

MODELLING SYNCHROTRON RADIATION FROM REALISTIC AND IDEAL LONG UNDULATOR SYSTEMS

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Abstract

An analytic description of the synchrotron radiation from electrons with short-period helical trajectories is given by the Kincaid equation [1]. A new code is under development which generates an analytical description of an arbitrary magnetic field with the ability to include non-linear and higher-order multipole (fringe field) components. The magnetic field map of a short-period undulator was modelled, including field errors, and its analytical field description has been used to compare the resulting synchrotron radiation output with that from electrons with an ideal trajectory. The results demonstrate how numerical inaccuracies in the particle tracking can affect the accuracy of the calculated synchrotron output. The affect of field errors on the synchrotron radiation from undulator systems is studied and the techniques to optimise the efficiency of the calculation are discussed.

INTRODUCTION

A numerical code has been developed to rapidly calculate the synchrotron output from long undulator systems [2]. The code makes use of analytic Lie maps to perform the tracking of the particle trajectory which allows tracking through the magnet system in an arbitrarily large step size with no loss of accuracy [3, 4]. The frequency with which the synchrotron radiation is calculated then becomes the most important factor in the accuracy of the calculated frequency spectrum. In this paper the effect of varying the step size (both for particle tracking and calculating the radiation) on the resultant spectrum is investigated. The effect of field errors is also reported.

IDEAL UNDULATOR SYSTEMS

The transverse on-axis magnetic field due to a single pair of current carrying wires wound in a bifilar helix, with characteristic periodicity λ_0 , can be calculated analytically [1] and results in a helical magnetic field. The orbit of a relativistic electron in such a field is also a helix with radius $r = \lambda_0^2/2\pi\rho$, where ρ is the cyclotron radius. For this investigation an ideal helical undulator field was used as an input for the synchrotron code. The Hamiltonian of an electron in such a field is then analytically integrated to produce a set of Lie maps, capable of transporting the electron through the undulator system. The accuracy of the tracking as a function of the integration step size and its effect on the final frequency spectrum of synchrotron radiation is then assessed. Throughout this paper the field parameters were taken from the baseline ILC design for the positron

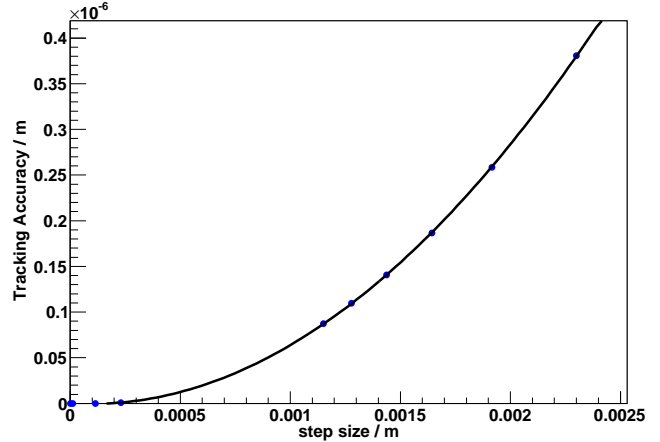


Figure 1: Tracking an electron through a 155 period undulator. The electrons deviation from an analytic trajectory at the undulator exit is plotted as a function of the step size used to integrate the equations of motion.

source: $B_0 = 0.86$ T, the energy of the electrons $E_0 = 150$ GeV and a period length of 0.0115 m, with 155 periods making up each section of the undulator (length per period=1.79 m). These parameters were chosen to maximise the production of 10 MeV photons in the synchrotron energy spectrum.

Tracking Accuracy

A second order symplectic integrator [5] was used to produce the Lie maps. Theoretically the accuracy of such an integrator is of order h^2 where h is the step size. The Hamiltonian was integrated over 155 periods of the undulator with varying step sizes (from 5 steps per 0.0115 m period to 10,000 steps per period). Electrons were then tracked through this system and their distance from the axis at the exit of the undulator was compared to the radius of the ideal electron trajectory. The results are shown in figure 1 where the order h^2 dependence on step size can be clearly seen. Using 10,000 steps per period, the deviation from the ideal trajectory is better than 1 nm over 155 periods with total length 1.79 m.

Tracking Accuracy and Synchrotron Radiation

As the electron progresses through the undulator, synchrotron radiation is emitted in the direction of the particles velocity vector into a cone with an opening angle of $1/\gamma$ (3.4×10^{-6} radians in this example) At an observation point ahead of the electron the 'spotlight' of radiation will

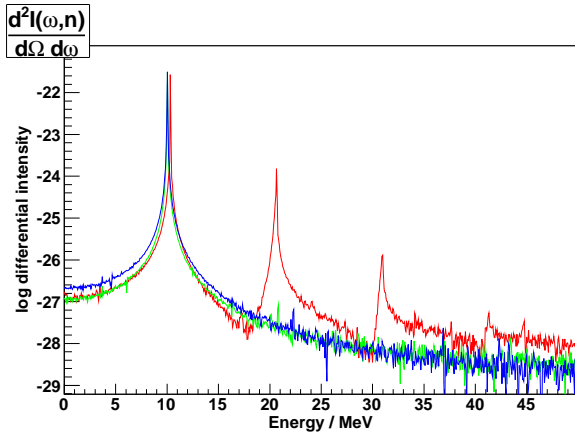


Figure 2: The energy spectra calculated when tracking an electron through a 1255 period undulator. In each case the radiation was calculated at 10 points per period, although the particle tracking was performed using 10 (red), 100 (green) and 1000 (blue) steps per period. Note that by using Lie maps, each of the 3 calculations took the same amount of time despite the increase in tracking accuracy.

flash across the observation point as the electron swerves in the magnetic field, and given the narrow opening angle at this energy it can be seen that a correspondingly high degree of accuracy is required when calculating the electrons instantaneous position. To investigate the effect of the tracking accuracy on the output radiation spectrum, three sets of Lie maps were calculated using 10,100 and 1000 steps per period. The synchrotron radiation emitted when an electron passes through 155 periods of the undulator, observed on-axis 0.21 m from the undulator exit was calculated at 10 points per period for each set of Lie maps. The results are shown in figure 2. With a tracking rate of 10 steps per period it can be seen that the peak of

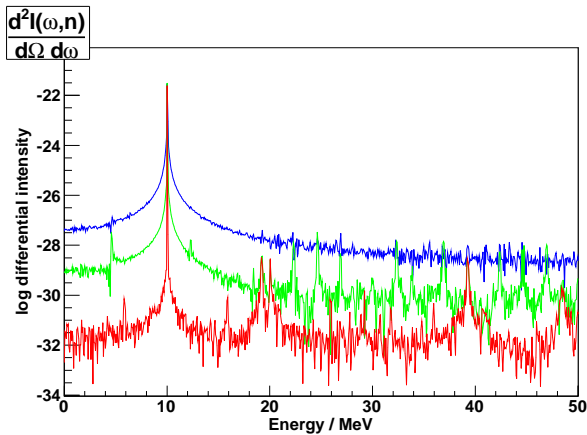


Figure 3: The energy spectra observed from an electron passing through a 155 period undulator. The emitted radiation was calculated at a rate of 10 (blue), 100 (green) and 1000 (red) steps per period.

the energy spectrum is shifted to the the right of the target 10 MeV position. Harmonics at 20 and 30 MeV are also present. These harmonics arise from alterations in the angle between the observation point and the electrons trajectory and observed from an on-axis position, it is expected th no harmonics will be generated [6]. The presence of the harmonics suggest that the calculated trajectory is deviating from the ideal trajectory. Using tracking rates of 100 and 1000 steps per period both calculations reproduce the expected spectrum.

The Synchrotron Calculation

Figure 2 assess the accuracy of the particle tracking on the output spectrum. The effect of the sampling rate at which the synchrotron radiation is calculated is shown in figure 3. In this plot the tracking was performed using 10,000 steps per period, but the emitted radiation was calculated at 10,100 and 1000 points per period. Figure 3 shows that all three spectra agree with regard to the peak position and intensity of the energy spectrum, although finer sampling of the radiation gives finer detail in the energy spectrum - note that sampling the radiation at 1000 points per period reveals the harmonics at an intensity over ten orders of magnitude below the peak intensity.

REALISTIC UNDULATOR SYSTEMS

Tracking particles through ‘ideal undulators’ whether described by an analytic expression or a numerical field map fails to capture the reality of magnet design and construction. Field errors will be present in any ‘real’ undulator system and the code described here can include these errors in an analytical description of the magnetic field. In this case the total field strength was varied every half-period. The field was constructed such that over the length of the undulator the field is smooth and continuous every-

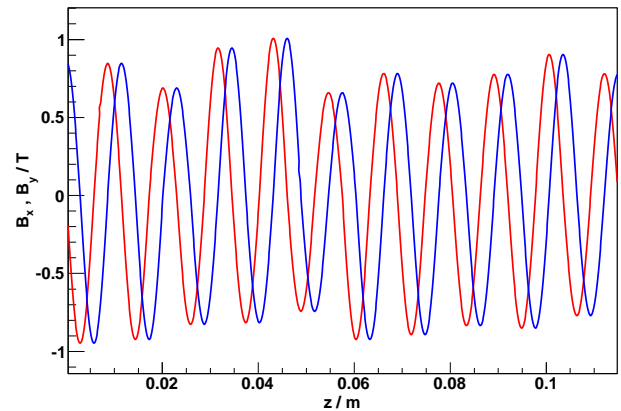


Figure 4: The magnetic field B_x (red) and B_y (blue) in a section of a helical undulator. Field errors which vary every half period are included in the field description. In this case $\Delta B/B_0 = 0.10$ is used for illustration only

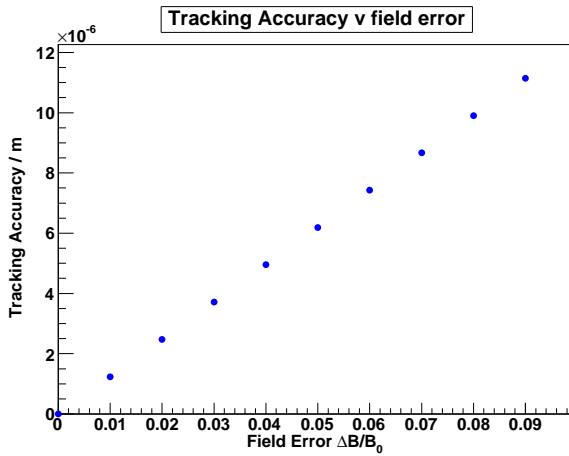


Figure 5: The effect of introducing field errors on the accuracy achieved (compared to an ideal trajectory) when tracking an electron through 155 periods of an undulator.

where. A section of the undulator's magnetic field is shown in figure 4 where the size of the fluctuations follows a Gaussian distribution with mean $\mu = B_0$ ($=0.84$ T) and width $\sigma = B_0/10$ (this value of σ is chosen for illustrative purposes only). In all the calculations in this section the tracking was performed using 100 integration steps per period with the synchrotron radiation calculated at every integration step.

The effect on the accuracy of tracking through a 155 period section of the undulator is shown in figure 5. The deviation of the trajectory varies linearly with the magnitude of the field error. Comparing the absolute values of the deviation with those shown in figure 1 it can be seen that even a relatively small field error will cause a large deviation in

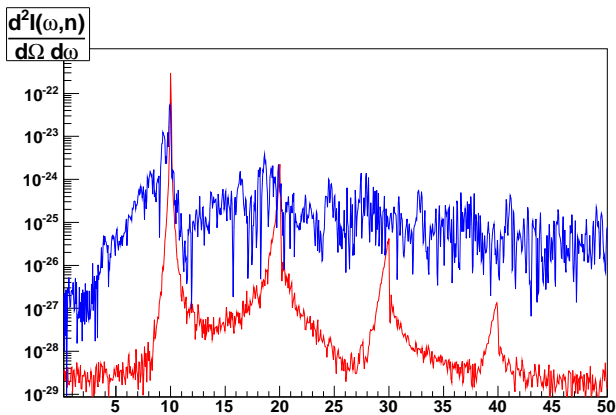


Figure 6: Introducing field errors to the undulator field causes a deterioration of the calculated energy spectrum of synchrotron radiation. Field errors of magnitude $\Delta B/B_0 = 0.001$ (red) and 0.01 (blue) were used in these calculations c.f. the green points in figure 3 for comparison with the spectrum calculated with no field errors.

the electrons trajectory. It may be expected that this will have a deleterious affect on the calculated spectrum of synchrotron radiation and this is shown in figure 6. In this figure field errors of magnitude 0.1% and 1% were introduced to the undulator field and the synchrotron radiation is plotted, as observed at a point 0.21 m (on-axis) from the exit of a 155 period undulator, with total length 1.79 m. It can be seen that with errors at the 0.1% level the energy spectrum is very similar to that calculated with no errors. However, when the magnitude of the field errors is increased to just 1% of the mean field the peak intensity drops by an order of magnitude and the peak shifts from 10.0 MeV to 9.9 MeV. The level of background radiation increases to around two orders of magnitude below the peak intensity. Calculations with increased field errors show that this trend of decreasing peak intensity at decreasing energy is continued as the field errors grow larger.

CONCLUSION

Calculating the synchrotron radiation output from long undulators is a computationally expensive process. By understanding how both the numerical tracking and the sampling of the emitted radiation effect the output spectrum, this process can be expedited. The inclusion of field errors on the calculated output has been shown to have a deleterious affect and the conclusion must be that such errors have to be included in the undulator model if a realistic output spectrum is to be achieved. All the simulations referred to in this paper modelled a 1.79 m section of the ILC helical undulator and although the ILC design report calls for an undulator of total length 147 m, the techniques used in this paper to estimate the accuracy of the output spectrum will be relevant for much longer undulators. It is expected that over longer undulators the optimum number of steps to track the particles and sample the synchrotron radiation will need to be reassessed. The application of this code to longer (~ 50 m) undulators is reported in reference [2].

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