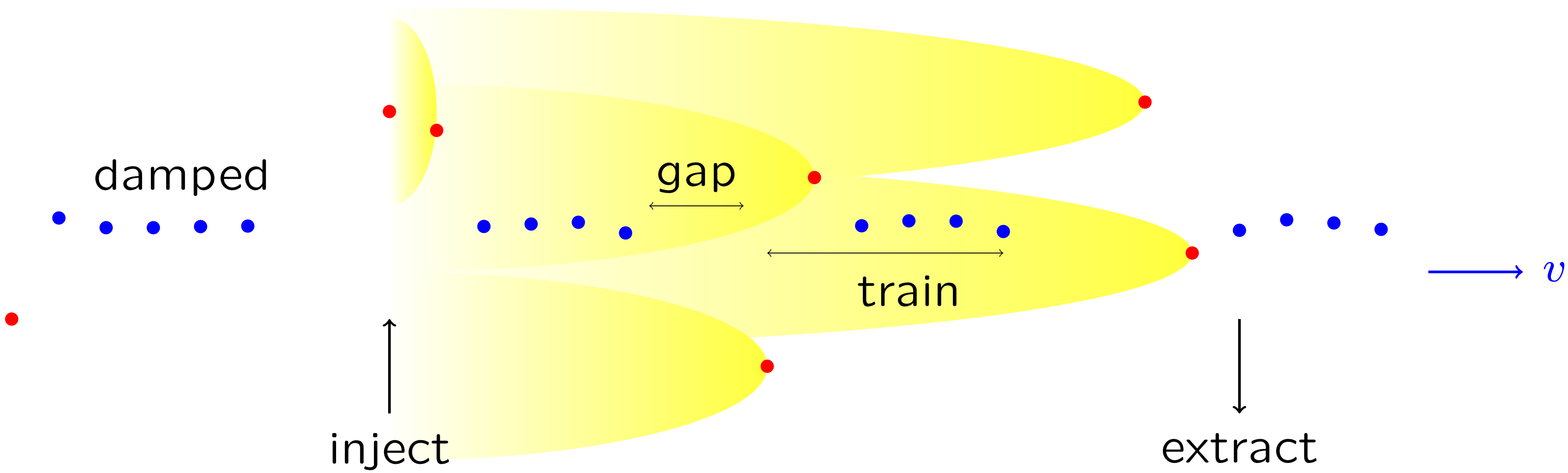
Simulation using the actual distrbution of focusing magnets around the lattice is very computationally intensive, so it would be of interest to check if the lattice does indeed make a difference to the jitter.





The figures above show clearly that the lattice gives rise to a jitter that is about 2 times larger than the case of uniform focusing. This confirms that it is important to take the lattice into account.

Injecting over 5000 random bunches and determining the disturbance on the damped bunches is a complex problem. It is therefore surprising that an analyic solution is in fact possible.



The solution relies mainly on three assumptions:

1. The bunches effectively travel at the speed of light.
2. Wake forces among the damped bunches are negligible.
3. Uniform focusing.

Within the turn n_t, the displacement of a damped bunch y_m is:

$$\frac{d^2 y_m}{dt^2} + 2\zeta \frac{dy_m}{dt} + \omega_\beta^2 y_m = Af(n_t T_0) e^{-(t-n_t T_0)/T_f} e^{-i\omega_\beta t}$$

On the left, ζ is the feedback damping factor, ω_β^2 is the averaged focusing strength. On the right, $Af(n_t T_0)$ is wake force at the start of turn n_t, T_f is feedback damping time, and T_0 the revolution time.

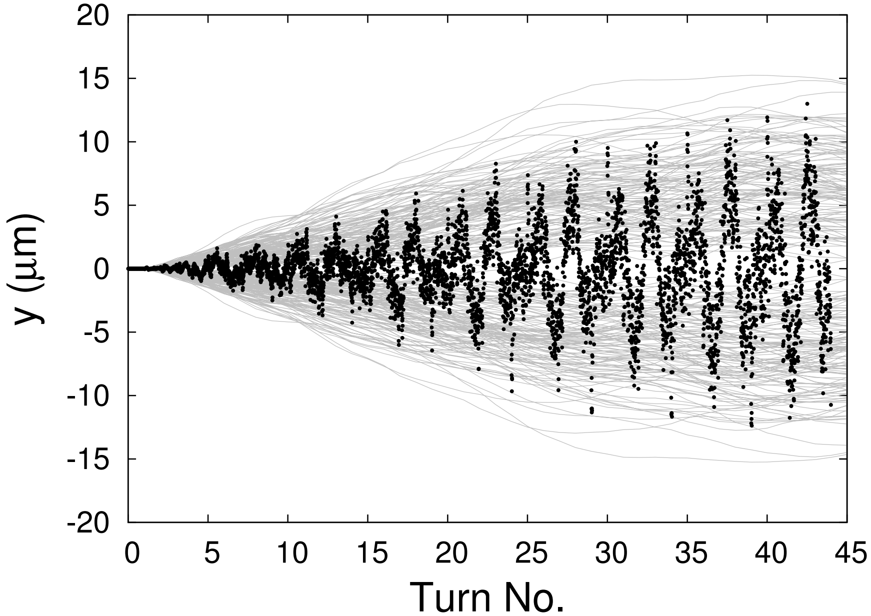
This can be solved analytically, and the solution is given by:

$$y_m(t) = A_{n_t} e^{(-\zeta + i\omega)t} + B_{n_t} e^{(-\zeta - i\omega)t} + p_{n_t} e^{(-\zeta - i\omega_\beta)t} \quad (1)$$

where $p_{n_t} = -a_{n_t} \exp(\zeta n_t T_0)/\zeta^2$ and $\omega = \sqrt{\omega_\beta^2 - \zeta^2}$.

A_{n_t} and B_{n_t} are unknown constants that can be obtained using the condition that the damped bunch has zero displacement and velocity initially.

For a simple estimate, it is sufficient to track only the bunches that are extracted in the final turn. The initial displacements of these bunches are likely to be close to those of the bunches that are extracted earlier.



The grey curves are the amplitudes of the oscillations of these bunches, obtained analytically. The distribution of these curves are very close to the distribution of black dots from simulation.

Conclusion

1. The lattice is shown to have a significant impact on the stability of bunces coupled by wake fields.

2. An analytic solution for the jitter is derived and shown to agree with simulation in the uniform focusing case.

3. A finite thickness of 2 mm of the damping ring wall is shown to have a small effect on the jitter.

4. The impedance is shown to peak when half wavelength in the wall is equal to its thickness. Thus a thinner wall would have a bigger impact on the jitter.

5. The jitter is significant compared with the specified emittance. The betatron action of positron would have to be reduced, or a control system installed to suppress the jittter.