The rapid calculation of synchrotron radiation from long undulator systems

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Abstract

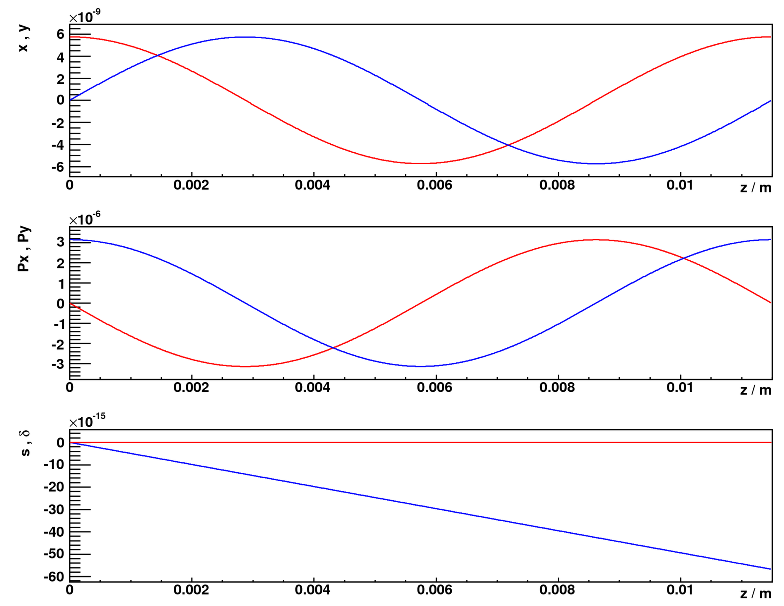
Recent designs for third-generation light sources call for undulator systems with a total length of greater than one hundred metres. Calculating the synchrotron radiation output from bunches of charged particles traversing such a system using numerical techniques takes an unfeasibly long time even on modern multi-node computer clusters. Analytical formulae (i.e. the Kincaid Equation) provide a more rapid solution for an idealised system necessarily fail to produce the non-ideal response which is under investigation. A new code is described which generates an analytic description of an arbitrary magnetic field and uses differential algebra and Lie methods to describe the particle dynamics in terms of series of transfer maps. The synchrotron radiation output can then be calculated using an arbitrarily large step size with no loss of accuracy in the trajectory. The code is easily adapted to perform parallel calculations on multi-core machines. Examples of the radiation output from several long magnet systems are described and the performance is assessed.

Particle tracking and synchrotron radiation

A numerical code has been developed to rapidly calculate the synchrotron radiation output from long undulator systems. To demonstrate the code the parameters for a helical undulator are taken from the baseline ILC design for the positron source, designed to produce 10 MeV photons:

B_0=0.84 T (on-axis field strength)
E_0=150 GeV (electron energy)
λ_0 = 0.0115 m (period length)
Section length=1.79 m (155 periods)

To track the particle through the magnet system a second-order symplectic integrator [3] is used. This technique is used to analytically integrate the Hamiltonian with respect to the dynamical variables x, P_x, y, P_y, s and δ, where s represents the longitudinal deviation, relative to a reference particle, and δ is the energy deviation. The other variables have their usual meaning. By integrating the particle through N integration steps (with step size h=magnetlength/N) , a set of N Lie maps are derived each of which can transport the particle from the entrance of the magnet to the position nh (n=0...N).



The plot above shows the evolution of the dynamical variables as an electron is tracked through one period of a helical undulator. Top: x (red) and y (blue). Centre: P_x (red) and P_y (blue). Bottom: δ (red) and s (blue). The longitudinal deviation, s, is used to calculated the phase slip between radiation emitted at different positions and its accurate computation is particularly critical. Also note that energy losses are not yet included in the model but will be added in the near future..

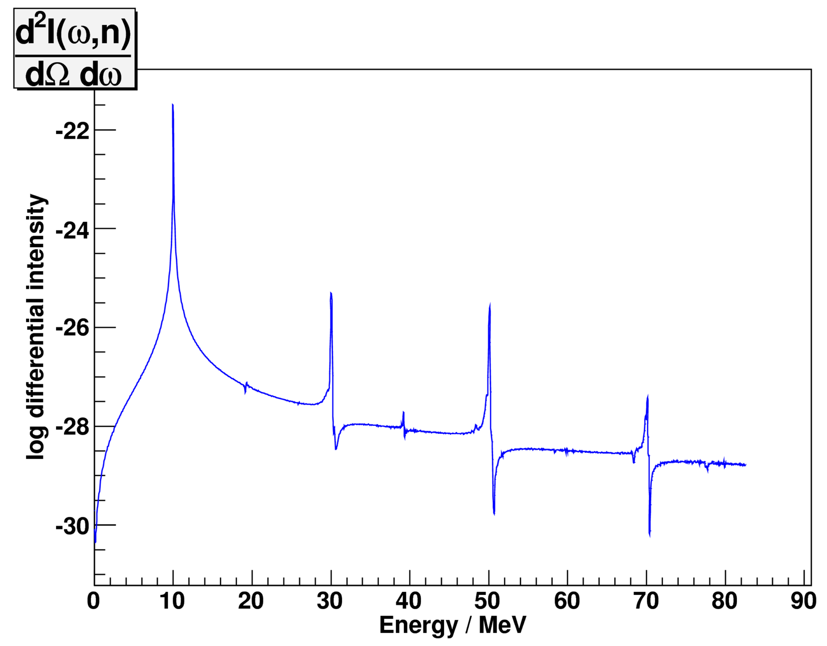
By using Lie maps to track the particle, once the Hamiltonian is integrated, an electron can be tracked through the undulator using arbitrarily large step size, calculating the syjnchrotron radiation emitted at each step. The electric field, at any observation point, due to an accelerated charge is given by:

$$\vec{E}(t) = \frac{e}{4\pi\epsilon_0 c}\frac{1}{(1-\vec{n}\cdot\vec{\beta})^3}\times\left(\frac{(1-|\vec{\beta}|^2)(\vec{n}-\vec{\beta})}{|\vec{R}|^2}+\frac{\vec{n}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{c|\vec{R}|}\right)_{RET}$$

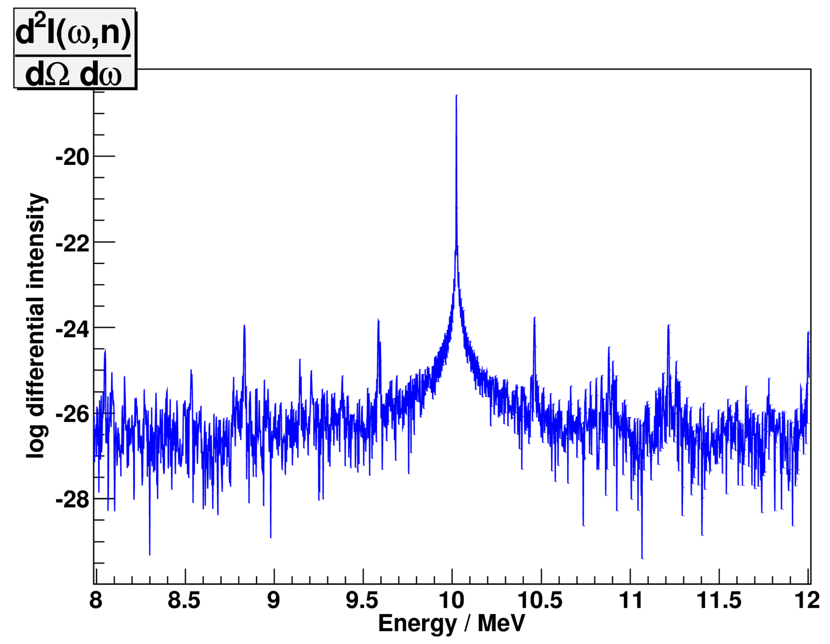
The elecric field is calculated at the retarded time, where, R is the vector from the charged particle to the observation point, with unit vector n, and the other symbols have their usual meaning. The dynamical variables β and its time derivative can be calculated directly from the phase space variables P_x, P_y and the magnetic field B. A Fourier transform of the resultant field gives the frequency spectrum of the observed radiation.

Synchrotron radiation from long helical undulators

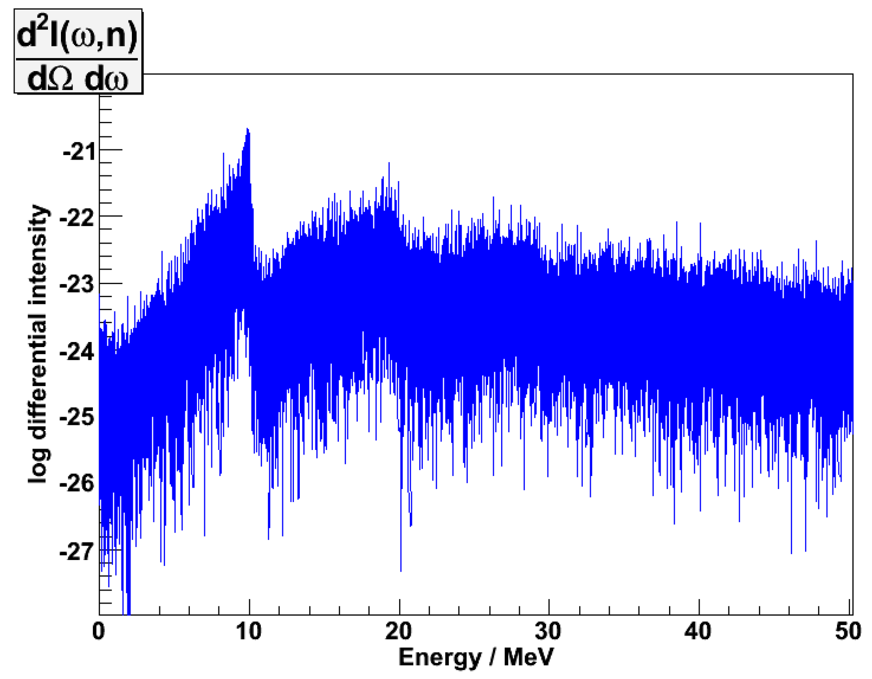
A 150 GeV electron was tracked through a helical undulator of periodicity 0.0115 m and on-axis field strength 0.86 T. The tracking was achieved by integrating the Hamiltonian over a single period using 10, 000 integration steps per period. The synchrotron radiation was calculated every tenth step. Results are shown for a 1.79 m long undulator (155 periods), a 53.7 m undulator (4650 periods) and a 53.7 m long undulator with 1% field errors. The field errors were defined such that the strength of the magnetic field fluctuates every half-period. The field was constructed such that over the length of the undulator the magnetic field is smooth and continuous everywhere.



The plot above shows the synchrotron radiation emitted into a 2 μm aperture 0.21 m from the end of a 1.79 m long helical undulator. The fundamental frequency is at 10 MeV with an intensity two orders of magnitude above the higher order harmonic frequencies.



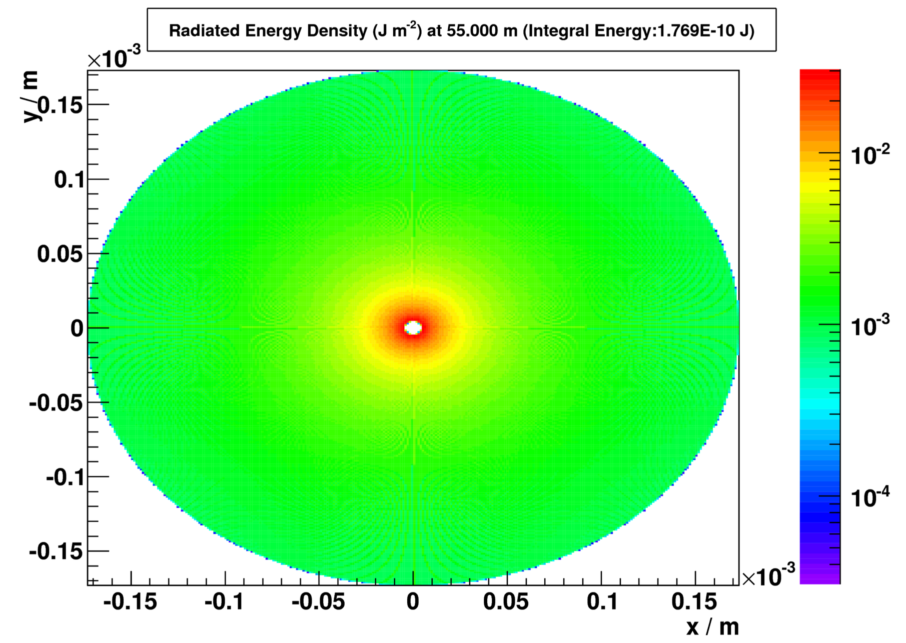
The plot above shows the synchrotron radiation emitted into a 2 μm aperture, 1.3 m from the end of a 53.7 m long helical undulator. The fundamental frequency is at 10 MeV with an intensity eight orders of magnitude above the background. The higher order harmonic frequencies are all heavily suppressed and are not shown in this plot.



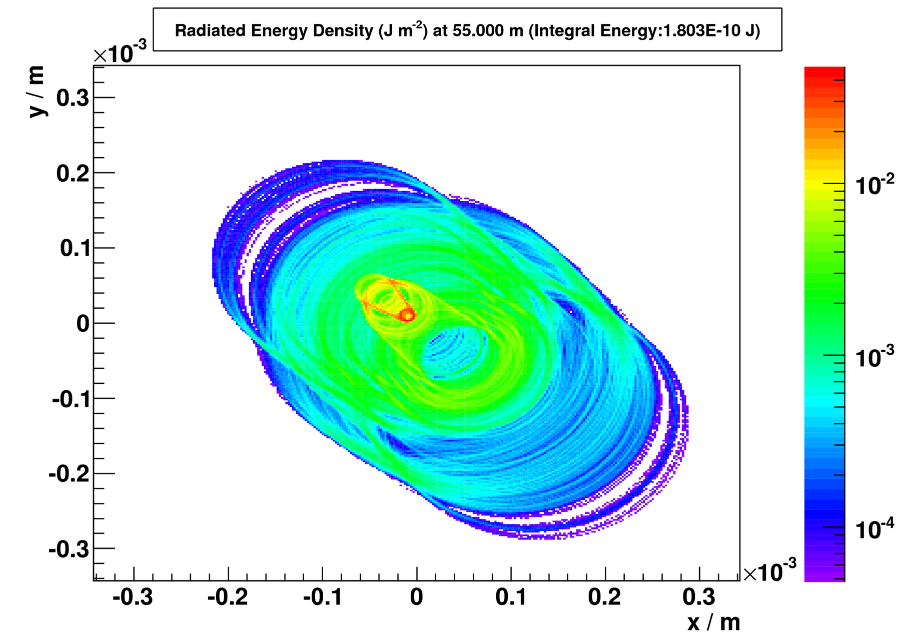
The plot above shows the synchrotron radiation emitted into a 2 μm aperture, 1.3 m from the end of a 53.7 m long helical undulator with 1% field errors. Using quadrupoles to steer the beam back on to the reference trajectory as it traverses the undulator, which would increase the peak intensity and give a cleaner spectrum.

The power output from long undulator systems

The power output from long undulator systems is calculated during the particle tracking. The radiation is emitted predominantly in the direction of th electron's velocity vector, into a cone with opening angle 1/gamma. In the following plots the power emitted by a single electron as it traverses a 53.7 m undulator (4,650 periods), observed on in a perpendicular plane 1.3 m from the exit of the undulator. In these plots the width of the cone of radiation is not taken into account



The plot above shows the energy emitted by an electron as it traverses a 53.7 m undulator. The small spot size (~0.3 mm) demonstrates the quality both of the particle tracking and the model magnetic field. Note that the total energy loss is around 0.7% (~ 1,104 MeV) of the electron's total energy.



Introducing field errors at the 1% level causes the electron to deviate from it's ideal trajectory, as can be inferred from the plot above.

Conclusion

The code described here is capable of rapidly calculating the synchrotron radiation emitted from long undulator systems. The calculation using an analytic or numerical field map for a 50 m undulator takes around five minutes to track the particle and calculate the dynamical variables. The calculation introducing random field errors takes longer, ~90 minutes, to calculate the dynamical variables, although this section of the code has not yet been optimised and much scope for improvement exists. Energy losses are not yet included in the model but will be implemented shortly. The use of Lie maps means each step of the calculation is independent so the code is ideally suited to parallel processing on multi-node clusters.

References

[1] M. Venturini and A.J. Dragt, ``Accurate computation of transfer maps from magnetic field data'', NIM A, 429(1999), 387-392

[2] D. Newton and A. Wolski, ``Fast, accurate calculation of dynamical maps from magnetic field data using generalised gradients'', Proc PAC09

[3] Y.Wu. E. Forest and D.S.Robin, ``Explicit higher order symplectic integrator for s-dependent magnetic field'', Phys. Rev. E, 6804(2001), 4,2

[4] J. D. Jackson, ``Classical Electrodynamics'', third edition, John Wiley \& Sons, 1999, Chapter 14

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