# Particle Tracking and Synchrotron Radiation

David Newton

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# Previous Work

- Characterise an arbrtrary magnetic field in terms of it's multipole expansion and generalised gradients to produce an analytical description of field as a fuction of the longitudinal coordinate
- Use the analytical expression a differential algebra or Lie algebra code to generate a Taylor or Lie (symplectic) map for the dynamics in the wiggler.
- Evaluate the analystical expressions to perform a numerical integration giving a fast particle tracking code to describe the evolution of the canonical coordinates within the magnet.
- The C++ code that has been has been written has a modular structure which facilitates extending the code
- A Synchrotron Radiation Module is being implemented which calculates the synchrotron emission from a particle into an arbitrary aperture
- eg ILC Helical undulator

## Synchrotron Radiation Calculation

Accelerated charges radiate energy. The observed electric field of the emitted radiation is:

$$\vec{E}(t) = \frac{e}{4\pi\epsilon_0 c}\left(\frac{c(1-|\vec{\beta}|^2)(\vec{n}-\vec{\beta})}{|\vec{R}|^2(1-\vec{n}\dot{\vec{\beta}})^3} + \frac{\vec{n}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{|\vec{R}|(1-\vec{n}\dot{\vec{\beta}})^3}\right)_{RET}$$

$$\frac{d^2U}{d\Omega dt} = \epsilon_0 c E^2 r^2 = \vec{G}(t)^2$$

$$\vec{G}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\vec{G}(t)\exp(\imath\omega t)dt$$

$$\frac{d^2I}{d\Omega d\omega} = |\vec{G}(\omega)|^2 + |\vec{G}(-\omega)|^2$$

# Implementation

An arbitrary number of onserving points are defined at initialisation. At each step of the tracking code, the Electric Field is by estimating the radius of curvature of the particle (using the two adjacent integration steps) and calculating

$$\vec{\beta},\ \dot{\vec{\beta}} \text{ and } \vec{n}.$$

At the end of the integration the differential intensity is found by performing a DFT on the electric field components.

# Benchmarking

Consider an electron accelerated by a constant magnetic field - > circular motion ($\vec{F} = q\vec{v} \times \vec{B}$).

For an electron following a trajectory of radius R, in the xz plane, The frequency distribution of the emitted radiation is

$$\frac{d^2 I}{d\Omega d\omega} = \frac{3}{4}\frac{q^2}{4\pi\epsilon_0 c}\gamma^2\left(\frac{\omega}{\omega_0}\right)(1+\gamma^2\Psi^2)^2\left[K^2_{2/3}(\zeta) + \frac{\gamma^2\Psi^2}{1+\gamma^2\Psi^2}K^2_{1/3}(\zeta)\right]$$

K is a fractional modified bessel function,

$\omega_0 = \frac{|\vec{v}|}{R}$

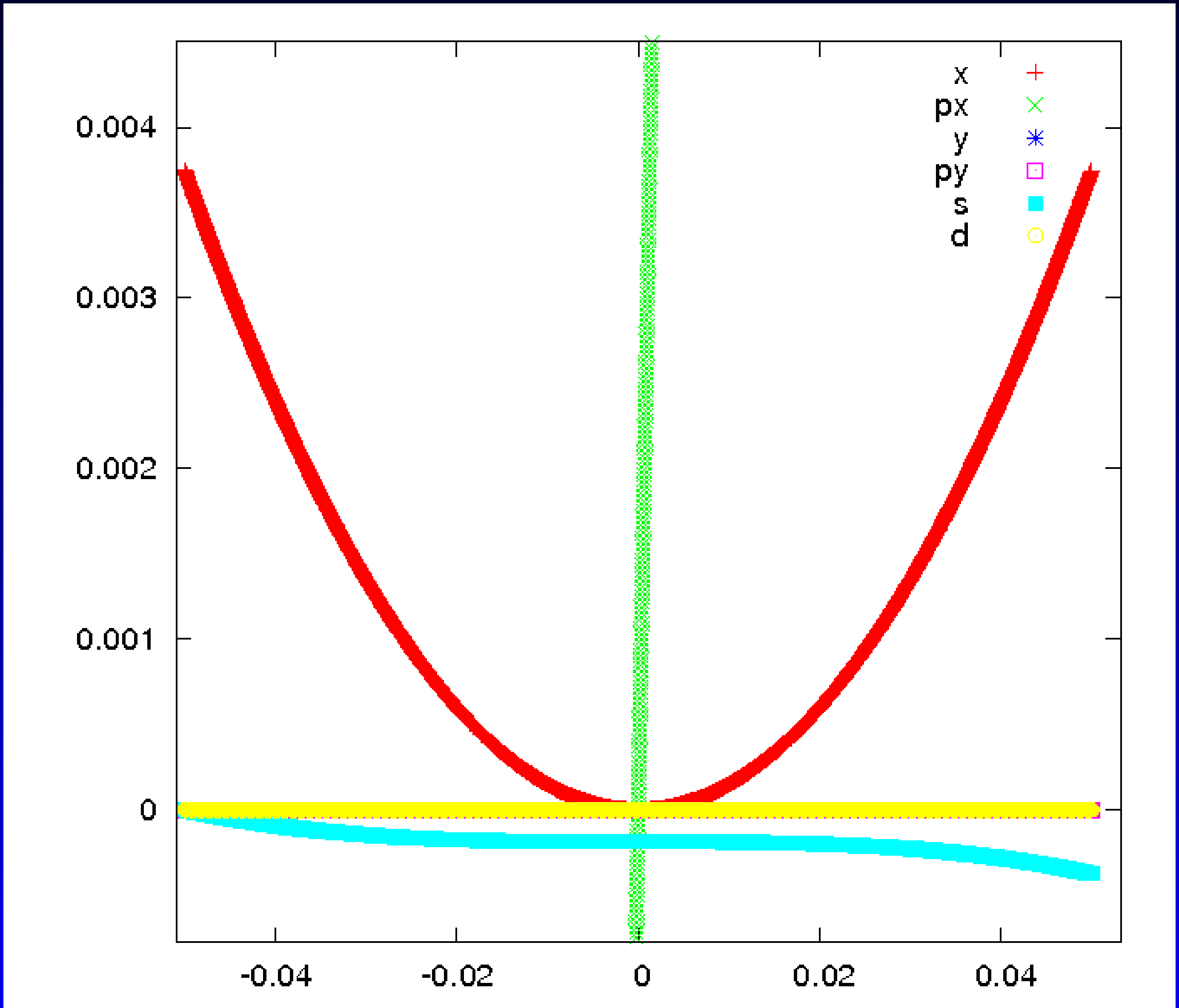
and

$\zeta = \frac{1}{2}\frac{\omega}{\omega_c}(1+\gamma^2\Psi^2)^{3/2}$

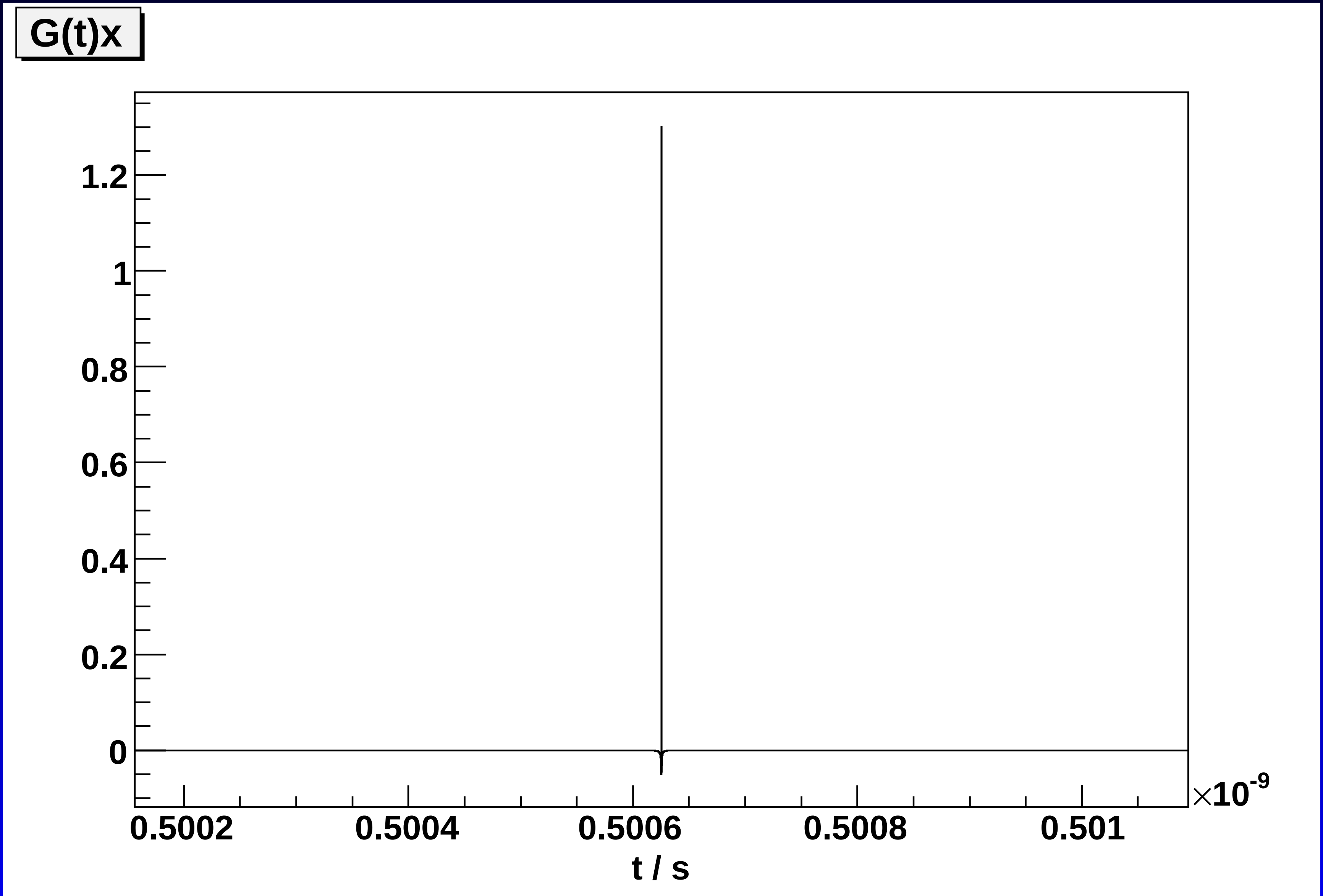
# Benchmarking Procedure

- Starting with a constant magnetic field ($B_y = 1$ T), model the field in terms of it's generalised gradients, produce analytical descriptions of the field and numerically integrate the motion of the particle through the field.
  - $E = 100$ MeV
  - R=33.3333 cm
  - L=10 cm
  - 10000 integration steps
- At each integration step calculated the electric field observed in an aperture, fourier transform the field to produce the frequency distribution and compare with the analytical description

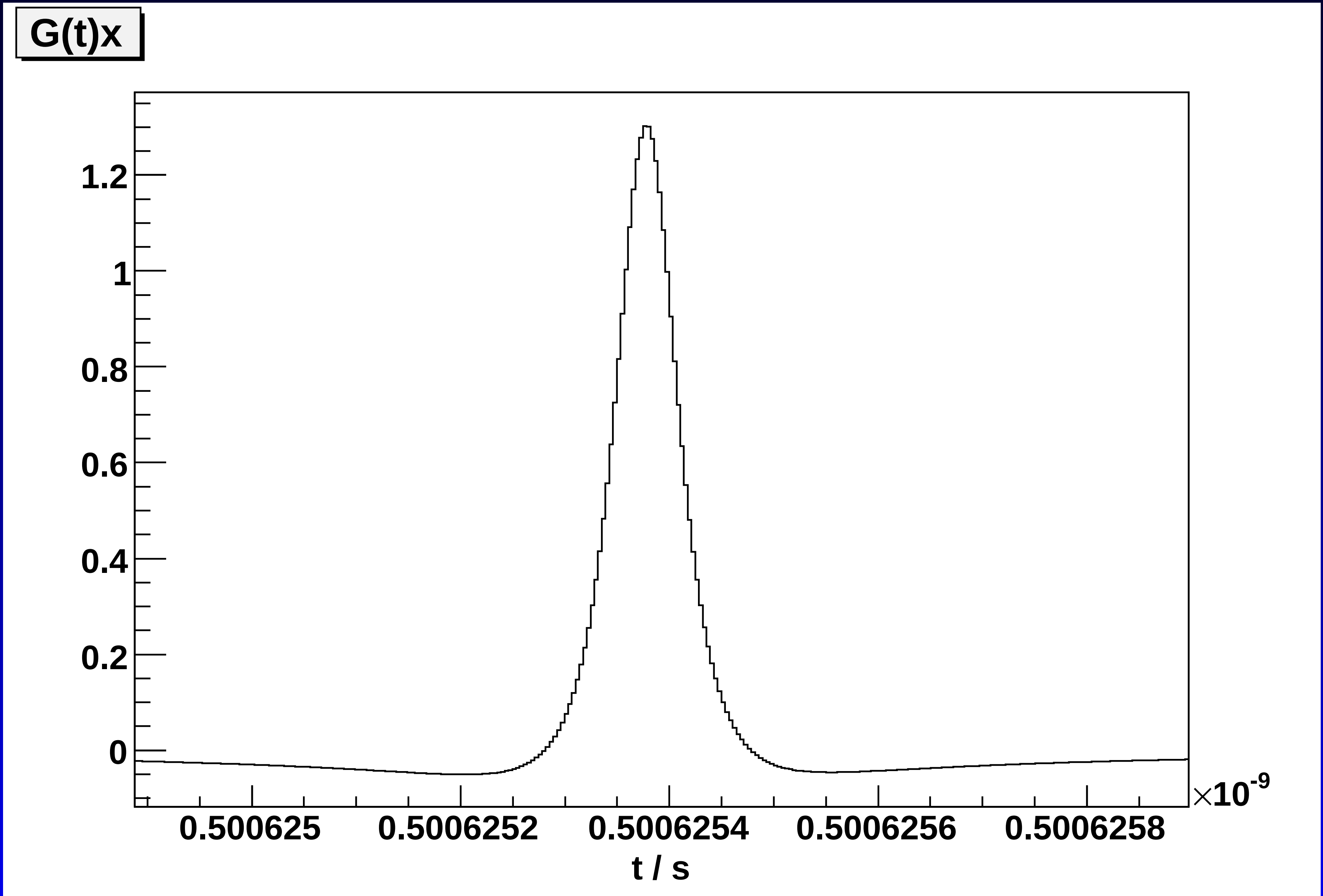
# Particle Tracking



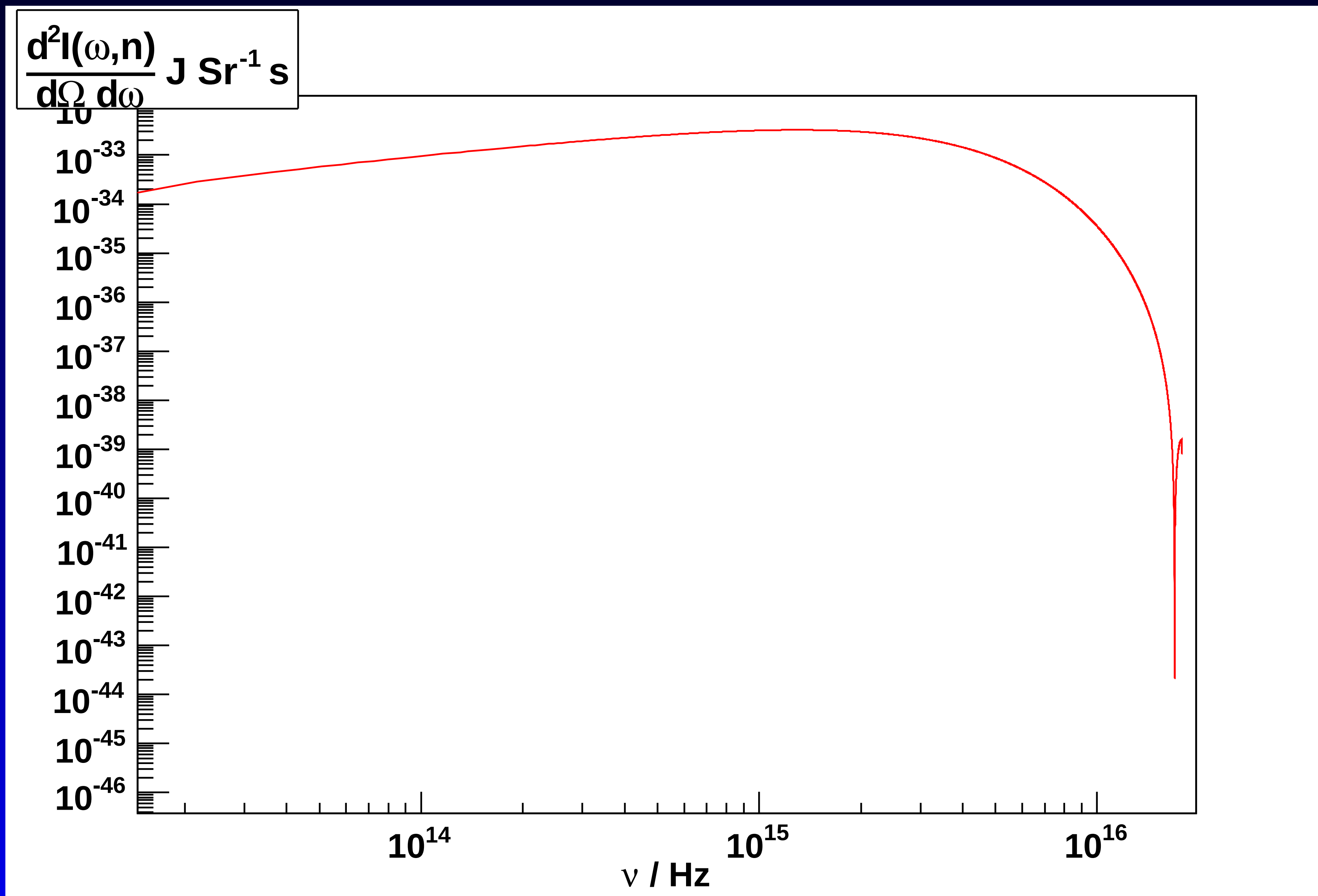
# Electric Field



# Electric Field

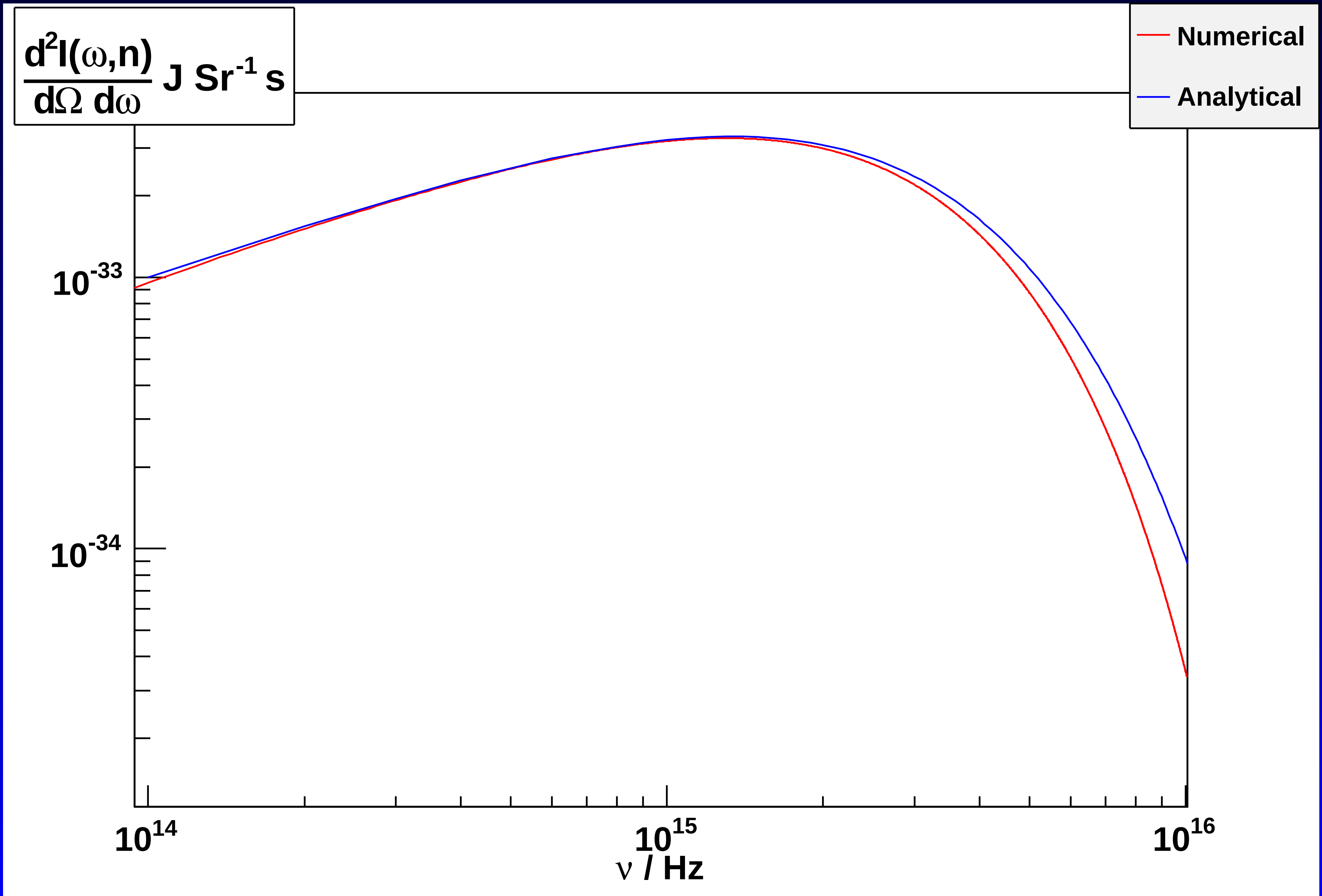


# Frequency Distribution



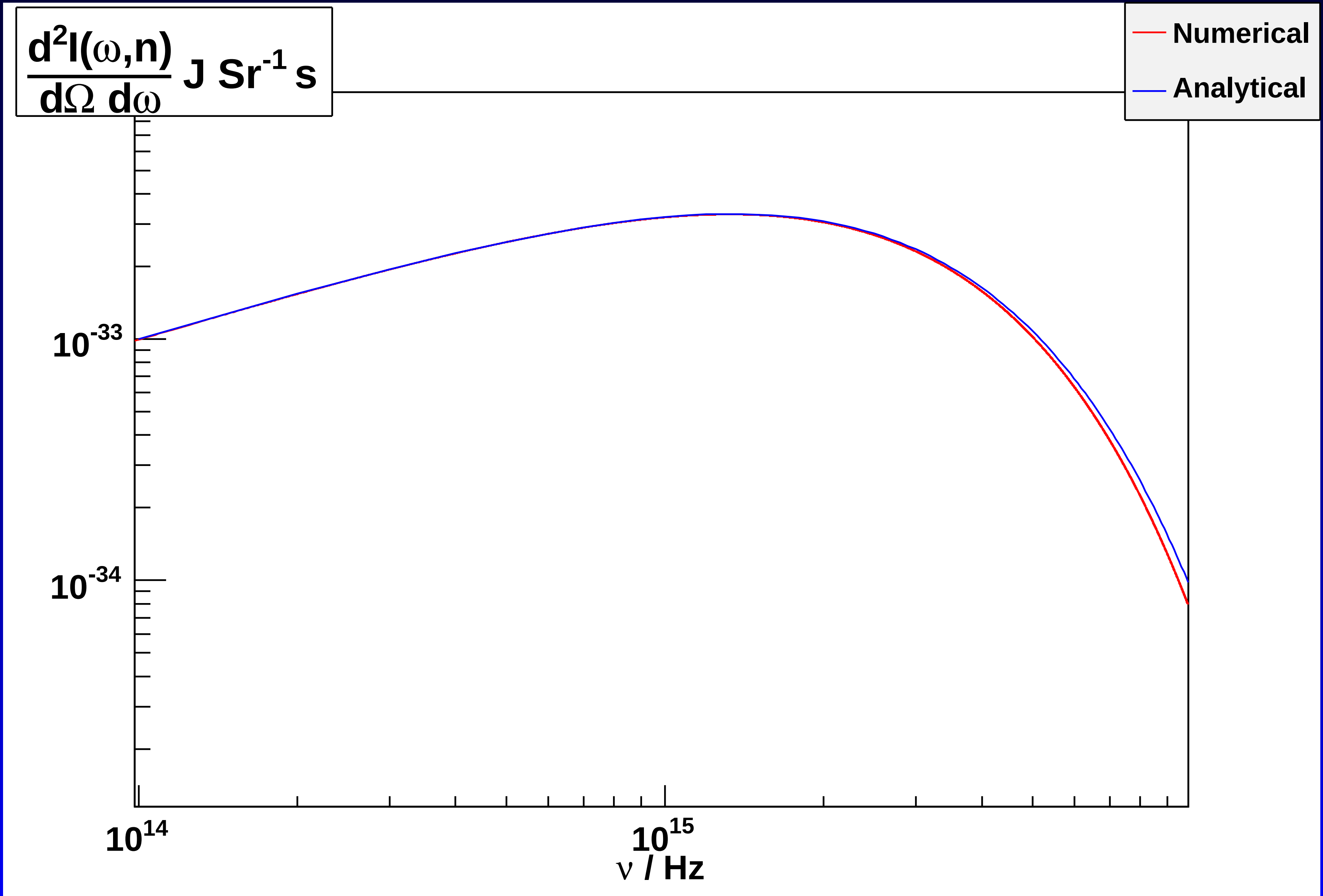
# Frequency Distribution

N=4882



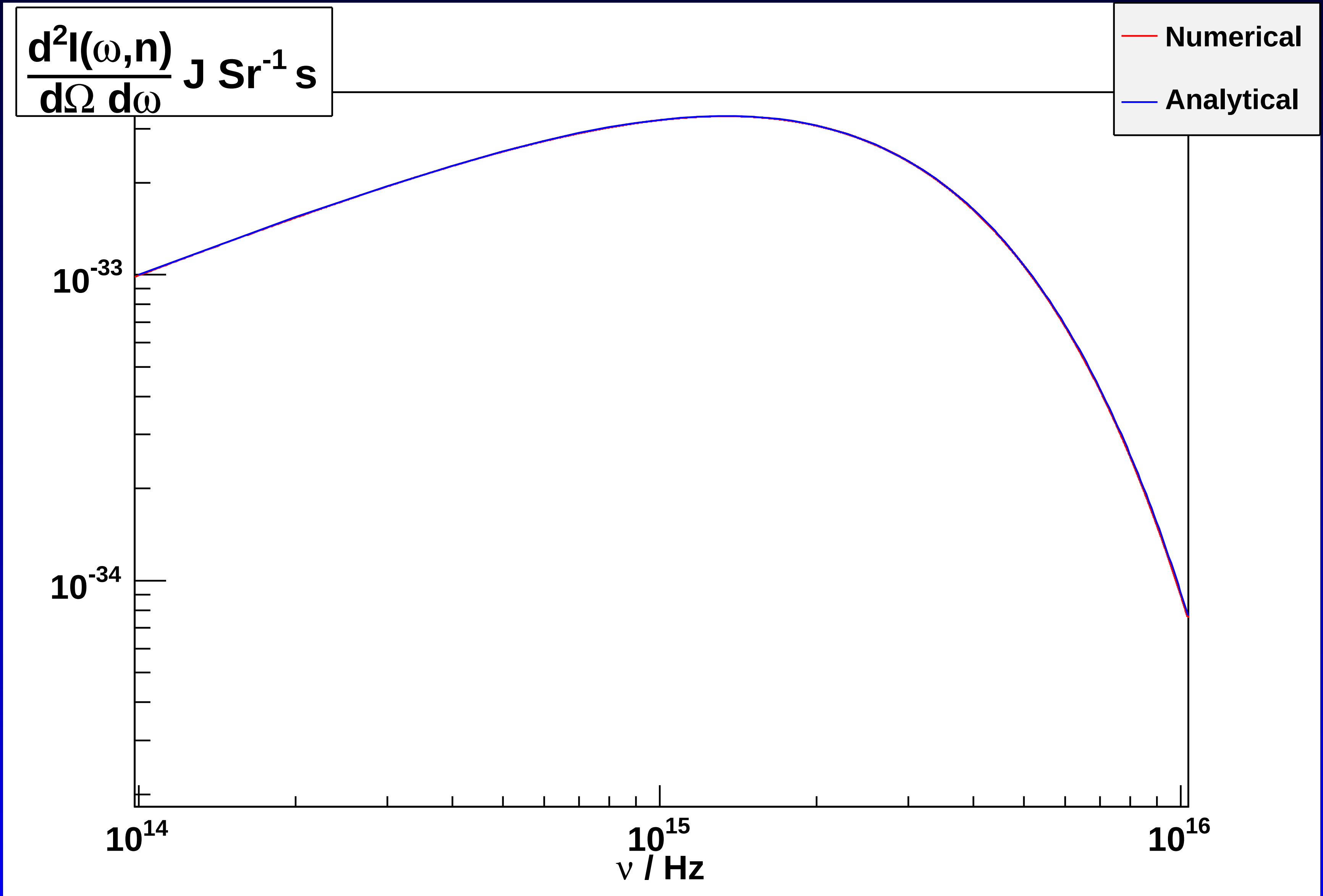
# Frequency Distribution

N=65536



# Frequency Distribution

N=262144

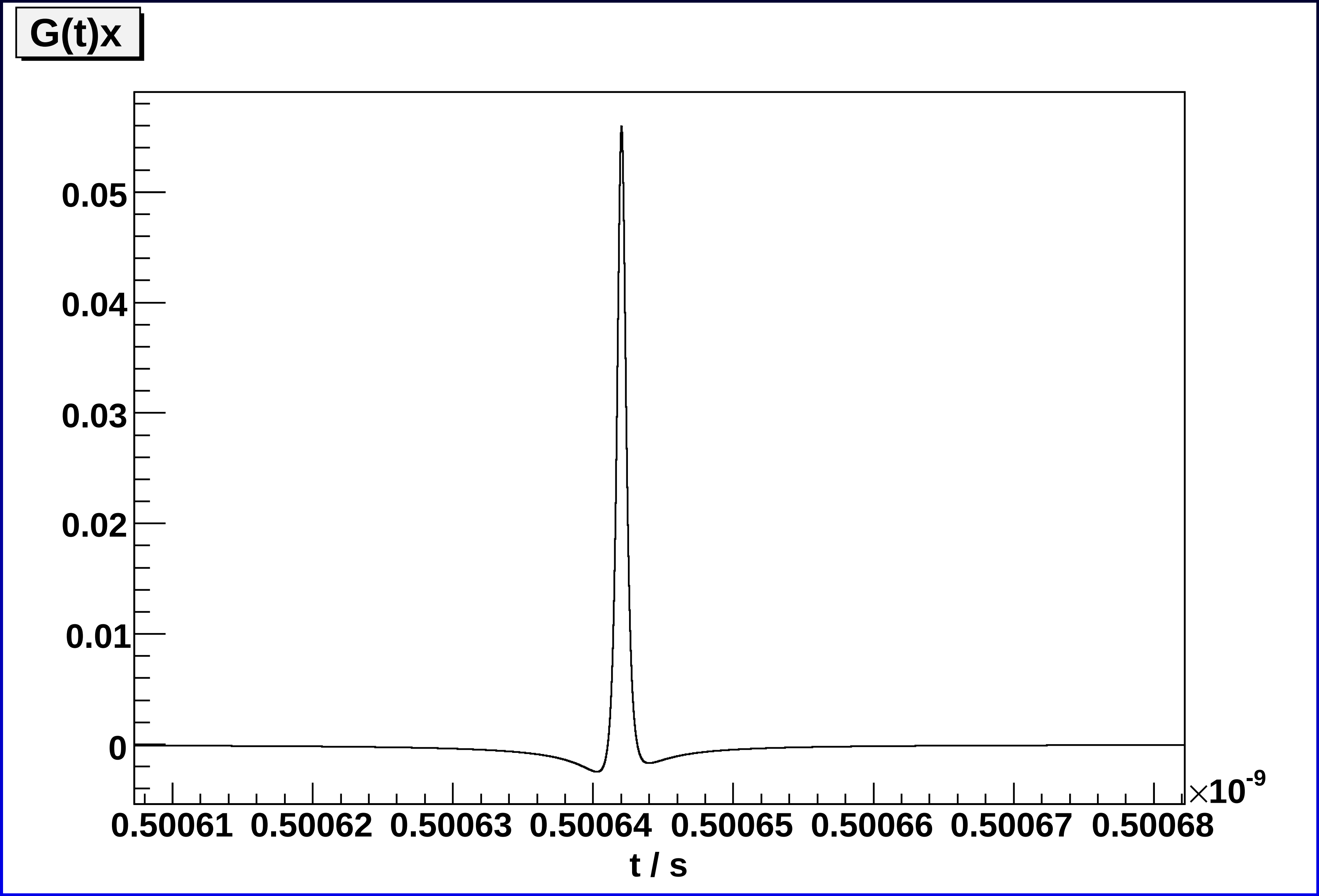


# Fast Fourier Transform

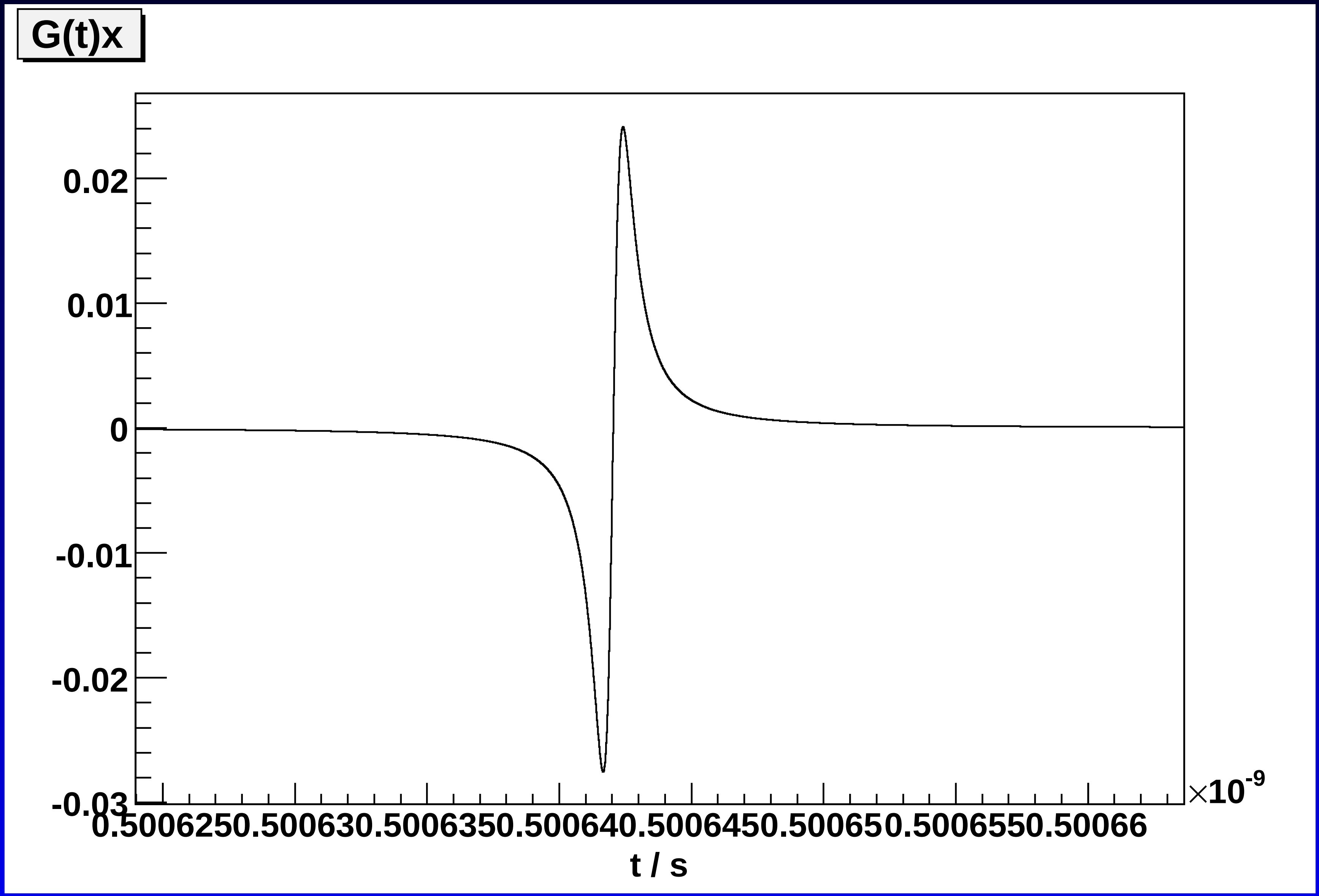
- The standard C++ library FFTW is used to perform the fourier transform.
- Typical transform times for N steps:
- N=262144, 4 seconds
- N=$35,000,000$, 13 minutes

Using N=262144 steps (analytical values in brackets)
Peak intensity: $3 \cdot 298.10^{-33}\,(3 \cdot 299.10^{-33}) JSr^{-1}$
Peak Frequency: $1 \cdot 273.10^{15}\,(1 \cdot 297.10^{15}) s$

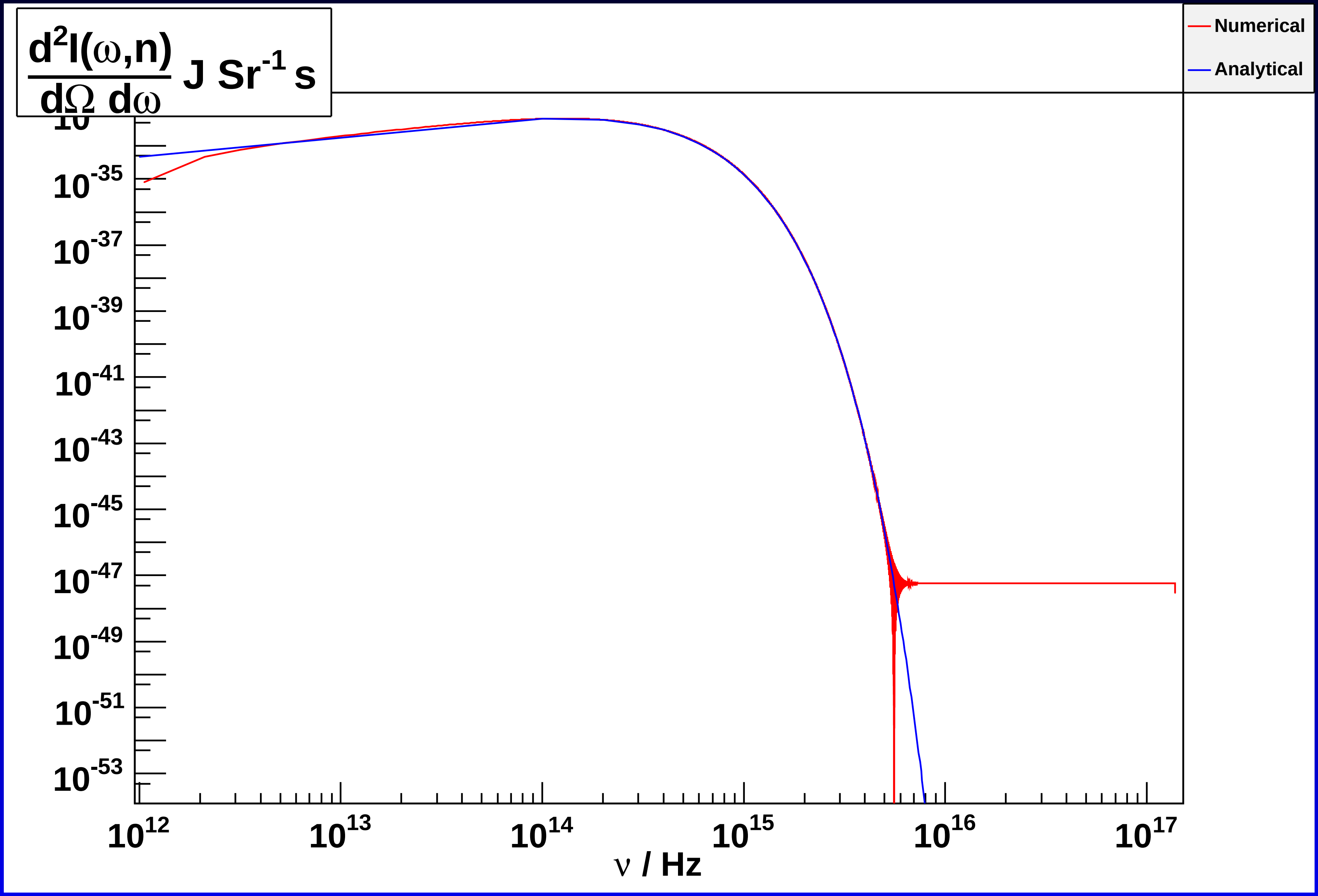
# Ex


G(t)x
0.05
0.04
0.03
0.02
0.01
0
×10-9
0.50061 0.50062 0.50063 0.50064 0.50065 0.50066 0.50067 0.50068
t / s

# Ey

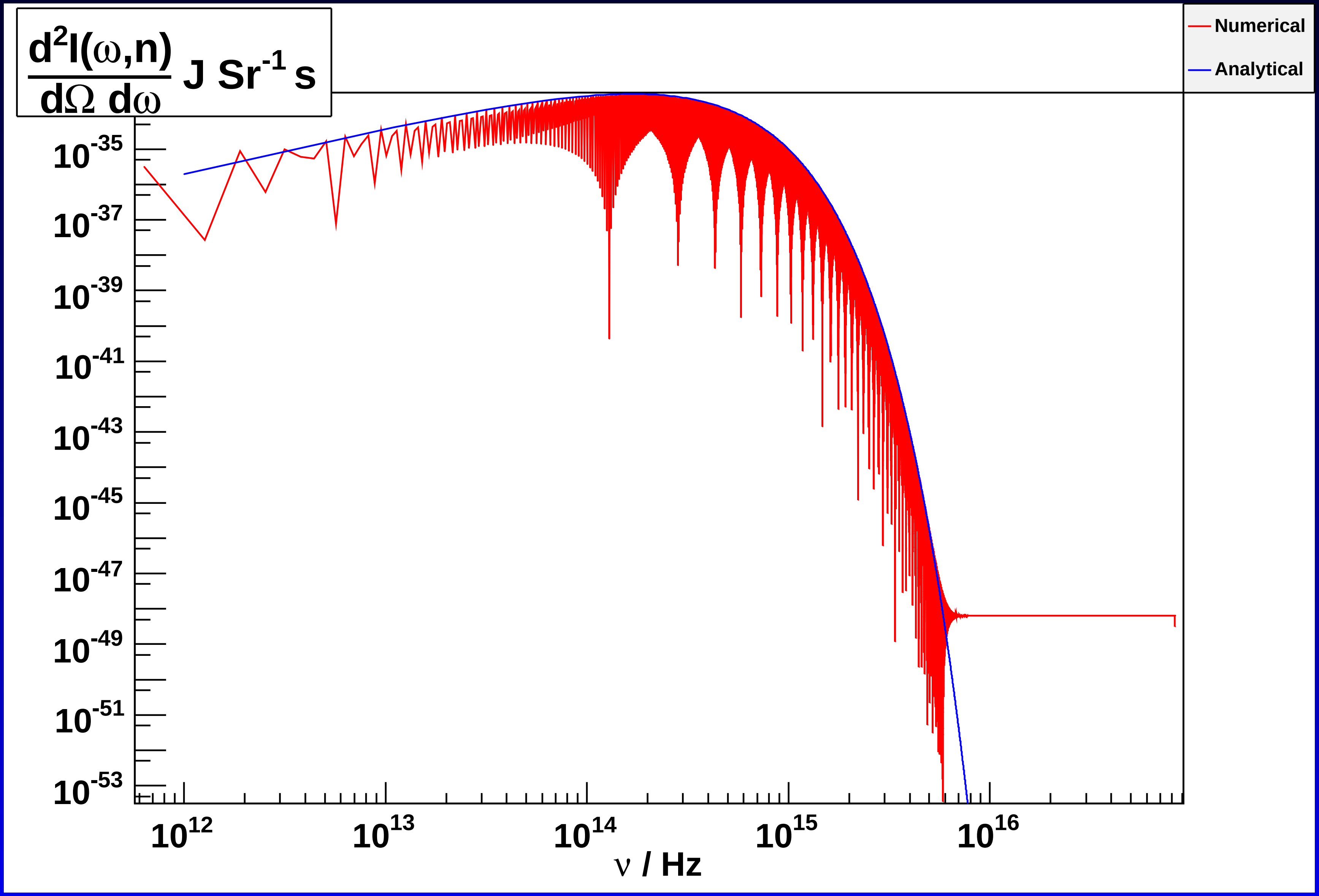

G(t)x
0.02
0.01
0
-0.01
-0.02
-0.03
0.500625
0.50063
0.500635
0.50064
0.500645
0.50065
0.500655
0.50066
×10^-9
t / s

# $\nu_x$



d²I(ω,n) / dΩ dω J Sr⁻¹ s
Numerical
Analytical
10^-35
10^-37
10^-39
10^-41
10^-43
10^-45
10^-47
10^-49
10^-51
10^-53
10^12
10^13
10^14
10^15
10^16
10^17
ν / Hz

# $\nu_y$



d²I(ω,n) / dΩ dω J Sr⁻¹ s
Numerical
Analytical
10^-35
10^-37
10^-39
10^-41
10^-43
10^-45
10^-47
10^-49
10^-51
10^-53
10^12
10^13
10^14
10^15
10^16
ν / Hz

# Conclusion

- Problem with strobing in the fourier transform
- No idea why this is - need to investigate
- Also present in the x transform if not done carefully

# Conclusion

- Synchrotron calculation gives a good result compared to an analytical calculation
- Problems still with the fourier transform (not robust) which need solving
- Lots of scope to optimise the algorithm
    - Radiation is beamed in a cone with angle $\approx \gamma^{-2}$ ... donj't need to calculate each point at each step.
    - Implement spline (fast) fourier transform? Don't need to worry about unequal step size in t
    - Need to optimise the parameters for a specific field map
- Benchmark on the ILC Helical Undulator