Application of a new code to compute transfer maps and describe synchrotron radiation in arbitrary magnetic fields

David Newton* and Andy Wolski
University of Liverpool, The Cockcroft Institute

Abstract

An analytic tracking code has been developed to describe an arbitrary magnetic field in terms of it's generalised gradients [1] and multipole expansion, which is used with a second order symplectic integrator [2] to calculate dynamical maps for particle tracking. The modular nature of the code permits a high degree of flexibility and allows customised modules to be integrated within the code framework. Several different applications are presented and the speed, accuracy and flexibility of the algorithms are demonstrated. A module to describe synchrotron radiation emission is described and it's application to a helical undulator is demonstrated.

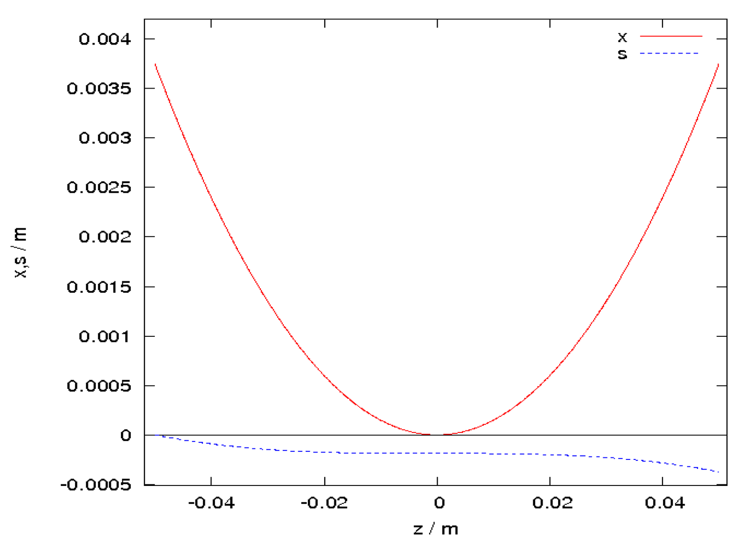
Benchmarking the synchrotron calculation

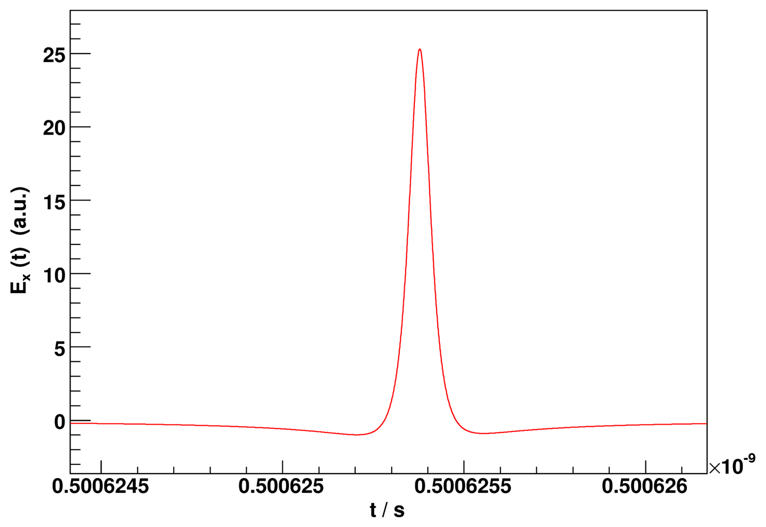
The frequency distribution of radiation emitted by a charged particle in circular motion can be calculated analytically [4], allowing a comparison with the numerical calculation.

$$\frac{d^2 I(\omega,\hat{n})}{d\Omega d\omega} = \frac{3}{4}\frac{q^2}{4\pi^3\epsilon_0 c}\gamma^2\left(\frac{\omega}{\omega_c}\right)(1+\gamma^2\psi^2)^2\left[K^2_{2/3}(\zeta)+\frac{\gamma^2\psi^2}{1+\gamma^2\psi^2}K^2_{1/3}(\zeta)\right]$$

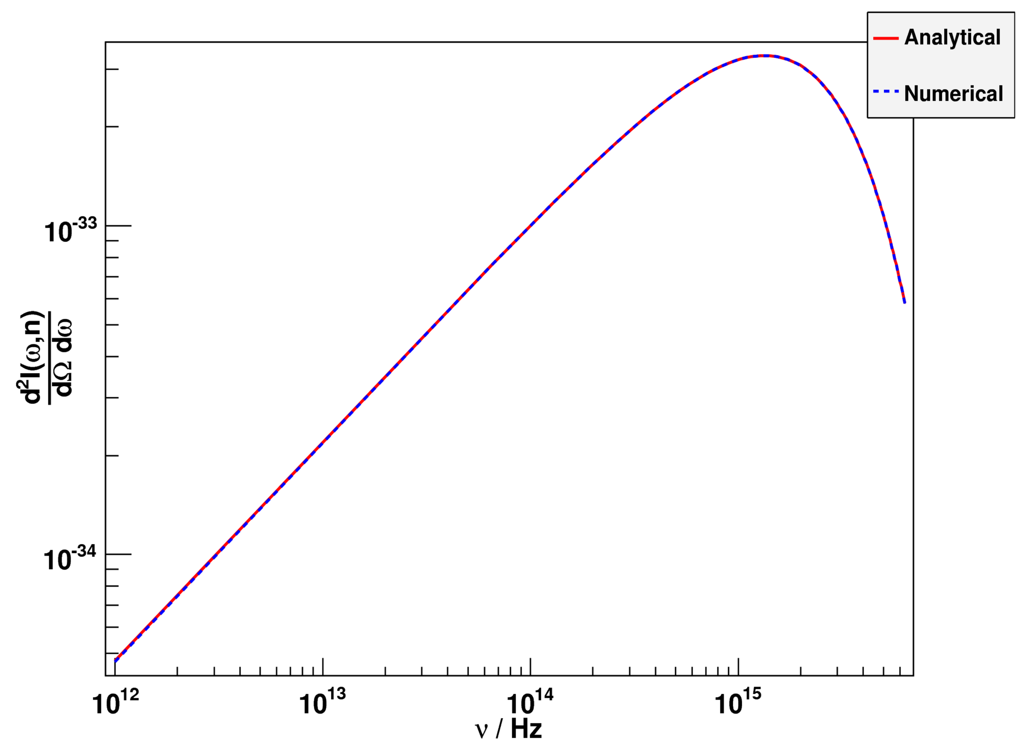
$\omega_c = \frac{3}{2}\gamma^3\frac{|\vec{c}|}{R}$, $\zeta = \frac{\omega}{2\omega_c}(1+\gamma^2\psi^2)^{3/2}$, ψ is the angle between the z coordinate and $\vec{n}$ (0 in this case) and K are the fractional modified Bessel functions.

Using the procedure given in [3] a normal 1 T dipole field was characterised and an electron of energy 100 MeV was tracked through 10,000 integration steps. In such a field, the particle describes an arc with radius ~0.33 m. At an observation point, in the same plane as the arc, and at a distance of 10 cm, tangential to the arc, the frequency distribution of the synchrotron radiation was calculated analtyically and compared to the numerical result.





The phase space variable x and s (the path length relative to the reference trajectory) are shown above (left). The electron describes an arc of radius ~0.33 m in the dipole field. The electric field observed at an observation point 10 cm from the electron is shown above (right). Because the observation point is in the same plane as the acceleration vector of the electron only the E_x component of the field is present.

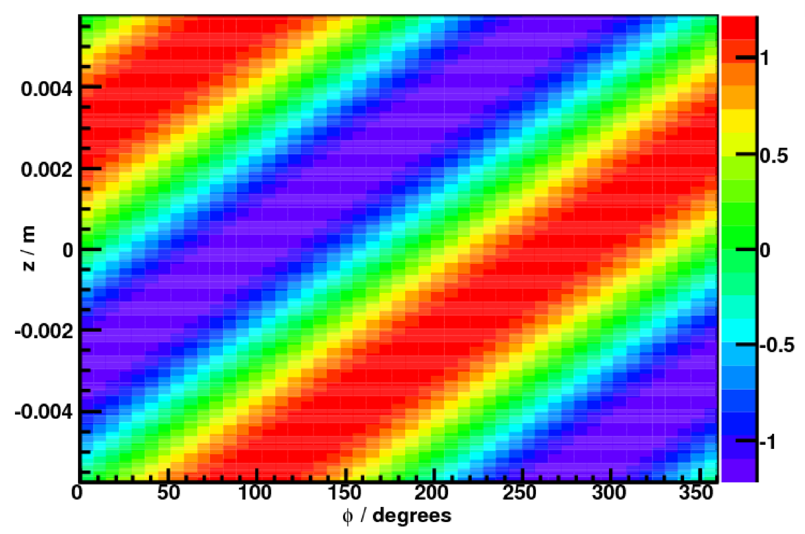


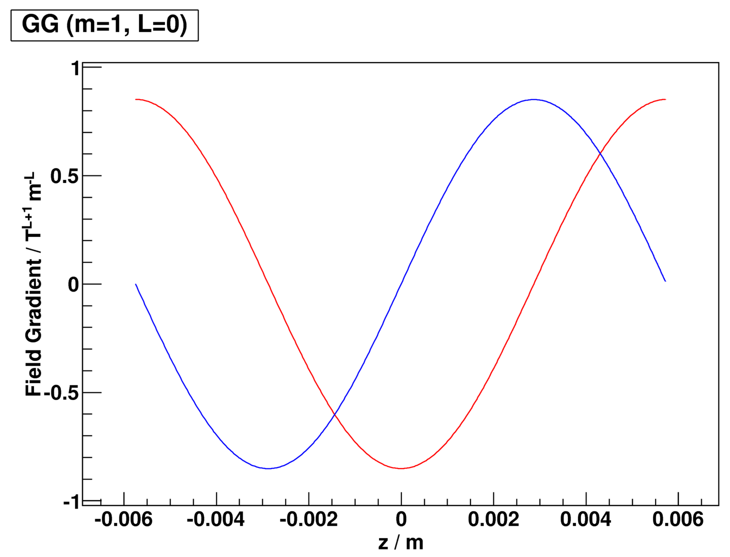
Above is shown the differential power spectrum (J Sr^{-1}), of an electron in a circular orbit, radiated into unit frequency interval. The analytical curve has a peak power of 3.300×10^{-33} J Sr^{-1} at 1.399×10^{15} Hz. The numerical curve shows a maximum discrepancy of (1.6×10^{-36} J Sr^{-1}) which occurs at this peak frequency. The curves agree to better than 0.05%.

Application to the ILC helical undulator

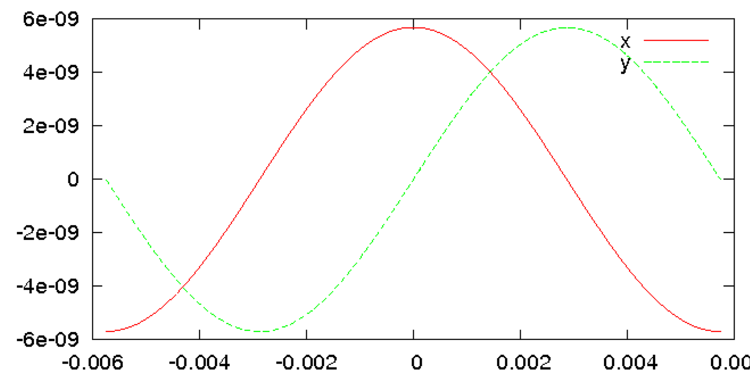
The ILC Reference Design Report [5] calls for a 150 GeV electron beam to be transported through a 150 m helical undulator, emitting 10 MeV synchrotron photons which will be used to produce electron-position pairs. The helical undulator will have a periodicity of 1.15 cm, with each module containing 155 periods, giving a module length of 1.79 m.

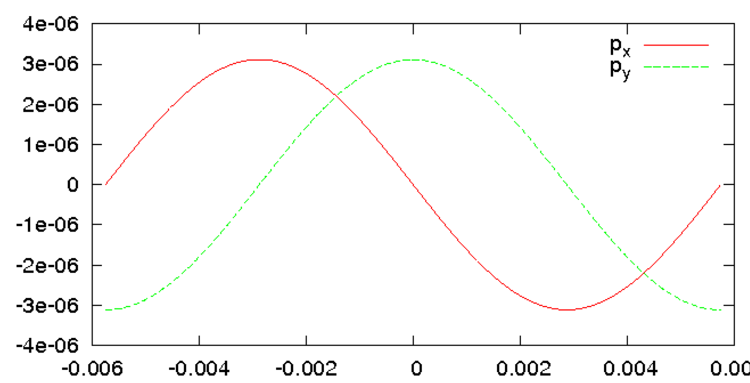
A numerically calculated field map of a single period of the undulator system was used as an input map for the code. Following the procedure outlined in [3], the generalised gradients were calculated and used to generate an analytical description of the magnetic vector potential. Finally the phase space vector (x,p_x,y,p_y,s,δ) was integrated over the length of the undulator system.

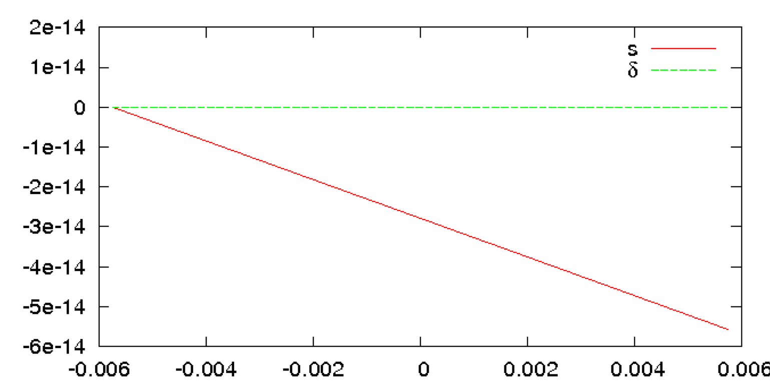




The plot above (left) shows the radial component, Bρ, of the magnetic field, projected onto the surface of a cylinder, with radius 0.0019 m. This field map was used to generate the generalised gradients, of which $C^{[0]}_1$, the normal (red) and skew (blue) dipole components are shown above (right). These components correspond to the on-axis B_y and B_x field.







A second order symplectic integrator was used to track the phase space vector over a single period using 10,000 integration steps (above).

At each integration step, the electric field, induced by an accelerated charge, is given by:

$$\vec{E}(t) = \frac{e}{4\pi\epsilon_0 c}\frac{1}{(1-\vec{n}\cdot\vec{\beta})^3}\times\left(\frac{(1-|\vec{\beta}|^2)(\vec{n}-\vec{\beta})}{|\vec{R}|^2}+\frac{\vec{n}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{c|\vec{R}|}\right)_{RET}$$

where, $\vec{\mathbf{R}}$ is the vector from the charged particle to a predefined observation point, with unit vector **n**.
A Fourier transform of the resultant field gives the frequency distribution:

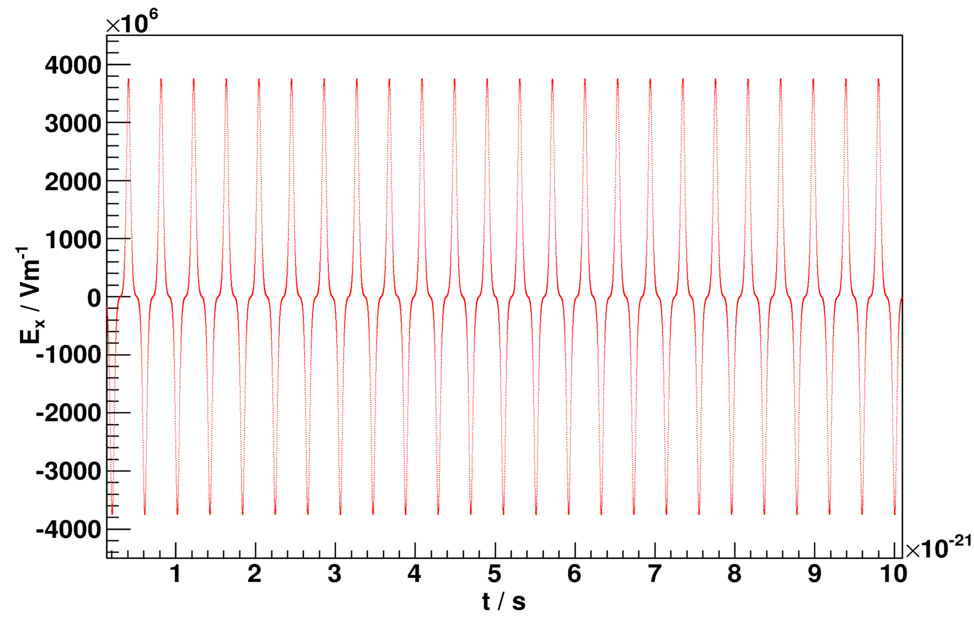
$$\frac{d^2U}{d\Omega dt} = \epsilon_0 c E^2 r^2 = \vec{G}(t)^2$$

$$\vec{G}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\vec{G}(t)\exp(\iota\omega t)dt$$

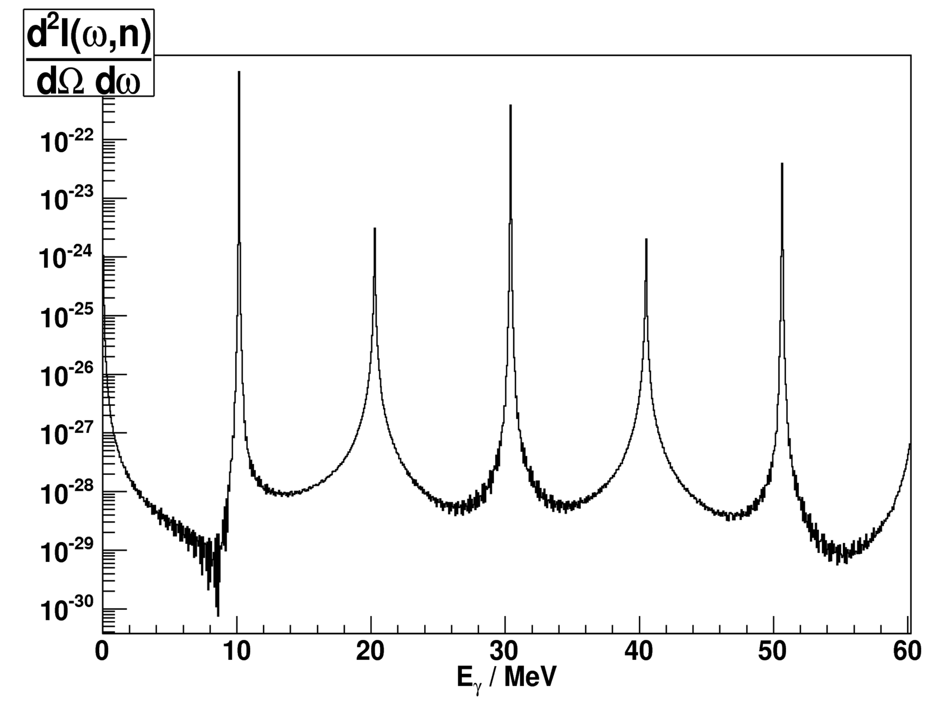
$$\frac{d^2 I}{d\Omega d\omega} = |\vec{G}(\omega)|^2 + |\vec{G}(-\omega)|^2$$

Synchrotron emission in the ILC helical undulator

An analytic description of a single period of the ILC helical undulator was used to track the phase space vector of a 150 GeV electron through the whole undulator module (155 periods) – a total length of 1.79 m. The electric field at a point 10 m from the end of the module (on the undulator axis) was calculated, and a Fourier transform was performed to find the frequency distribution of the emitted radiation.



The plot above shows a portion of the electric field (x-component) measured 10 m from the end of the undulator module. The total pulse has a duration of 8×10^{-20} s.



The photon energy distribution of radiation emitted by a 150 GeV electron passing through one section (1.79 m, 155 periods) of a helical undulator. The first harmonic occurs at an energy of 10.1 MeV

Conclusion

The C++ code described here and in [3] is designed to give a fast, accurate method of characterising magnetic fields, performing numerical particle tracking and calculating analytical transfer matrices. The code is still in the development phase – the synchrotron radiation module in particular is not yet optimised. The modular design allows additional functionality to be implemented with minimum reprogramming. The ability to calculate analytic transfer maps means that the synchrotron radiation module can be optimised to reduce the number of calculations, with no loss of accuracy in the tracking code. The calculation can also be started at any point within the magnet system, making the code ideally suited to parallel or pseudo-parallel (e.g. Condor) applications.

References

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[2] Y Wu, E. Forest, and D. S. Robin, “Explicit higher order symplectic integrator for s-dependent magnetic fields”, Phys. Rev. E, 68 (2003), 046502
[3] D. Newton and A. Wolski, “Fast, accurate calculation of dynamical maps from magnetic field data using generalised gradients”, these proceedings
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[5] ILC Reference Design Report (2007)
http://www.linearcollider.org/cms/?pid=1000437

* david.newton@stfc.ac.uk, This work was supported by the Science and Technology Facilities Council, UK