APPLICATIONS OF A NEW CODE TO COMPUTE TRANSFER MAPS AND DESCRIBE SYNCHROTRON RADIATION

D. Newton and A. Wolski, Liverpool University, The Cockcroft Institute.

Abstract

An analytic tracking code has been developed to describe an arbitrary magnetic field in terms of its generalised gradients [1] and multipole expansion, which is used with a 2nd-order symplectic integrator [2] to calculate dynamical maps for particle tracking. The modular nature of the code permits a high degree of flexibility and allows customised modules to be integrated within the code framework. Several different applications are presented, and the speed, accuracy and flexibility of the algorithms are demonstrated. A module to simulate synchrotron radiation emission is described and its application to an 'ILC-type' undulator system is demonstrated.

INTRODUCTION

Analytical descriptions of magnetic fields offers many advantages over numerical maps. Explicitly describing field components allows a deeper understanding of charged particle dynamics, and the ability to generate transfer maps gives huge speed advantages in particle tracking and lattice analysis. However, accurately describing higher-order (non-linear) components of 'real' magnets has historically been a challenging field. Often an assumption is made that the magnet is ideal or pseudo-ideal, and often this assumption is valid, but realistic analytic descriptions of magnetic fields can be of particularly importance for novel magnet designs where non-linear terms give a significant contribution to the total field, or for long undulator systems, with strong fringe field contributions, where numerical tracking schemes can be laborious. A technique to analytically describe an arbitrary magnetic field is given in reference [1] and has been implemented in a C++ code described in reference [3], along with a second order symplectic integrator [2] allowing fast, accurate numerical or analytic particle tracking through arbitrary magnetic fields. In this paper, some applications of the code are discussed. A module to describe synchrotron radiation has been added and some results using both ideal and 'real' magnet systems are presented.

SYNCHROTRON RADIATION EMISSION

A module has been developed to describe the synchrotron radiation emitted by a charged particle as it is accelerated by a magnetic field. The particle is numerically tracked in the manner described in reference [3], but at each integration step, the effective radius of curvature of the particle is estimated, and the resulting electric field is calculated at a pre-defined observation point. The electric field induced by an accelerated charge is given by:

$$\vec{E}(t) = \frac{e}{4\pi\epsilon_0 c}\frac{1}{(1-\vec{n}\cdot\vec{\beta})^3} \times \left(\frac{(1-|\vec{\beta}|^2)(\vec{n}-\vec{\beta})}{|\vec{R}|^2} + \frac{\vec{n}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{c|\vec{R}|}\right)_{RET} \quad (1)$$

where, $\vec{R}$ is the vector from the charged particle to the observation point, with unit vector $\vec{n}$ [4], and the other symbols have their usual meaning. A Fourier transform of the resultant field gives the frequency spectrum:

$$\frac{d^2U}{d\Omega dt} = \epsilon_0 c E^2 r^2 = \vec{G}(t)^2$$

$$\vec{G}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\vec{G}(t)\exp(\imath\omega t)dt$$

$$\frac{d^2I}{d\Omega d\omega} = |\vec{G}(\omega)|^2 + |\vec{G}(-\omega)|^2$$

The electric field data, including the polarisation state, are calculated at each step and saved in a ROOT TTree object [5]. This format is designed for storing large quantities of data and is optimised to reduce disk space and enhance access speed. It also offers a flexible method of inspecting and plotting the raw data and periodically writes to a file during long calculations, safeguarding against system failures and reducing the necessary memory cache.

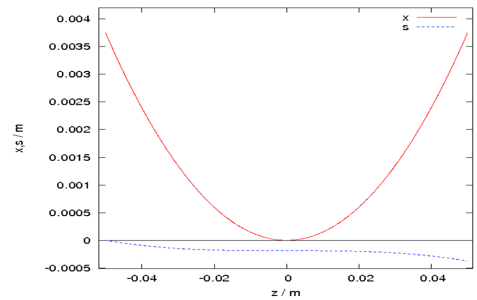

Figure 1: The phase space coordinates x and s (the longitudinal coordinates relative to the reference trajectory) for a 100 MeV electron in a dipole field. The frequency distribution for an electron in a circular orbit can be calculated analytically, allowing the accuracy of both the numerical tracking routine and the synchrotron calculation to be benchmarked.

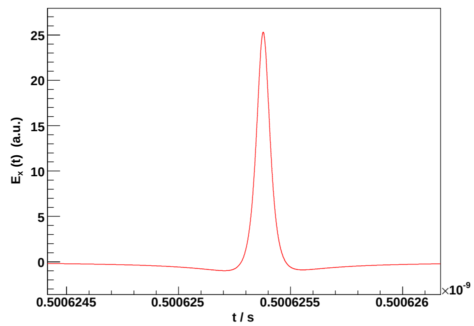

Figure 2: The calculated electric pulse due to an electron in a circular orbit. The observation point is in the same plane as the electron's path, so only the x-component of the electric field is present.

Comparison with an analytic calculation

The frequency distribution of radiation emitted by a charged particle in circular motion can be calculated analytically [4], allowing a comparison with the numerical calculation. Using the code described in reference [3], an ideal normal dipole field with strength 1 T was characterised and an electron of energy 100 MeV was tracked through 10,000 integration steps using a second order symplectic integrator. Figure 1 shows the evolution of the variables x and s (the longitudinal coordinate relative to a reference particle). In such a field, the particle describes an arc of radius $\sim$ 0.33 m in the xy plane. At an observation point $(x = 0, y = 0, z = 0.1)$, in the coordinate frame of figure 1, the frequency distribution was calculated analytically and compared to the numerical result.

Figure 2 shows the electric field as a function of time, calculated using equation 1, and after a Fourier transform into the frequency domain the differential spectrum is shown in figure 3. The analytical result is also plotted here. The analytical spectrum has a peak value of $3.300\times10^{-33}\,J\,Sr^{-1}\,s^{-1}$ at $1.339\times10^{15}Hz$, which is also where the maximum discrepancy between the two curves occurs. The calculated value differs by $\sim\,1.6\times10^{-36}$, giving an agreement between the two of better than 0.05%.

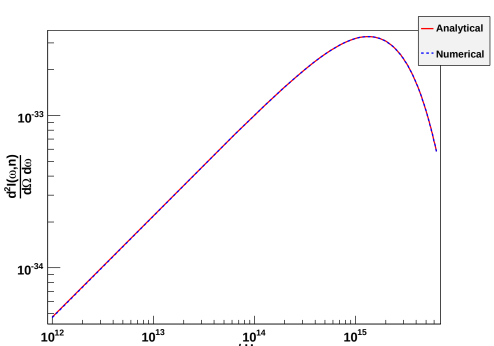

Figure 3: The differential power spectrum for an electron in a circular orbit, measured in $J\,Sr^{-1}$ radiated into unit frequency interval. The analytical and numerical curves agree within 0.05%.

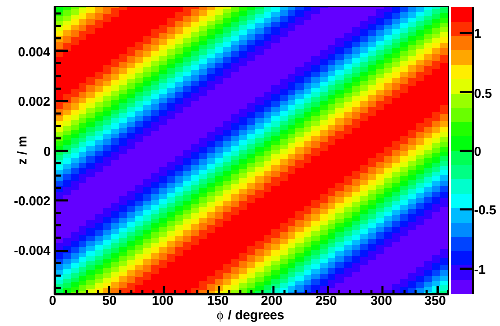

Figure 4: The radial component of the magnetic field, for a helical undulator with periodicity 0.115 m, projected onto the surface of a cylinder of radius 0.0019 m.

APPLICATION TO THE ILC HELICAL UNDULATOR

The ILC Reference Design Report calls for a 150 GeV electron beam to be transported through a 150 m helical undulator emitting 10 MeV synchrotron photons which will be used to produce electron-position pairs. The helical undulator will have a periodicity of 1.15 cm, with each module containing 155 periods giving a module length of 1.79 m.

A numerically calculated field map of a single period of the undulator system (figure 4) was used as an input map for the code. Following the procedure outlined in reference [3], the generalised gradients were calculated and used to generate an analytical description of the vector potential. Finally the phase space vector $(x, p_x, y, p_y, s, \delta)$ was integrated over the length of the undulator system. The normal and skew on-axis generalised gradients C_1^0 are shown in figure 5, corresponding to the on-axis field components B_y (normal) and B_x (skew) and figure 6 shows the result of

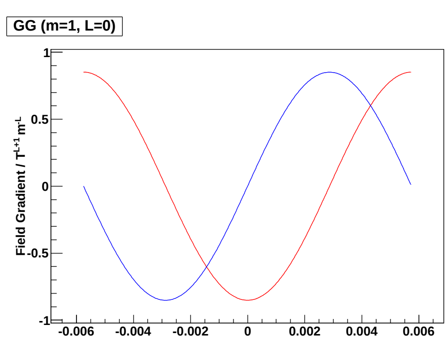

Figure 5: The normal (red) and skew (blue) dipole components of the generalised gradient for the helical undulator. These components are identified with the on-axis fields B_y and B_x

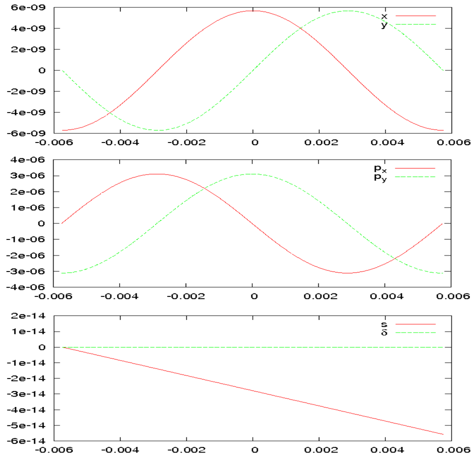

Figure 6: The evolution of the phase space vector $(x, p_x, y, p_y, s, \delta)$ as a 150 GeV electron traverses one period of the helical undulator.

numerically tracking the phase space vector across a single period of the undulator. The phase space coordinates were integrated over one period using 10,000 integration steps, although the electric field was only calculated every 10th step. The speed of this operation could be improved by analytically integrating a transfer matrix over every 10 steps and writing the result to a file. Once this is done the numerical integration can be achieved using only 1000 steps with no loss of accuracy for any initial phase space coordinates.

The frequency distribution of synchrotron radiation emitted by a 150 GeV electron, traversing a 1.79 m helical undulator, of 155 periods, observed an observation point 10 m (on-axis) from the end of the undulator was calculated using 1.55×10^6 integration steps, with the electric field calculated every 10^{th} step. The start-to-end calculation took just under 15 minutes on a 2.66 GHz processor. A section of the x-component of the field is shown in figure 7. The total pulse had a duration of 8^{-20} s. Following a Fourier transform the energy spectrum is shown in figure 8, with a strong harmonic at 10.1 MeV.

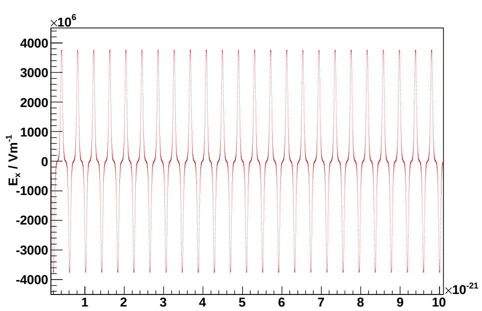

Figure 7: A portion of the x-component of the electric field, observed 10 m from the end of a 1.79 m helical undulator (155 periods).

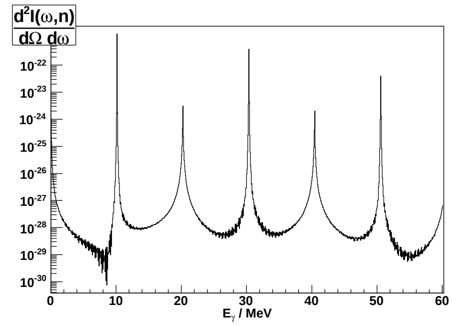

Figure 8: The photon energy distribution of radiation emitted by a 150 GeV electron passing through one section (1.79 m, 155 periods) of a helical undulator. The first harmonic occurs at a photon energy of 10.1 MeV.

CONCLUSION

The C++ code described here and in reference [3] is designed to give a fast, accurate method of characterising magnetic fields and numerical particle tracking and calculating analytic transfer maps, although it is still in the development phase - the synchrotron module in particular is not yet optimised. The modular design allows additional functionality to be implemented with minimum reprogramming. The ability to calculate analytic transfer maps means the synchrotron radiation add-on can be optimised to reduce the number of calculations with no loss of accuracy in the tracking code. The calculation can also be started at any point within the magnet system, making the code ideally suited to parallel or pseudo-parallel (i.e. Condor) applications.

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