CALCULATION OF COUPLED LATTICE FUNCTIONS FROM TURN-BY-TURN TRAJECTORY DATA IN STORAGE RINGS *

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Abstract

BPMs capable of high resolution turn-by-turn bunch position measurements are becoming increasingly widely used in electron storage rings. Analysis of the data from a set of such BPMs following the excitation of a coherent betatron oscillation can yield useful information for tuning the optics and improving machine performance. This approach to optics measurement has the benefits that the data collection is very fast, and analysis can be local, so that application is as easy for a large ring as for a small one. Here, we describe a technique for using turn-by-turn BPM data to determine lattice functions that describe the local coupling in a storage ring; this may be helpful, for example, for achieving low vertical emittance. We discuss the principles of the technique, give some examples, and consider possible limitations arising, for example, BPM gain and coupling errors.

INTRODUCTION

Tuning the optics is a key step in optimising the performance of a storage ring, and there are well-established techniques for measuring the lattice functions (Twiss parameters and dispersion) that are generally used to characterise the optics. By adjusting magnet strengths to bring the measured values of the lattice functions close to the design values, it is frequently possible to improve machine performance.

One common procedure for measuring the beta functions at the location of a particular quadrupole is to observe the variation in betatron tunes with respect to changes in strength of the quadrupole. For example, if the change in quadrupole strength is known, the horizontal beta function β_x at the location of the quadrupole can be determined from the equation:

$$\Delta\nu_x = \frac{1}{4\pi}\beta_x \, \Delta k_1 L,$$

where $\Delta\nu_x$ is the change in horizontal tune resulting from a change $\Delta k_1 L$ in the integrated normalised strength of the quadrupole. However, to determine the beta functions at a large number of quadrupoles using this technique is a lengthy process. An alternative technique is to measure, at a set of beam position monitors (BPMs), the coherent betatron oscillations resulting, for example, either from a "kick" given to a stored bunch, or from resonant excitation of the beam. In the absence of coupling and processes (such as decoherence, and synchrotron radiation) that damp the amplitude of the coherent oscillations, the horizontal coordinate at a given BPM on a turn t can be written as:

$$x(t) = \sqrt{2\beta_x J_x}\cos\left(2\pi\nu_x t + \phi_{x0}\right),$$

where β_x is the beta function at the BPM, ν_x the betatron tune, J_x the betatron action resulting from the kick given to the bunch, and ϕ_{x0} the initial (turn $t = 0$) betatron phase of the bunch at the BPM. Even if the betatron action is not known, it is possible, from measurement of the oscillation amplitude at each BPM, to determine the *relative* beta functions.

The phase of the betatron oscillation at each BPM also provides information on the optics. This can be useful since, with measurements over a reasonable number of turns, the phase advance between any two BPMs may be determined with very good accuracy. Phase advance data are already used very successfully for optics measurements and correction in storage rings [1, 2]. Since high-bandwidth BPMs with good resolution are now becoming widely available, it is interesting to consider different approaches to analysis of turn-by-turn BPM data, with the aim of making maximum use of the information that may be obtained.

Use of turn-by-turn BPM measurements for measurement of global coupling parameters has already been demonstrated [3]. Here, we discuss an analysis technique aimed at determination of the *local*, *coupled* lattice functions from turn-by-turn data. Our technique is based on using data from a set of three (or more) BPMs together with a model of the lattice between them, to determine the lattice functions at the BPMs. In principle, it is possible to fit BPM gains and magnet strengths to the data, as well as the lattice functions. However, simulations suggest that the BPM data may have a rather poor sensitivity to errors on individual magnets. This is both a drawback, in that it makes it difficult to determine individual magnet errors from the data; and an advantage, in that the magnet strengths should generally be known well enough to justify use of a model in fitting the data.

In the following sections, we first outline the relevant theory, then present some results from simulations of the Accelerator Test Facility (ATF) damping ring at KEK. Presently, only a small number of BPMs at the ATF have the capability of making turn-by-turn measurements; however, an upgrade of the BPM system in the ATF is planned for later this year, and we hope that it will soon be possible to carry out practical tests of this technique at the ATF.

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THEORY

There are various definitions in use for the lattice functions in a coupled beamline. For the present purpose it is convenient to use the definition [4]:

$$\beta_{ij}^k = N \cdot T^k \cdot N^{\mathrm{T}}$$

where (in n degrees of freedom) N is a $2n \times 2n$ matrix that normalises (i.e. transforms into a pure rotation) the single-turn transfer matrix at a given point in the lattice, i and j are indices taking values from 1 to $2n$ (corresponding to components of the cartesian phase space vector), $k = \mathrm{I}, \mathrm{II}...$ (corresponding to the normal mode of oscillation), and the matrices T^k have components $T_{ij}^k = 1$ for $i = j = 2k$ and $i = j = 2k-1$, and $T_{ij}^k = 0$ otherwise. With these definitions, the beam distribution at any point in the lattice is given by:

$$\langle x_i x_j \rangle = \sum_{k=\mathrm{I},\,\mathrm{II}...} \beta_{ij}^k \varepsilon_k,$$

where ε_k are the emittances, invariant under transport around the ring, given by:

$$\text{eigenvalues}\,(\Sigma) = \pm i\varepsilon_k,$$

where Σ is a matrix constructed from the second-order moments $\langle x_i x_j \rangle$ of the beam distribution.

The normalising transformation N_A at a point A in the lattice relates the cartesian variables to the action-angle variables. From now on, we consider two degrees of freedom, so we can write:

$$\begin{pmatrix} x \\ p_x \\ y \\ p_y \end{pmatrix} = N_A \cdot \begin{pmatrix} \sqrt{2J_{\mathrm{I}}}\cos\phi_{\mathrm{I}} \\ -\sqrt{2J_{\mathrm{I}}}\sin\phi_{\mathrm{I}} \\ \sqrt{2J_{\mathrm{II}}}\cos\phi_{\mathrm{II}} \\ -\sqrt{2J_{\mathrm{II}}}\sin\phi_{\mathrm{II}} \end{pmatrix},$$

where J_{I} and J_{II} are the invariant actions giving the amplitudes of the oscillations in each normal betatron mode, and ϕ_{I} and ϕ_{II} are the corresponding phase angles.

The normalising matrix N_A represents a transformation between two sets of canonical variables, and must therefore be symplectic: a symplectic matrix in two degrees of freedom can be specified by 10 parameters. However, we can choose the reference phase angles with respect to the cartesian coordinates by specifying values for $N_{A,12}$ and $N_{A,34}$. With a given choice of reference for the phase angles, the normalising matrix can be specified with 8 parameters. If the transfer matrix to some other point in the storage ring is known (from the strengths and positions of the magnets between the two points), then the same 8 parameters also specify the normalising matrix at this second point.

Measurements of phase and amplitude data using BPMs at three different locations (labelled A, B and C) provide 14 constraints. For each mode (I and II), we have a phase advance from BPM A to BPM B, and a phase advance from BPM B to BPM C. Assuming that mode I is associated predominantly with horizontal motion, we would identify further constraints from measurements of the amplitude of an oscillation in mode I, relative to the "x" amplitude at BPM A: specifically, we measure the "y" amplitude at BPM A, and the "x" and "y" amplitudes at BPMs B and C. Finally, assuming that mode II is associated predominantly with vertical motion, we identify the final constraints from measurements of an oscillation in mode II, relative to the "y" amplitude at BPM A: specifically, we measure the "x" amplitude at BPM A, and the "x" and "y" amplitudes at BPMs B and C.

If we know the transfer matrices between the BPMs, and we do not include BPM gains as variables, then there are 8 variables (parameters of the normalising transformation at any of the BPMs) to fit 14 constraints: the fit is over-constrained. In principle, we can use the "surplus" constraints to determine BPM gain errors or magnet focusing errors.

In a linear approximation, the BPM gains may be represented as a 2×2 matrix relating the measured beam co-ordinates to the actual beam co-ordinates. Each BPM gain matrix therefore potentially adds four variables to the fit, although if we do not know the absolute amplitudes of the induced oscillations in the two normal modes, we can exclude from the set of variables two of the components of the gain matrix of one BPM (i.e. we would fit gains relative to these components). With three BPMs, there are therefore ten additional variables; including all of these in the fit would result in the fit becoming under-constrained. We can include at most four BPM gains.

There are two different methods that we have tried for performing the fit. In the first method, we fit eight variables that parameterise the normalising transformation at A, and determine the normalising transformations at B and C by applying transfer matrices computed from an "ideal" model. In the second method, we fit 24 variables that parameterise the normalising matrices at all three BPMs, and apply the transfer matrices computed from the ideal model as "additional" constraints. In practice, we find that the second method is a little more robust, and converges more quickly, than the first method.

To perform the fit, we need a parameterisation of a symplectic matrix. The matrix exponential representation provides a convenient parameterisation:

$$N = \exp\left(S \cdot Q\right),$$

where S is a $2n \times 2n$ matrix with block diagonals:

$$S_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

If Q is any *symmetric* $2n \times 2n$ matrix, then N will be symplectic, though not necessarily (in the case of two degrees of freedom) with $N_{12} = N_{34} = 0$. The conditions on N_{12} and N_{34} cannot be simply expressed in terms of constraints on Q; however they can easily be added as additional constraints on the fit.

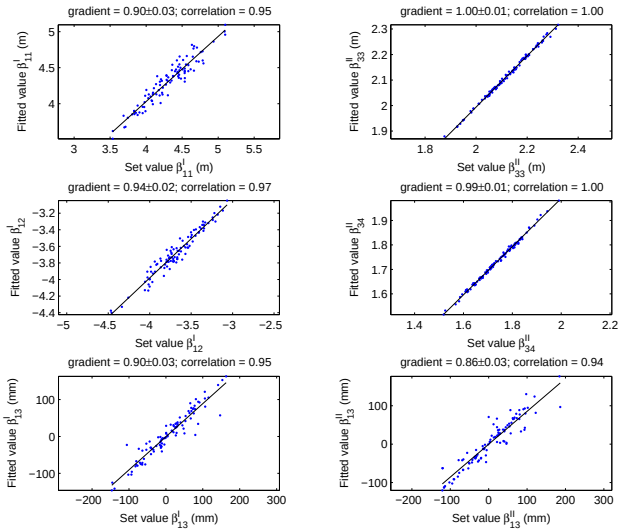

Figure 1: Selected lattice functions at the central BPM from a set of three providing data for a fit. The fitted values from turn-by-turn data (vertical axis) are plotted versus the set values resulting from applied normal and skew quadrupole strength errors. No BPM gain errors applied.

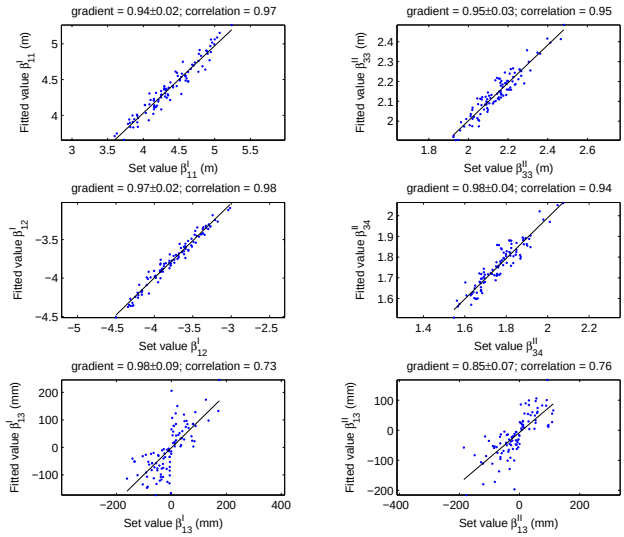

Figure 2: As Fig. 1, but with 2% rms BPM gain errors applied.

SIMULATIONS

Data were simulated by tracking particles 600 turns in a model of the ATF damping ring lattice, with initial conditions corresponding to each of the "transverse" normal modes in turn. In practice, corresponding data would be collected by recording turn-by-turn data at the BPMs, while resonantly exciting the beam at a frequency corresponding to one or other of the betatron tunes. For the results shown here, strength errors corresponding to 0.5% rms variation in the gradient were applied to all the normal quadrupoles. Random strengths with rms $0.04\,\mathrm{m}^{-1}$ integrated normalised gradient were applied to the skew quadrupoles, to generate some coupling. The coupling generated in this way would correspond, roughly, to vertical orbit offset with rms 1 mm in the sextupoles. The errors were applied only for generating the orbit data: fitting (using the second of the two methods described above) was performed using a model with the errors removed.

Fig. 1 shows the correlation between the actual and fitted values of some of the lattice functions at one BPM (BPM B, i.e. the central BPM of the set of three that provide data for the fit). The lattice functions shown are, on the left hand side of the figure: β^{I}_{11} (corresponding to β_x in the uncoupled case); β^{I}_{12} (corresponding to $-\alpha_x$); β^{I}_{13} (which gives the beam tilt $\langle xy \rangle$ dependence on ε_{I}). On the right hand side of the figure are shown: β^{II}_{33} (corresponding to β_y in the uncoupled case); β^{II}_{34} (corresponding to $-\alpha_y$); β^{II}_{13} (which gives the beam tilt $\langle xy \rangle$ dependence on $\varepsilon_{\mathrm{II}}$). We see that there is a good correlation between the actual and fitted values, particularly for β^{II}_{33} and β^{II}_{34}. The correlation for the lattice functions describing local coupling, β^{I}_{13} and β^{II}_{13} is perhaps less good than in the other cases; though this may simply reflect that these lattice functions are significantly smaller than the others.

Fig. 2 shows the results of a simulation using the same conditions as for Fig. 1, but with 2% rms BPM gain errors applied. While the correlations for the "in-plane" lattice functions are still good, there is significant deterioriation for the cases of β^{I}_{13} and β^{II}_{13}. This may again be a consequence of the magnitude of these functions, compared to the others.

The accuracy of the fit relies on having a good model of the lattice. It may be possible to improve the reliability of the fit by carrying out the fitting iteratively. For example, once the lattice functions have been fitted at different locations around the ring, the quadrupole strengths leading to these lattice functions can be determined. Then, fitting can be repeated using the "improved" model. There remains scope for further improving the quality of the fit that can be obtained: for example, data from a larger number of BPMs could be used to provide additional constraints on the fit.

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