Modeling synchrotron radiation from realistic and ideal long undulator systems

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Abstract

An analytic description of the synchrotron radiation from electrons with short-period helical trajectories is given by the Kincaid equation [1]. A new code is under development which generates an analytical description of an arbitrary magnetic field with the ability to include non-linear and higher-order multipole (fringe field) components. The magnetic field map of a short-period undulator was modeled, including field errors, and its analytical field description has been used to compare the resulting synchrotron radiation output with that from electrons with an ideal trajectory. The results demonstrate how numerical inaccuracies in the particle tracking can effect the accuracy of the calculated synchrotron output. The affect of field errors on the synchrotron radiation from undulator systems is studied and the techniques to optimise the efficiency of the calculation are discussed.

Accurately tracking particles through long undulator systems with Lie maps

Kincaid [1] gives an analytic description of the magnetic field due to a single pair of bifilar current carrying wires which results in a helical magnetic field, and the orbit of a relativistic electron in such a field is also a helix. For this investigation an analytic helical undulator field was used. The Hamiltonian of an electron in such a field is then analytically integrated to produce a set of Lie maps[3,4]. A second order symplectic integrator [5] was used to produce the Lie maps, which gives an numerical accuracy of order h^2, where h is the size of the integration step. Once the Lie maps are produced a particle can be tracked through the undulator with any step size >= h, allowing rapid particle tracking.

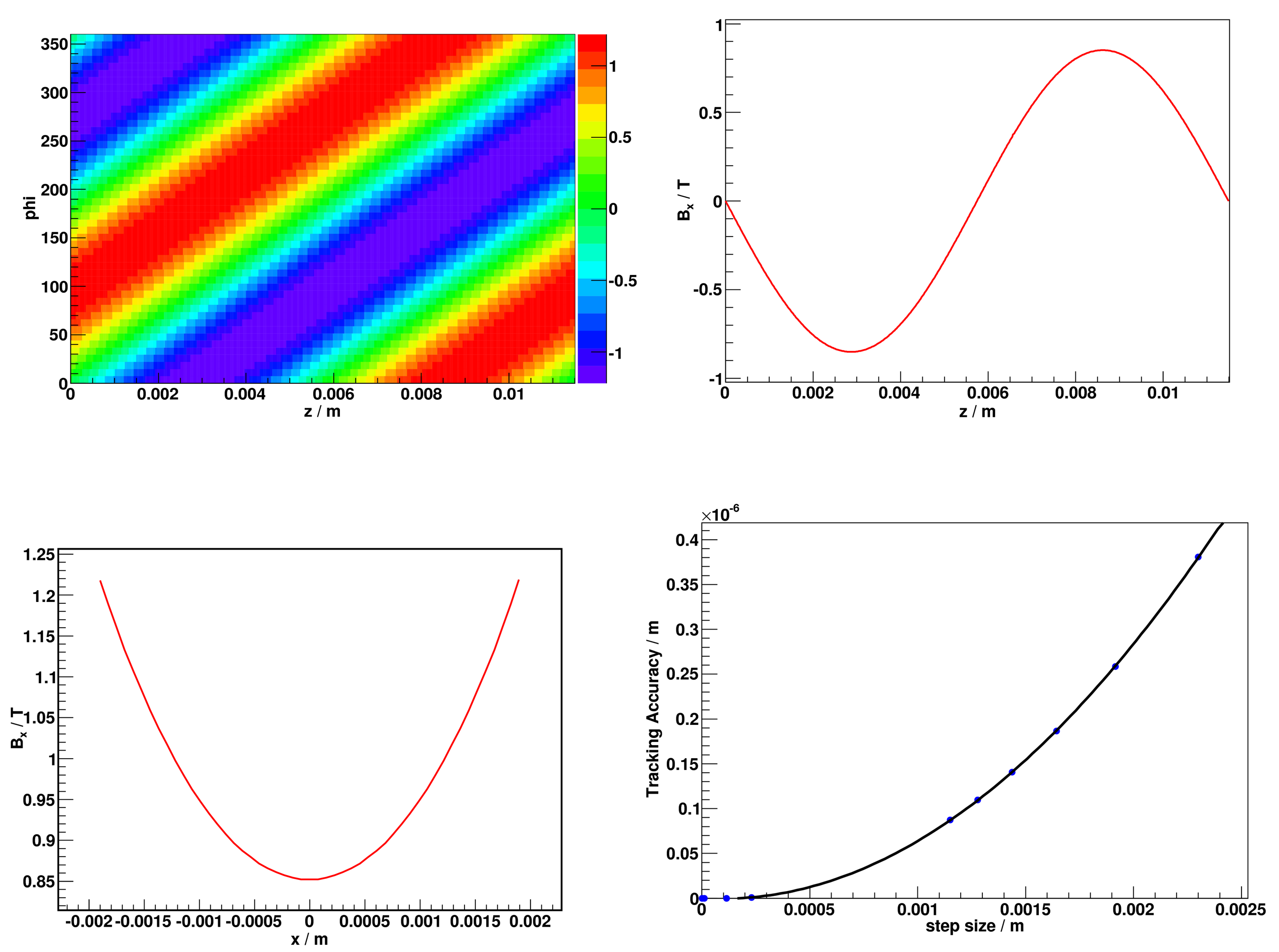
The parameters of the helical undulator are taken from the baseline ILC design for the positron source, designed to produce 10 MeV photons:

B_0=0.84 T (on-axis field strength)

E_0=150 GeV (electron energy)

λ_0 = 0.0115 m (period length)

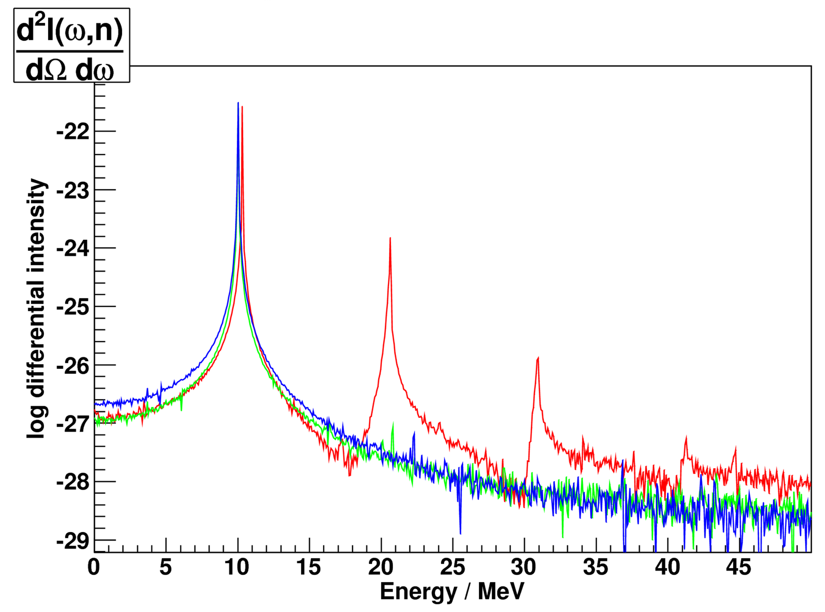
Section length=1.79 m (155 periods)



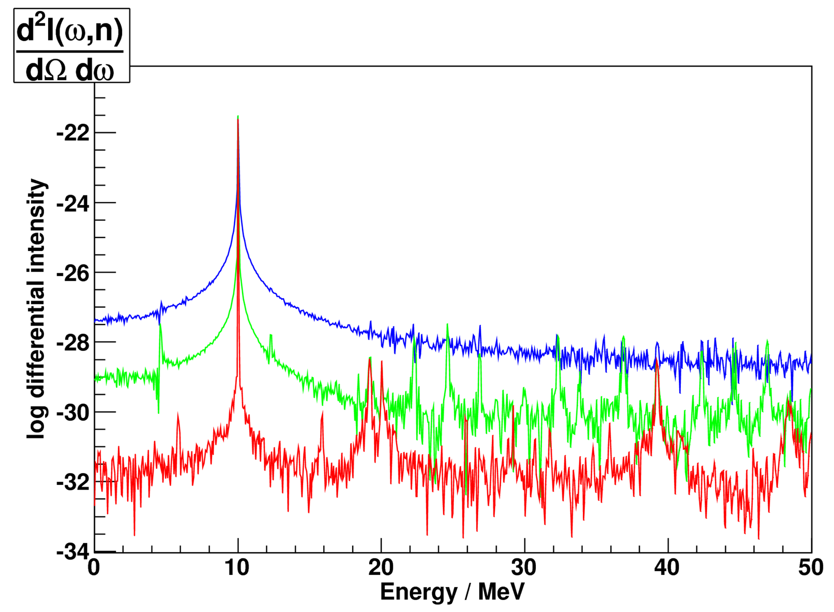
The radial field of the helical undulator is shown in the plot top left. Top right: the x-component of the magnetic field, on-axis, as a function of the period length (z). Bottom left: the x-component of the magnetic field as a function of the x-coordinate. Note that the field decreases towards the centre of the undulator, but close to the axis (x=0) the field is approximately constant at 0.86 Tesla. Bottom right: The accuracy achieved when integrating the equations of motion for an electron in the helical field as a function of the step size used on the integration. 155 periods were used to calculate the accuracy. The radial distance of the electron from the undulator axis at the exit of the undulator was compared to the analytic trajectory to produce this plot. The plot clearly shows the order h^2 dependency on the step size.

Synchrotron radiation from helical undulator systems

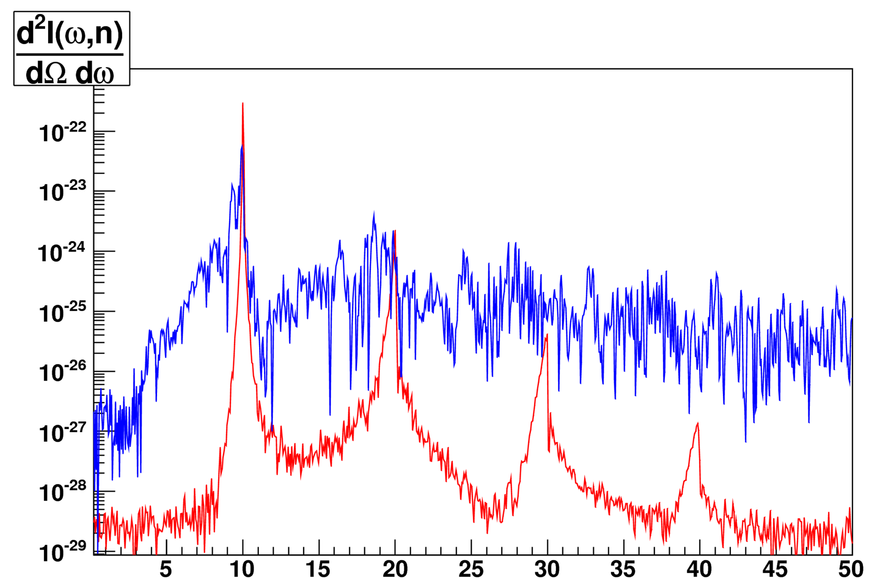
Calculating the synchrotron radiation emitted from long undulator systems is a computationally expensive process. The radiation from a relativistic electrom in an undulator is emitted in the direction of the particles velocity vector into a cone with an opening angle of 1/γ, around 3.4 x 10^{-6} radians in this example. At an observation point ahead of the electron the 'spotlight' of radiation will flash across the observation point as the electron swerves in the magnetic field, and given the narrow opening angle it can be seen that a high degree of accuracy is required when calculating the electrons instantaneous position. The rate at which the synchrotron radiation is emitted is also important. Obviously this calculation cannot be carried out more frequently than the number of tracking steps, but if the same spectra can be achieved, calculating the radiation every 10^{th} step, huge time savings can be made.



The plot above shows the synchrotron radiation (observed on-axis) emitted by an electron, calculated at 10 points per period as the electron traversed a 155 period undulator for three cases, tracking the electron at 10 (red), 100 (green) and 1000 (blue) steps per period. At 10 steps per period, the lack of accuracy in the tracking gives rise to the harmonics at multiples of 10 MeV.



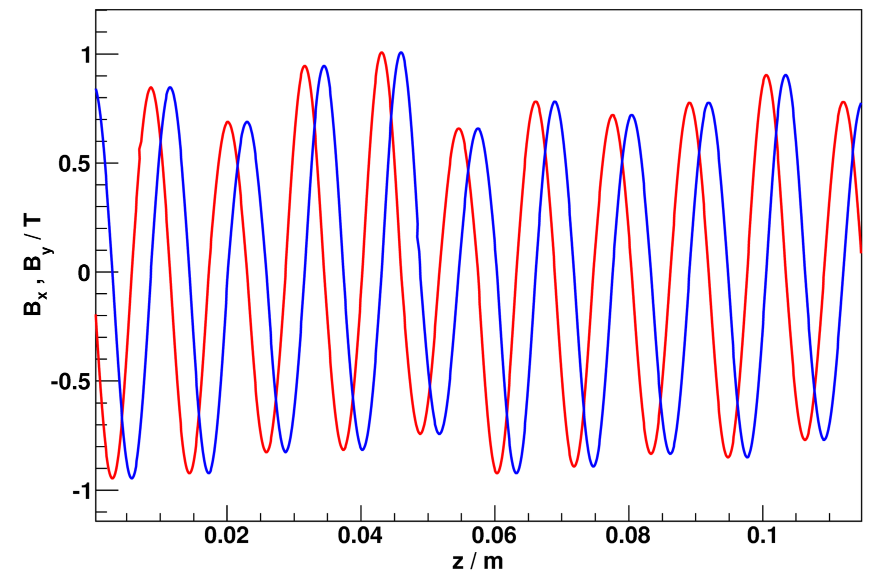
The plot above shows the synchrotron radiation (observed on-axis) emitted as an electron traverses a 155 period undulator. The particle tracking was calculated using 1000 steps per period, and the radiation was calculated at 10 (blue), 100 (green) and 1000 (red) steps per period. Sampling the radiation at finer scales gives more detail in the spectrum – in the lowest spectrum note the harmonics present at an intensity 10 orders of magnitude below the peak intensity.



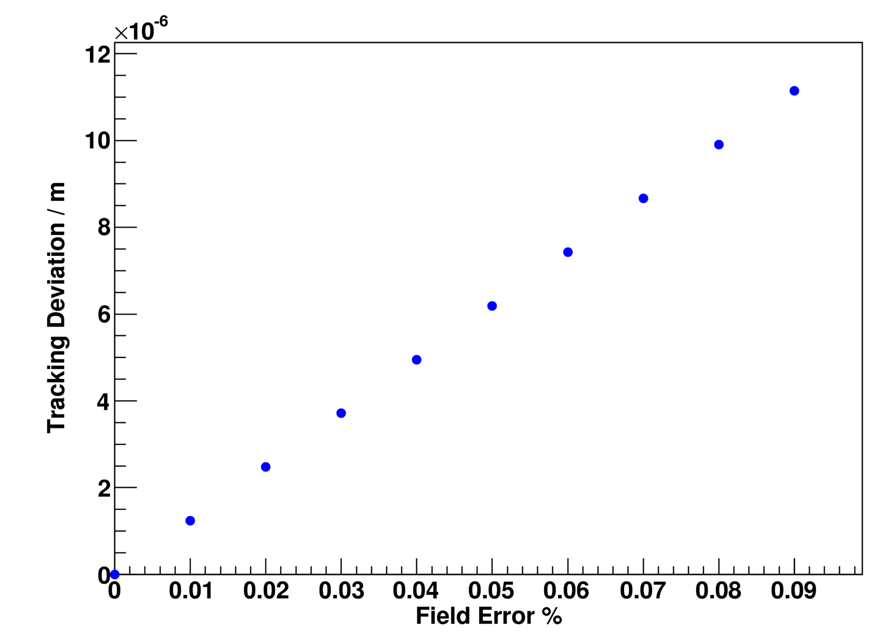
The plot above shows the synchrotron radiation (observed on-axis) emitted as an electron traverses a 155 period undulator. The particle tracking was calculated using 1000 steps per period, and the radiation was calculated at 100 points per period. Field errors were introduced to the undulators magnetic field with a magnitude of 0.1 % (red) and 1% (blue) of the total field strength. The presence of field errors in the magnetic field dominates the quality of the calculated intensity spectrum.

Realistic undulator systems with field errors

Field errors will be present in any real undulator system and the code described here can include these errors in the analytic description of the magnetic field. In this example the strength of the magnetic field is varied each half-period in such a way that over the length of the undulator the field is smooth and continuous everywhere.



The plot above shows the x and y components of the magnetic field in a helical undulator with field errors. For the purposes of the illustration 10 % field errors are used. The effect of field errors on the trajectory of an electron are shown in the plot below. Here an electron was tracked through 155 periods with field errors ranging from 0 to 10 %. The deviation of the electron from an analytical trajectory without field errors was calculated. This plot illustrates that field errors will be the major source of discrepancies between synchrotron radiation spectra calculated using ideal and realistic undulators, and must be included in any realistic calculation..



Conclusion

Calculating the synchrotron radiation output from long undulators is a computationally expensive process. By understanding how both the numerical tracking and the sampling of the emitted radiation effect the output spectrum, this process can be expedited. The inclusion of field errors on the calculated output has been shown to have a detrimental affect and the conclusion must be that such errors have to be included in the undulator model if a realistic output spectrum is to be achieved. All the simulations referred to in this paper model a 1.79 m section of the ILC helical undulator and although the ILC design report calls for an undulator of total length 147 m, the techniques used in this paper to estimate the accuracy of the output spectrum will be relevant for much longer undulators. It is expected that for longer undulators the optimum number of steps to track the particles and sample the synchrotron radiation will need to be reassessed. The application of this code to longer (~ 50 m) undulators is reported in reference [2].

References

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