TUNE MEASUREMENT IN NON SCALING FFAG EMMA WITH MODEL INDEPENDENT ANALYSIS*

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Abstract

The Non Scaling Fixed Field Alternating Gradient (NS-FFAG) EMMA accelerator has a purely linear lattice, and the crossing of resonances during acceleration is therefore a key characteristic of the beam dynamics. An accurate measurement of the tune is essential for a full understanding of the machine behaviour. However, commonly used measurement techniques require the beam to perform a large number of turns in the machine. Simulations have shown us that rapid decoherence of the beam requires a technique capable of providing a tune measurement from just one or two turns of the ring. Model independent analysis (MIA) has been investigated as a possible approach. The singular value decomposition of a matrix composed of BPM readings from the trajectories of different bunches provides information on the machine optics. Simulations indicate that it should be possible to derive an accurate value of the tune using MIA, even in the presence of BPM noise and beam decoherence.

INTRODUCTION

The Electron Model for Many Application (EMMA [1]) is the first Non Scaling Fixed Field Alternating Gradient ever built. The lattice is composed entirely of quadrupole magnets that are used simultaneously to steer and focus the beam: since there is no chromatic correction, the tune variation during the acceleration cycle (from 10 MeV to 20 MeV) is large. For the same reason, the energy spread on a bunch in the machine leads to rapid decoherence (within a couple of turns) of any coherent oscillations, making measurements of the betatron tunes very challenging.

In this paper, we describe an approach to tune measurements based on Model Independent Analysis (MIA [2]). The technique relies on identifying correlations between measurements made using different BPMs, from a number of different bunch trajectories. We first explain the principle, then describe its application to EMMA. We simulate the measurement technique in the ideal machine (without magnet or BPM errors), to assess the magnitude of systematic errors arising from certain limitations in the analysis technique. Finally, we consider the impact of BPM noise and beam decoherence on the accuracy of the tune measurement.

* Work supported by the Engineering and Physical Sciences Research Council, UK.
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THEORY

Our goal is to determine the tune in EMMA from BPM readings over one or (at most) two turns of a bunch performing coherent betatron oscillations. A set of BPM readings can be collected each time a bunch is injected into the machine. By injecting several bunches, we can construct an array of BPM readings, with each column corresponding to a separate BPM, and each row corresponding to a separate trajectory.

The trajectory of a bunch is determined by the initial values of the dynamical variables describing the position and momentum of the bunch centroid. If we consider (for simplicity) motion in just one degree of freedom, then the trajectory is determined by two parameters. It must then be possible to construct any row of the matrix of BPM readings by a linear combination of just two vectors; appropriate orthonormal vectors may be determined by performing singular value decomposition (SVD) of the matrix of BPM readings:

$$A = U \cdot S \cdot V^T, \tag{1}$$

where A is the matrix of BPM readings, S is a diagonal matrix, and U and V are orthonormal matrices. Any row of the matrix A can be constructed from a linear combination of the rows of V^T. In the case that only two vectors are needed to construct any row of A, S has only two non-zero components, and only the first two rows of V^T are significant.

A betatron oscillation with invariant amplitude J_0 and initial phase ϕ_0 can be written as:

$$x_n = \sqrt{2\beta_n J_0}\cos(\phi_n + \phi_0), \tag{2}$$

where x_n is the coordinate of the bunch centroid at the nth BPM, ϕ_n is the betatron phase advance from the start of the beamline to the nth BPM, and β_n is the beta function at the nth BPM. This can be written as:

$$\begin{aligned} x_n &= \cos(\phi_0 - \psi)\sqrt{2\beta_n J_0}\cos(\phi_n + \psi) \\ &\quad - \sin(\phi_0 - \psi)\sqrt{2\beta_n J_0}\sin(\phi_n + \psi). \end{aligned} \tag{3}$$

If we define two vectors $\vec{c}$ and $\vec{s}$ with components:

$$c_n = \alpha_c\sqrt{\beta_n}\cos(\phi_n + \psi), \tag{4}$$

$$s_n = \alpha_s\sqrt{\beta_n}\sin(\phi_n + \psi), \tag{5}$$

then it is clear that any trajectory (set of BPM readings) can be written as a linear sum of c_n and s_n (with coefficients determined by the betatron amplitude J_0, and initial phase ϕ_0). Furthermore, if $\vec{c}$ and $\vec{s}$ are orthonormal:

$$\sum_n \beta_n \cos(\phi_n + \psi)\sin(\phi_n + \psi) = 0, \qquad (6)$$

$$\alpha_c^2 \sum_n \beta_n \cos^2(\phi_n + \psi) = 1, \qquad (7)$$

$$\alpha_s^2 \sum_n \beta_n \sin^2(\phi_n + \psi) = 1, \qquad (8)$$

then we can obtain the components c_n and s_n from the first two rows of V^T.

If the matrix A is constructed from a number of measured bunch trajectories, SVD of A gives the values of c_n and s_n. If there are N BPMs, then Eqs. (4) – (8) express $2N + 3$ constraints on the values of the $2N + 3$ variables β_n, ϕ_n, ψ, α_c and α_s. Unfortunately, not all the constraints are independent, and there is therefore some degeneracy in the solution. However, if we start with values for the beta functions and phase advances taken from a model that is reasonably close to the machine, then we can find the minimal changes required to these values to fit the measured data. This should provide an improved model of the machine. Note that the analysis in this case is no longer truly "model independent".

In practice, a range of different trajectories may be produced by adjusting the steering in the injection line, or (in the horizontal plane) by adjusting the strengths of the injection kickers. In either case, variations may be systematic or random. There will, in any case, likely be some injection jitter resulting from variations in beam energy, shot-to-shot kicker strength, etc. If this jitter is sufficiently large, then it may be possible to make tune measurements from SVD without any deliberate variation in steering magnet or kicker strengths. On the other hand, a systematic variation in the injection trajectory may allow study of such effects as tune shifts with amplitude.

Generally, BPM readings will be subject to errors including systematic offset and gain errors, and random noise errors. Systematic offsets may be reduced by subtracting the mean from a set of readings at each BPM. Gain errors will mainly affect measurement of the beta functions. In the case that the BPM measurements are completely free of random noise, then only two vectors are needed to construct any row of A, and this will be reflected in the fact that the diagonal matrix S will contain only two non-zero values. Random variations in the BPM readings will result in additional non-zero values appearing in S; however, if the noise is not too large, then the betatron signals will dominate the components of A, and the largest values in S will correspond to the vectors c_n and s_n, as given above in Eqs. (4) and (5), in the first two rows of V^T. One of the advantages of the MIA technique is that it allows one to identify (and, if necessary, to exclude) "noisy" BPMs.

SIMULATIONS

In the first stage of EMMA commissioning, beam will be transported through only the first four sectors (2 injection cells plus 21 identical periodic cells) of the machine. Each cell is composed of a defocusing quadrupole (D) and a focusing quadrupole (F), each mounted on a mover that allows control of the radial position. Varying the positions and strengths of the magnets allows different lattice configurations to be studied. For the purpose of this paper, we limit our study to the baseline lattice [3]. We simulate the beam dynamics in EMMA using dynamical maps [4].

Each EMMA cell contains two BPMs. One is always situated between the D and F magnets ('DF' type); the other is alternatively positioned just after the F magnet ('AF' type) or just before the D magnet ('BD' type). For the present, we focus on the phase advance between two location separated by a cell length; therefore, only the 'DF' type BPMs are considered.

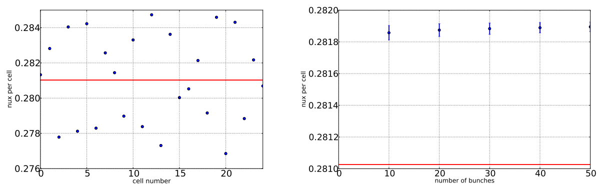

Figure 1: Simulated tune measurement using MIA in lattice without magnet or BPM errors. Left: Tune per cell, with 50 bunches per event. Right: Tune in cell 17, determined from events with different numbers of bunches. The red line shows the actual tune.

Ideal Model

In an ideal case, we consider all EMMA cells to be identical, and all BPM signals free of noise. We use a reference energy of 12 MeV; the calculated horizontal tune per cell (phase advance divided by 2π) at this energy in the baseline lattice is ν_x=0.2810. In Fig. 1 (left hand plot), we show the results of a simulation of the tune measurements. The points show an average over the number of events, N_E (one event consists in tracking a specified number of bunches with different trajectories), and the error bars show the standard deviation in the tune over all events, divided by $\sqrt{N_E}$ (with $N_E = 30$, for the plots shown here). We observe that in the ideal case (no magnet or BPM errors) the tune measurement has a random error (error bar) of order 10^{-4}, and a systematic error (difference between measured and nominal tune) of order 10^{-3}. Increasing the number of bunches per event decreases the random error (see Fig. 1, right hand plot), but a systematic error remains. This is likely to be a consequence of the degeneracy in solving the equations to determine the phase advances from the SVD modes. However, the precision of the measurement we obtain in this ideal case is sufficient for experimental studies. Note that over the energy range 10 MeV to 20 MeV, the tune varies from 0.36 to 0.16.

Effects of BPM Noise

In principle, the SVD analysis separates BPM signals corresponding to real beam motion from fluctuations in individual BPM readings arising from random noise. This is possible because the readings from different BPMs are correlated in the case that the signals are generated by betatron motion; but the noise signals are expected to be uncorrelated. However, with a finite number of trajectories, it is impossible to separate completely the noise from the signal corresponding to the betatron trajectories, and it is therefore likely that limited BPM resolution will affect the accuracy of the tune measurements.

The BPM resolution is expected to be in the range 70 μm to 20 μm, depending on the bunch charge (which will be in the range 10 pC to 30 pC, with the best resolution achieved with the highest bunch charge). In the simulations, noise was added to the BPM measurements simply by adding random numbers generated so as to have a normal distribution with width equal to the specified resolution.

Fig. 2 shows the effect of 50 μm BPM noise on tune measurements using MIA. The left hand plot shows the results of simulated tune measurements in all cells; the right hand plot shows how the tune measurement in cell 17 depends on the number of bunches (trajectories) used in a single event.

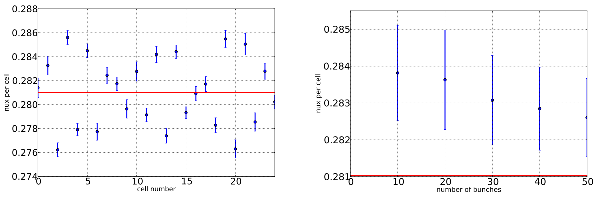

Figure 2: Effect of 50 μm BPM noise on tune measurements. Left: All cells (50 bunches per event). Right: Cell 17.

As expected, an increasing level of BPM noise leads to an increase in the spread of tune measurements obtained from multiple events – see Fig. 3.

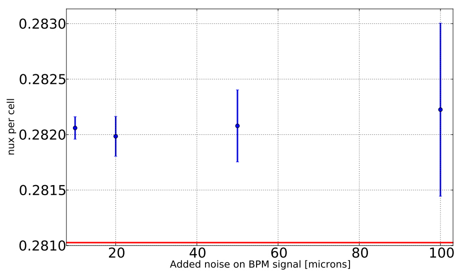

Figure 3: Effect of increasing BPM noise on simulated MIA tune measurement in cell 17 (50 bunches per event).

Effects of Energy Spread

The large chromaticity in the EMMA lattice means that the energy spread of particles in a bunch will lead to rapid decoherence of coherent betatron oscillations of the bunch. Since the BPMs record the centroid position of a bunch, decoherence will affect the BPM readings, and hence will impact the tune measurements.

The tune measurement in the first cells is little affected by the energy spread, since the beam has little chance to decohere. However, after less than one turn, the effects become noticeable. Fig. 4 shows the results of a simulated tune measurement in cell 23, with 0.5% rms energy spread on the injected bunch. We see there is a significant increase in the spread of the tune measurements compared with the case without energy spread (Figs. 1 and 2). Combined with BPM noise, the uncertainty in the tune measurement is of order 10^{-1}.

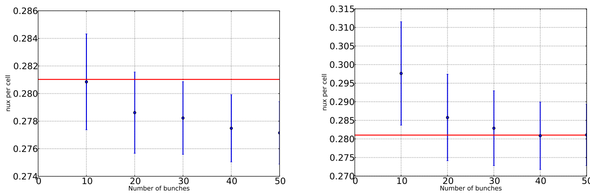

Figure 4: Effect of decoherence on MIA tune measurement in cell 23. Left: No BPM errors. Right: BPM resolution 50 μm.

SUMMARY

Conventional techniques for tune measurements cannot be applied in EMMA, because of the rapid decoherence of coherent betatron oscillations. A technique based on MIA holds promise for making reasonably accurate measurements of the phase advance between BPMs, although even in an ideal lattice (without magnet or BPM errors) systematic and random errors arise from intrinsic uncertainties in the analysis method. However, errors associated with limitations on the BPM resolution, and beam decoherence due to chromaticity, are likely to dominate the tune measurements.

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