THE RAPID CALCULATION OF SYNCHROTRON RADIATION OUTPUT FROM LONG UNDULATOR SYSTEMS

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Abstract

Recent designs for third generation light sources commonly call for undulator systems with a total length of several hundreds of metres. Calculating the synchrotron output from bunches of charged particles traversing such a system using numerical techniques takes an unfeasibly long time even on modern multi-node computer clusters. Analytical formulae (i.e. the Kincaid Equation) provide a more rapid solution for an idealised system but necessarily fail to produce the non-ideal response which is under investigation. A new code is described which generates an analytic description of an arbitrary magnetic field and uses differential algebra and Lie methods to describe the particle dynamics in terms of series of transfer maps. The synchrotron output can then be calculated using arbitrarily large step size with no loss of accuracy in the trajectory. The code is easily adapted to perform parallel calculations on multi-core machines. Examples of the radiation output from several long magnet systems are described and the performance is assessed.

INTRODUCTION

A numerical code has been developed to rapidly calculate the synchrotron output from long undulator systems. In developing the code three main requirements have been considered: 1) the code must track particles through long high-field magnets to a high degree of accuracy. 2) The dynamical variables β and $\dot{\beta}$ (required to calculate the synchrotron output) and their evolution through the magnet system must be calculated rapidly and accurately, and 3)The modelling of the magnetic field must be flexible, allowing analytic field descriptions or numerical field maps with the option of incorporating field errors.Numerical field maps can give a more realistic description of the field than simple analytic models, but with the disadvantage that to track particles through them is often a laborious process. In reference [1] a method of computing an analytic description of an arbitrary field is described. This technique describes a numerical field in analytical terms of the multipole modes and has been implemented in this code [2]. To demonstrate these techniques the results from tracking an electron through a long helical undulator system are presented here. Two descriptions of the field are used, an analytical description, and a numerical field map, and in both cases parameters similar to the design of the ILC positron source are adopted: $B_0 = 0.86$ T, the energy of the electrons $E_0 = 150$ GeV and a period length of 0.0115 m, with 155 periods making up each section of the undulator.

CODE DESCRIPTION

Particle Tracking

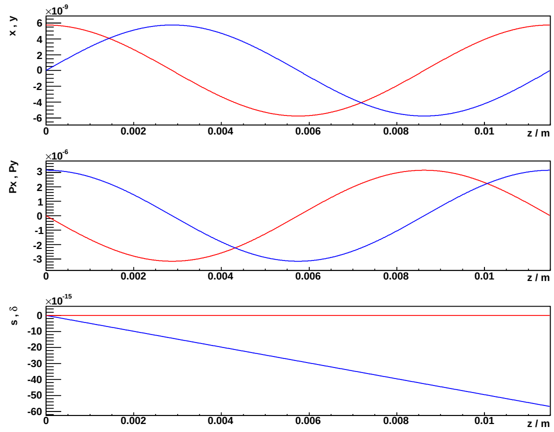

Figure 1: Particle tracking through one period of a helical undulator described by a numerical field map. Top: x (red) and y (blue). Middle: P_x (red) and P_y (blue). Bottom: δ (red) and s (blue) - note that energy losses are not yet included in the model.

To track the particle through the magnet system a second-order symplectic integrator [3] is used. This technique is used to analytically integrate the Hamiltonian with respect to the dynamical variables x, P_x, y, P_y, s and δ where s represents the longitudinal deviation, relative to a reference particle, and δ is the energy deviation. The other variables have their usual meaning. By integrating the particle through N integration steps (with step size h=magnetlength/N) , a set of N Lie maps are derived each of which can transport the particle from the entrance of the magnet to the position nh ($n = 0...N$). The use of Lie maps means that once the initial integration is completed, a particle with any arbitrary set of initial phase-space coordinates can be rapidly transported to any region within the magnet in one step. Figure 1 shows the evolution of the phase space variables through one period of the field map undulator. Any radiation emitted by the particle will be beamed in the direction of the velocity vector into a cone with opening angle of $1/\gamma$, $\sim 3.4 \times 10^{-6}$ radians in this example. For an observation point many tens of metres ahead of the particle, the 'spotlight' of radiation will flash across the point as the electron swerves in the magnetic field, and given the high degree of collimation at this energy it can be seen that a correspondingly high degree of accuracy is required when calculating the electrons instantaneous position. As the radiation is emitted in the forwards direc-

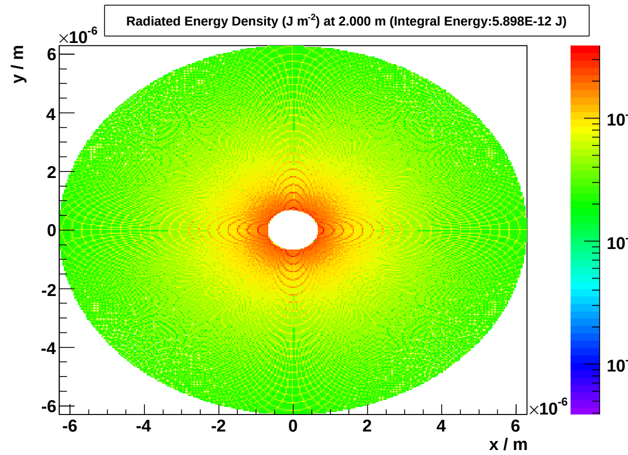

Figure 2: The energy emitted from a 1.79 m undulator by a single electron, onto a plane 0.21 m from the end of the undulator.

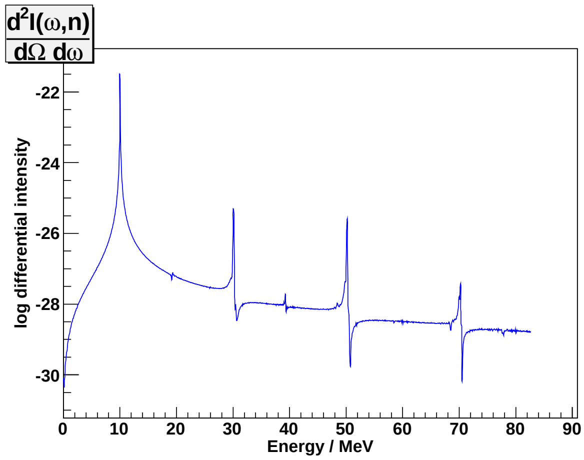

Figure 3: The energy spectrum of radiation emitted into a 2 μm aperture 0.21 m from the end a a 1.79 m long undulator.

tion it constructively interferes at a frequency determined by the undulator periodicity and the angle of observation. The dynamical variable s is used to calculate the phase-slip between the emitted radiation at different positions through the undulator and is therefore particularly critical.

Synchrotron Calculation

The electric field induced by an accelerated charge is given by:

$$\vec{E}(t) = \frac{e}{4\pi\epsilon_0 c}\frac{1}{(1-\vec{n}\cdot\vec{\beta})^3} \times \left(\frac{(1-|\vec{\beta}|^2)(\vec{n}-\vec{\beta})}{|\vec{R}|^2} + \frac{\vec{n}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{c|\vec{R}|}\right)_{RET} \tag{1}$$

where, $\vec{R}$ is the vector from the charged particle to the observation point, with unit vector $\vec{n}$, and the other symbols have their usual meaning. A Fourier transform of the resultant field gives the frequency spectrum [4].

The dynamical variable β can be extracted from the phase-space variables P_x, P_y and the acceleration variable $\dot{\beta}$ can be found from the cross-product of β and the magnetic field. All the information required to calculate the synchrotron emission is present in the Lie maps and the field description.

RESULTS

Single Undulator Section 1.79 m

A 150 GeV electron was tracked through a 1.79 metre long (155 periods) helical undulator of periodicity 0.0115 m and on-axis field strength of 0.86 T, described by a numerical field map. The tracking was achieved by integrating the Hamiltonian over a single period using 10,000 integration steps giving the equivalent of 1.55 million integration steps over the length of the undulator. The synchrotron radiation was calculated every tenth step. The mean distance of the particle from the undulator axis changed by only 2×10^{-8} m, indicating both the quality of the field map and the accuracy of the tracking algorithm. The energy emitted 0.21 m from the end of the undulator (2m from the entrance) is plotted in figure 2. The accuracy of the tracking is reflected in the small spot size of the emitted radiation ($< 10\,\mu$m). The frequency spectrum of radiation incident on a 2 μm aperture, on-axis, is shown in figure 3. Note the strong fundamental line at 10.0 MeV, 3 orders of magnitude above the background and two orders of magnitude above the higher order harmonics present at integer multiples of 10 MeV, although the even harmonics are heavily suppressed. These harmonics are due to slight alterations in the observation angle as the electron moves

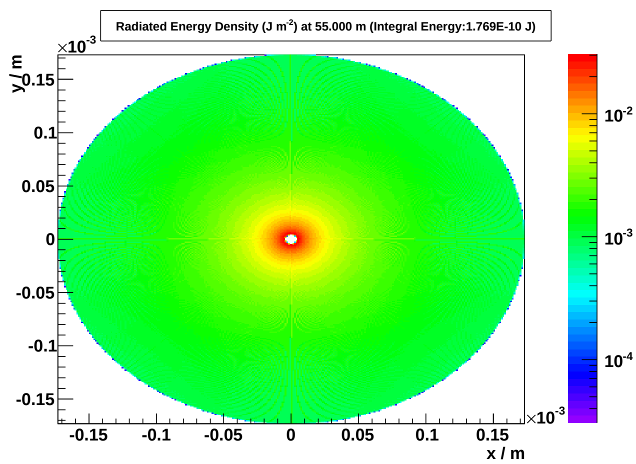

Figure 4: The energy emitted from a 53.7 m undulator by a single electron, onto a plane 1.3 m from the end of the undulator.

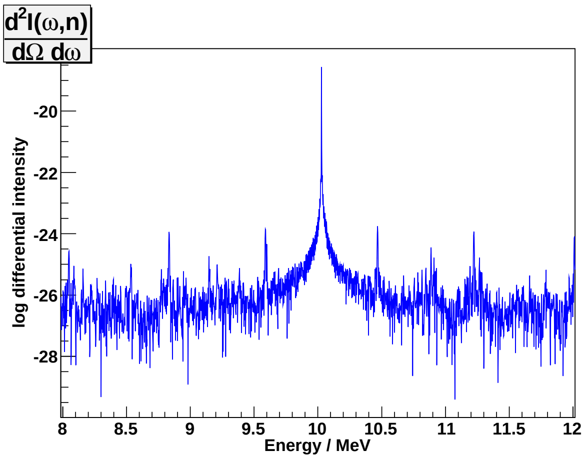

Figure 5: The energy spectrum of radiation emitted into a 2 μm aperture 1.3 m from the end a a 53.7 m long undulator.

in the magnetic field.

Multiple Undulator Sections 53.7 m

The energy emitted from the end of a 53.7 m long undulator (4,650 period, described by a numerical field map) is shown in figure 4 with the resulting energy spectrum in figure 5, observed in an on-axis 2 μm aperture. Both plots were calculated 1.3 m from the exit of the undulator. The size of the emission beam is ~ 0.3 mm and the peak of the energy spectrum at 10 MeV is 8 orders of magnitude higher then the background emission. The harmonics are all heavily suppressed and are not shown in this plot.

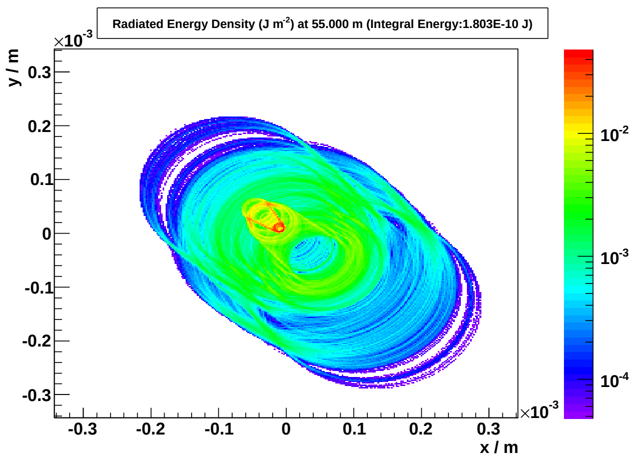

Figure 6: The energy emitted from a 53.7 m undulator by a single electron, onto a plane 1.3 m from the end of the undulator. Field errors (1%) are included in the undulator description

Multiple Undulator Sections 53.7 m with field errors

Tracking particles through 'ideal undulators' whether described by an analytic expression or a numerical field map fails to capture the reality of magnet design and construction. Field errors will be present in any 'real' undulator system and the code described here can include these errors in an analytical description of the magnetic field. As an example the 53.7 m undulator described above was adjusted such that the total field strength fluctuates every half period. The size of the fluctuation was 1% of the mean field ($B_0 = 0.86$ T) and the field was constructed such that over the length of the undulator the field is smooth and continuous everywhere. Figure 6 shows the energy emitted from 1.3 m from the end of such an undulator. Although the total spot size is similar to that from the ideal undulator, the movement of the electron in the random field can be clearly inferred. The total energy emitted is 1,104 MeV (1.769×10^{-10} J, around 0.7 % of the electron energy). The energy spectrum from this particle (not shown) still shows a peak at 10 MeV, though only ~ 5 times higher than the background and the peak width is of the order of several MeV. Focusing the beam, by resetting the particles position and momenta to its initial values every two undulator lengths (3.58 m) results in an energy distribution similar to that of figure 4 with an energy spectrum similar to that of figure 5.

CONCLUSION AND FURTHER WORK

The code described here is capable of rapidly calculating the synchrotron radiation emitted from long undulator systems. The calculation using an analytic or numerical field map for a 50 m undulator takes $\sim$ 5 minutes to track the particle and calculate the dynamical variables. The calculation introducing random field errors takes longer,~ 90 minutes to calculate the dynamical variables, although this section of the code has not yet been optimised and much scope for improvement exists. Energy losses are not yet included in the model but will be implemented shortly. The use of Lie maps means each step of the calculation is independent so the code is ideally suited to parallel processing on multi node clusters.

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