Beam Dynamics in EMMA:

Models and (some) Measurements

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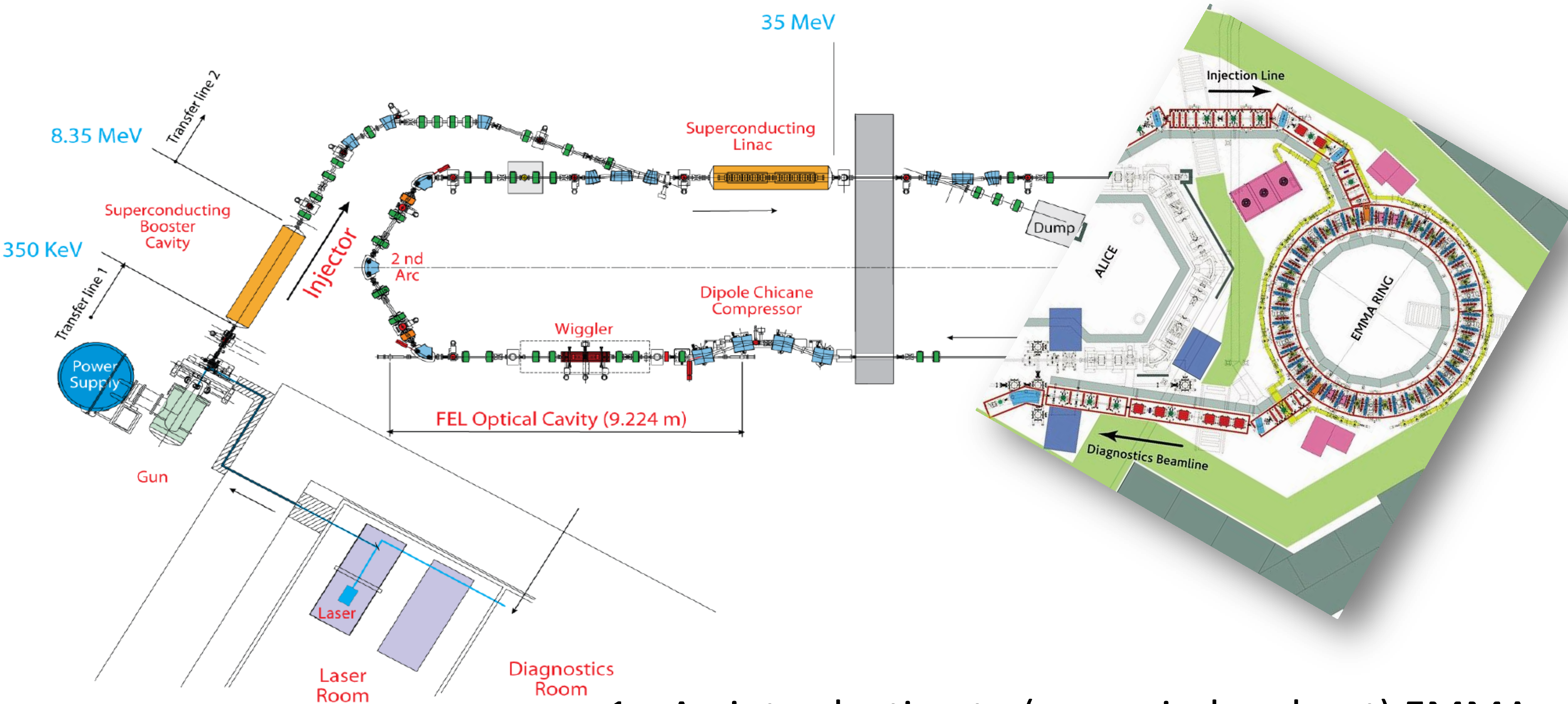


1. An introduction to (or reminder about) EMMA.
2. Beam dynamics in EMMA: what's so special?
3. How to build a computer model for EMMA.
4. How well does the model work?

EMMA: Electron Model for Many Applications

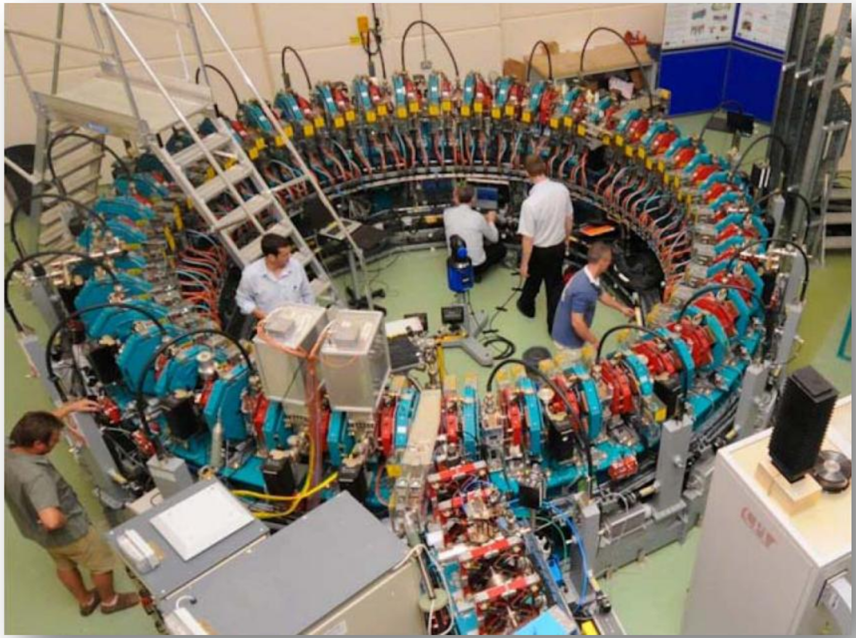
EMMA is a prototype non-scaling fixed-field alternating gradient (nsFFAG) accelerator.

- Alternating gradient: lattice composed of alternate focusing and defocusing quadrupoles (which also steer the beam).
- Fixed-field: magnet strengths are fixed during acceleration (nominally from 10 MeV to 20 MeV).
- Non-scaling: beam trajectory changes during acceleration.

Potential applications:

- Muon collider.
- Proton therapy.
- Accelerator-driven (thorium) reactors.

EMMA provides an "electron model" for the above (muon or hadron) applications.



Beam Dynamics in EMMA

A single bunch on each pulse is injected from ALICE.

- Bunch charge is approximately 60 pC.
- Energy is variable, but typically 12 MeV.
- Pulse repetition rate is 10 Hz.

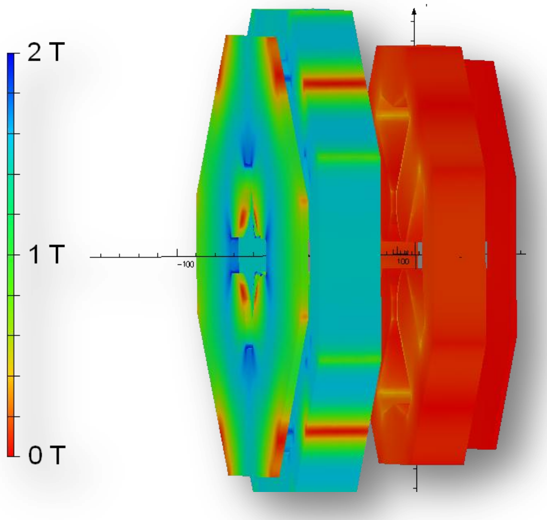
Quadrupole magnets in EMMA simultaneously steer and focus.

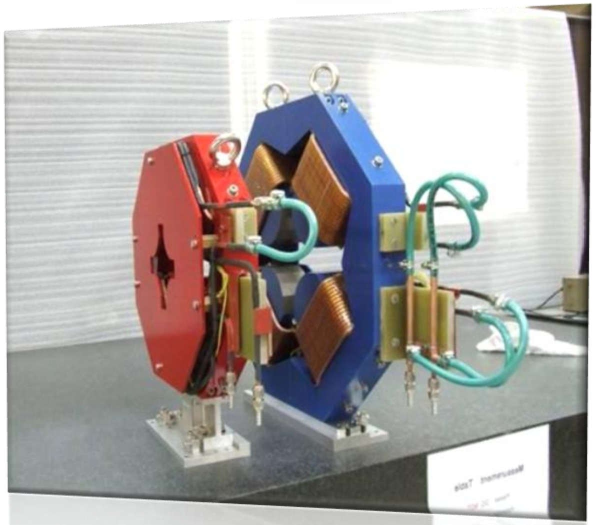
- Strengths and positions can be adjusted to control the lattice properties.

The bunch is accelerated over a few dozen turns, by a number of RF cavities placed around the ring.

The bunch is extracted into a diagnostics line.

Injection and extraction are tricky: we will only discuss what happens during acceleration, once the bunch is in the ring.



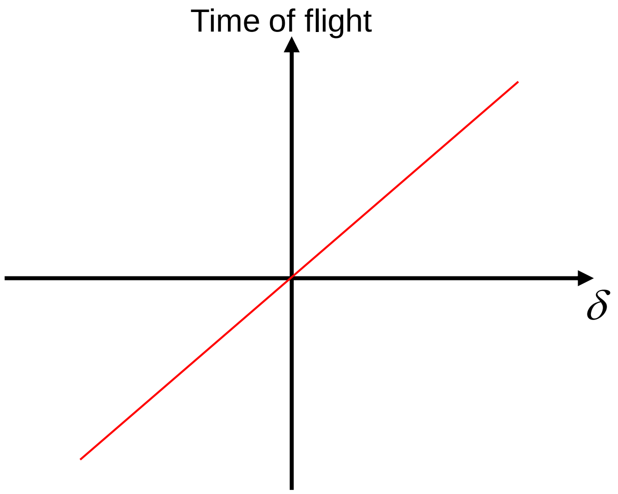


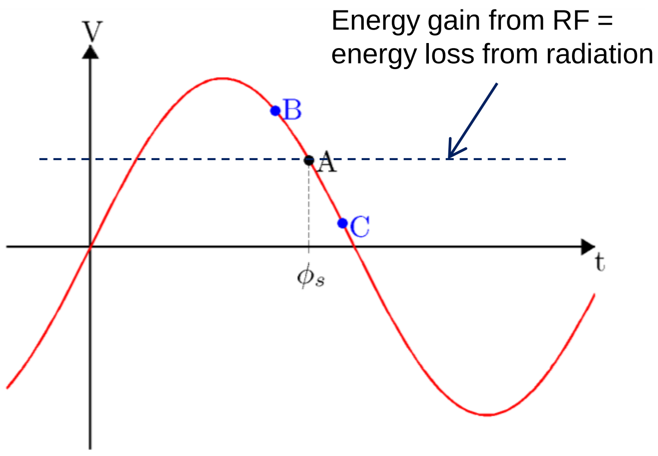
Longitudinal Dynamics in a "Normal" Synchrotron

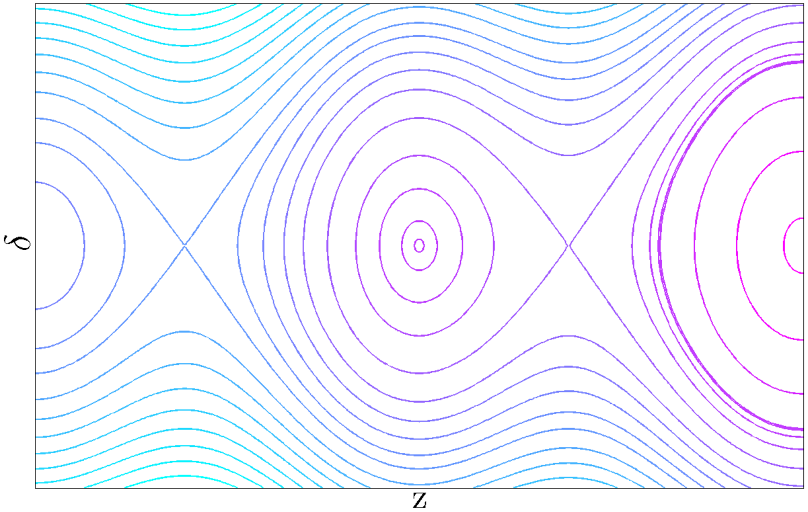
Time of flight (revolution period) increases with energy.

A particle arriving at an RF cavity later than it should receives less energy than it needs to replace synchrotron radiation losses.

If the particle is not too far from the "correct" phase, stable synchrotron oscillations result.





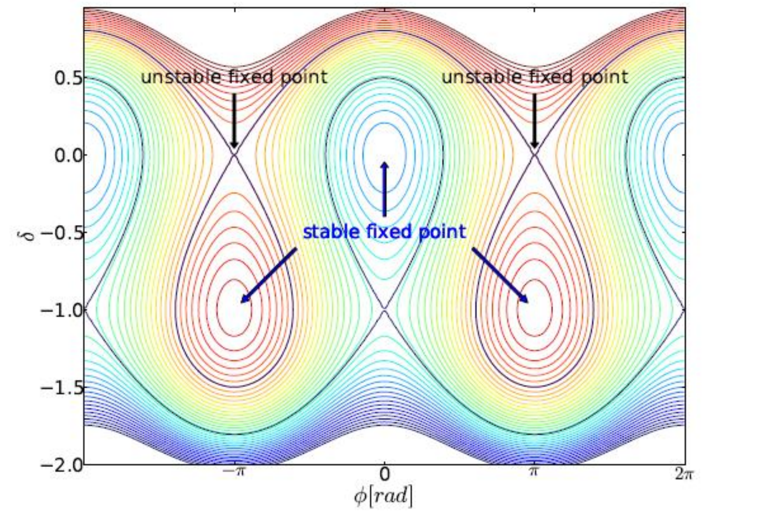


Longitudinal Dynamics in a Non-Scaling FFAG

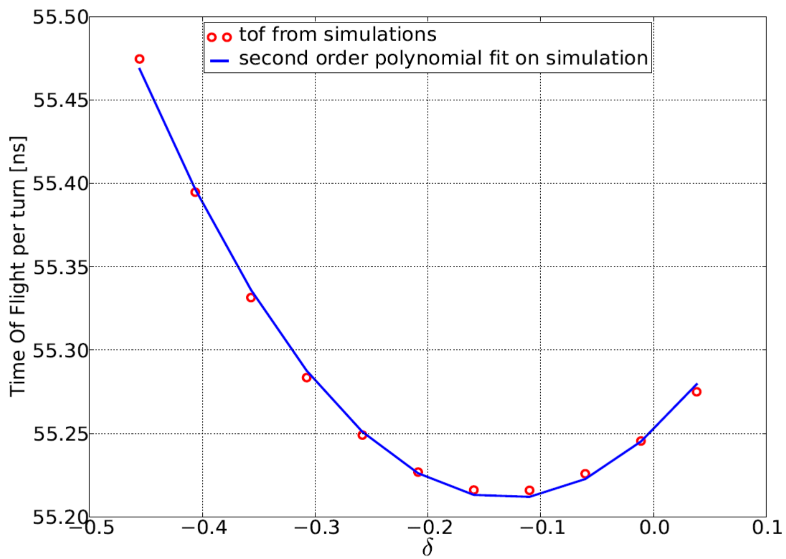
The lattice is designed so that the time-of-flight as a function of energy is a parabola.

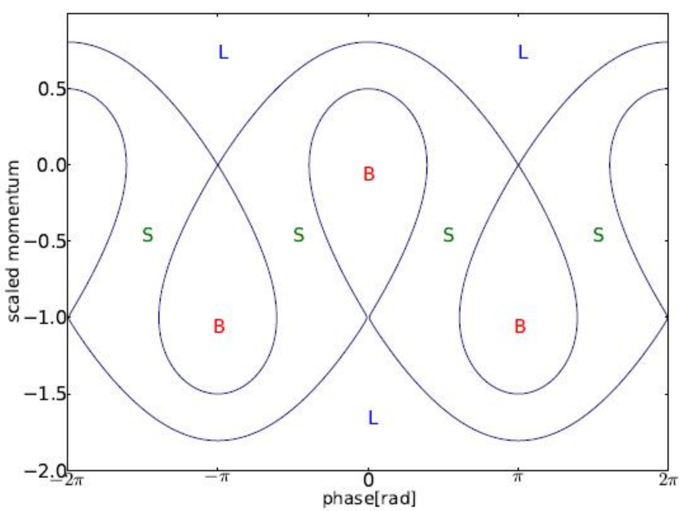
The shape of the longitudinal phase space is quite different.

There are still stable buckets: but now a "serpentine acceleration" channel can be opened.



(a)





Modelling the Beam Dynamics in EMMA

The goal is to construct a computer model that can predict the beam behaviour in EMMA for different lattice configurations (quadrupole strengths and positions).

Achieving this goal will:

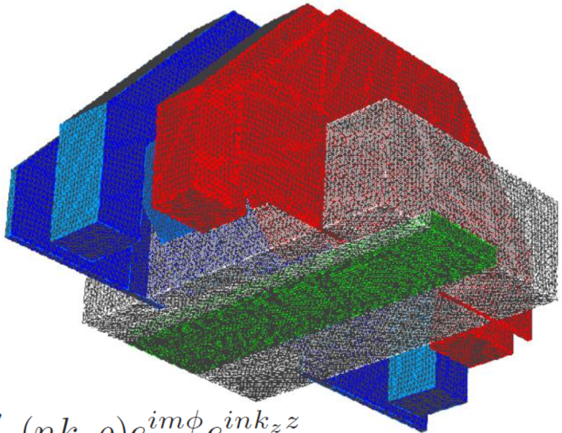
- demonstrate understanding of the beam dynamics in a ns-FFAG;
- provide a useful tool for lattice tuning and optimisation.

The challenges are:

- tracking particles with good accuracy through the complex magnetic field configurations in EMMA (short lengths and wide apertures);
- extracting the dynamical properties with sufficient computational efficiency, to provide a practical optimisation tool.

Constructing the Model in Four Steps

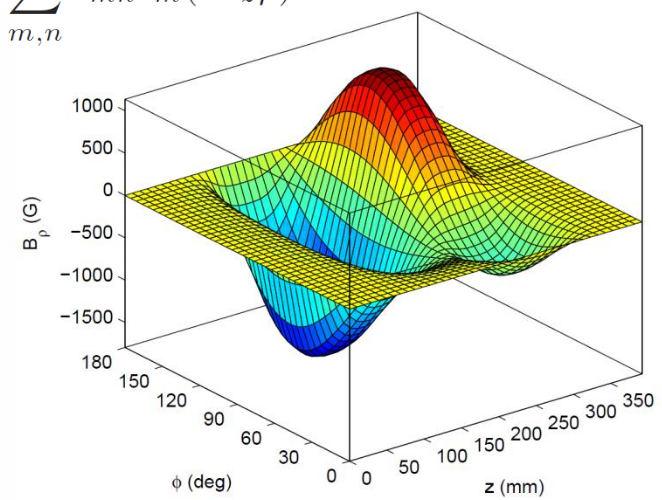
1. Model the magnetic fields in a single EMMA cell for a given lattice configuration.
   - i.e. solve Maxwell's equations numerically, using a finite-element code.
2. Fit an analytical series to the numerical field data.
   - Field components expressed as series in the co-ordinates: fitting involves finding the correct coefficients.
3. Construct a transfer map for the dynamics through the given field.
   - Final values of the phase space variables are expressed as functions (e.g. power series) of the initial values.
4. Analyse the transfer map to extract the dynamical properties of the lattice.
   - Trajectory, tunes, chromaticity, time-of-flight versus energy…



$$B_\rho = \sum_{m,n} a_{mn} I'_m(nk_z\rho) e^{im\phi} e^{ink_z z},$$

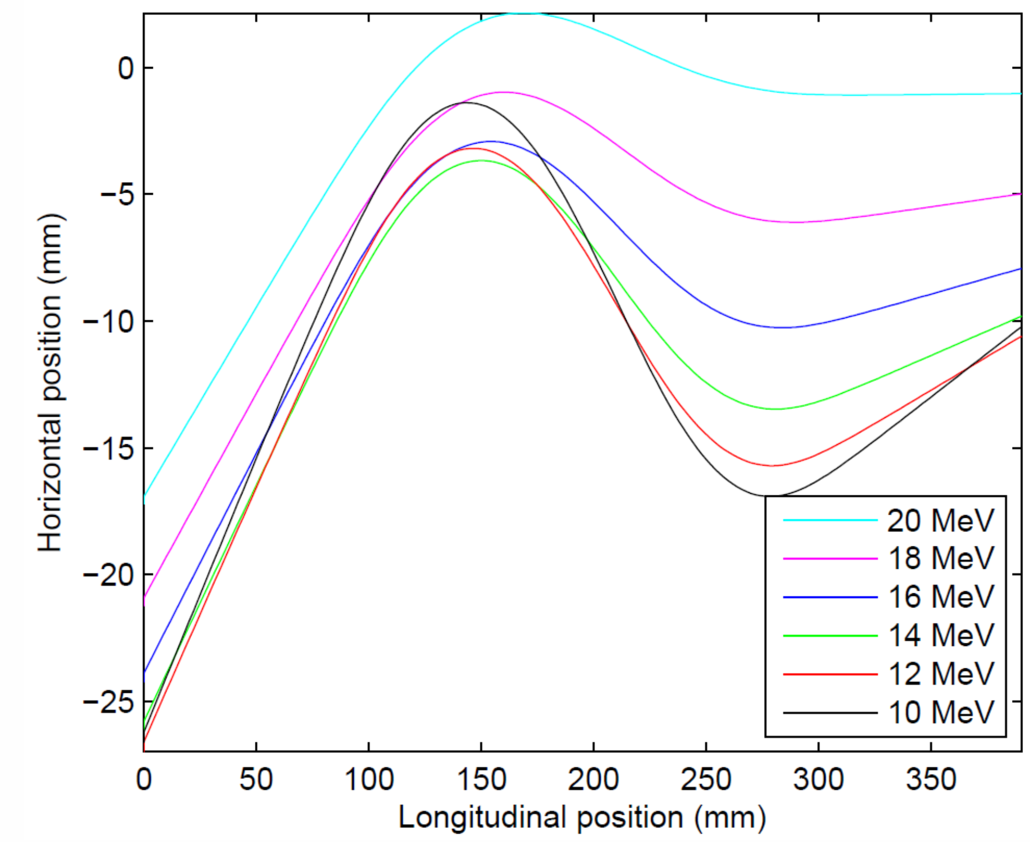
$$B_\phi = i \sum_{m,n} a_{mn} \frac{m}{nk_z\rho} I_m(nk_z\rho) e^{im\phi} e^{ink_z z},$$

$$B_z = i \sum_{m,n} a_{mn} I_m(nk_z\rho) e^{im\phi} e^{ink_z z}.$$



Trajectories Through an EMMA Cell

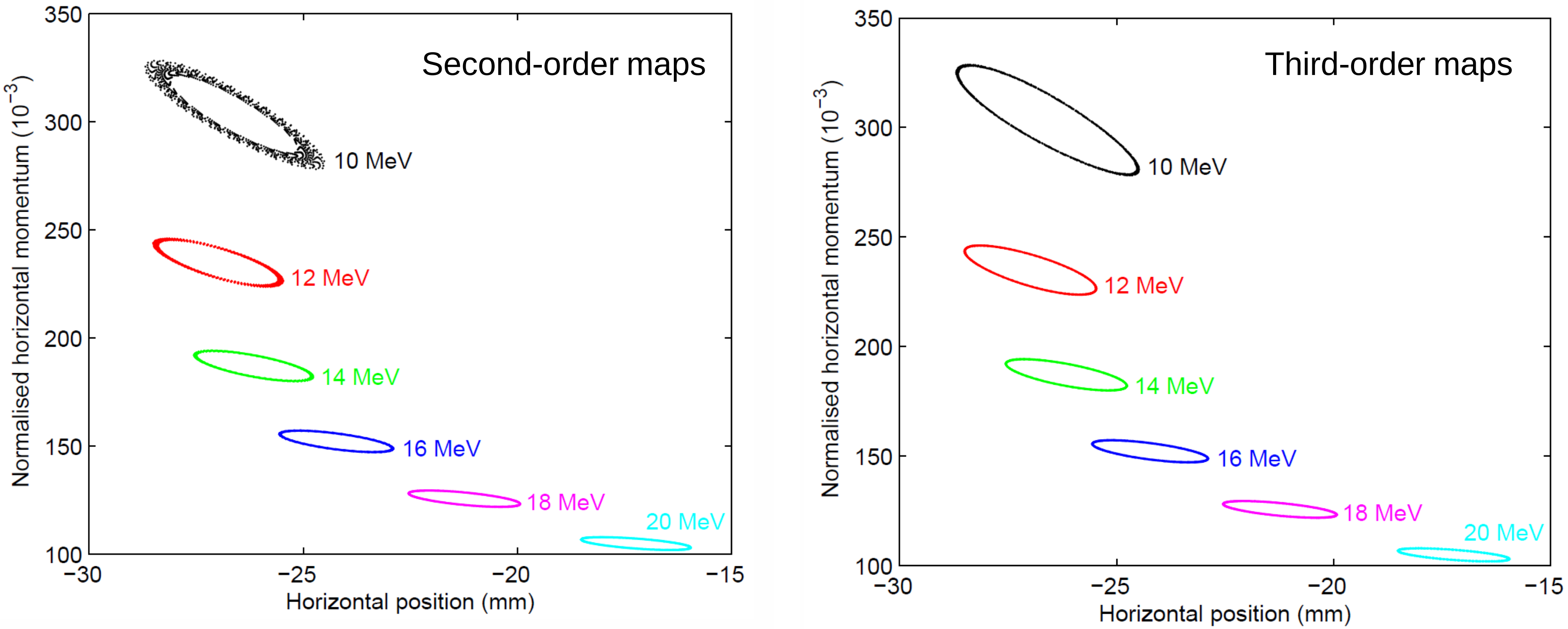
Controlling the trajectories through an EMMA cell is critical to achieving the required curve for time-of-flight versus energy (and hence for achieving acceleration).



Transverse Dynamics: Betatron Oscillations

Truncated power-series maps are (in general) non-symplectic.

The effects can be significant, especially if the maps are truncated at low order.

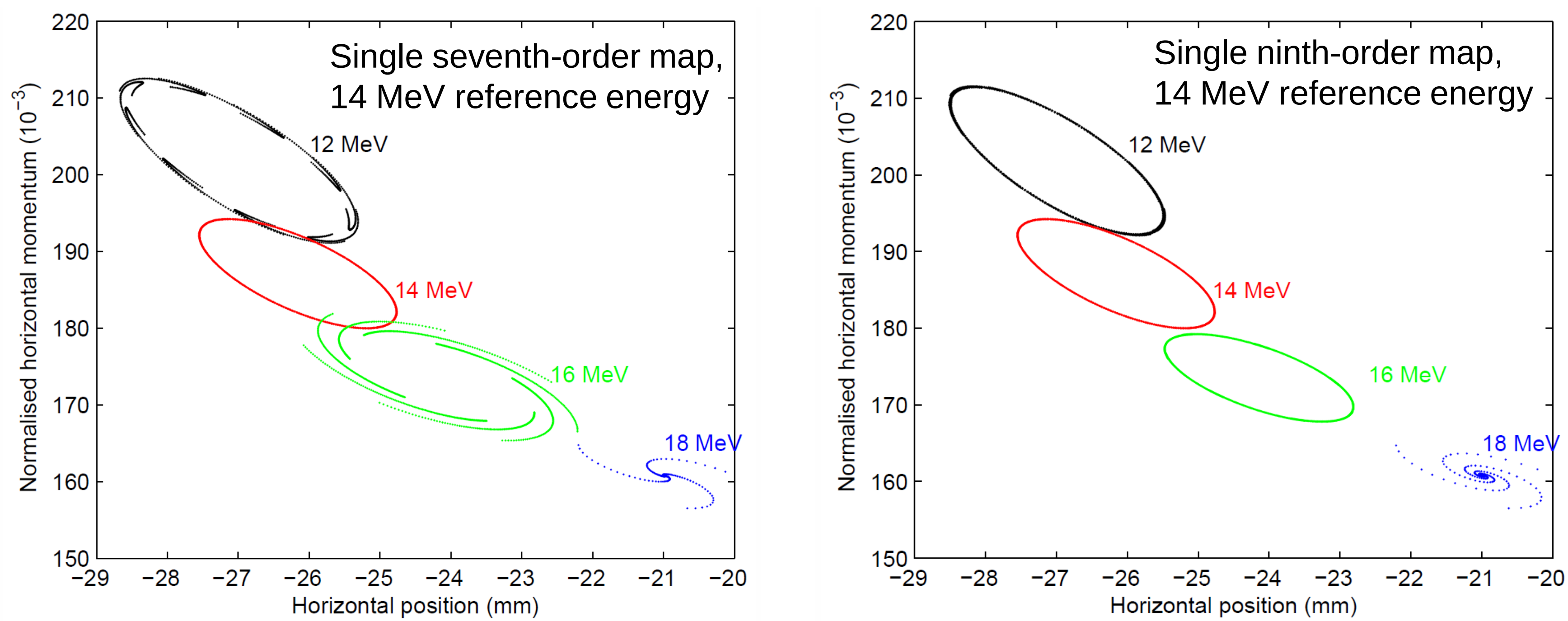


Note: the above figures use a separate map calculated for each energy.

To model the acceleration process, we need a single map covering a range of energies…

Maps with Energy Deviation

During acceleration in EMMA, the energy deviation (assuming fixed reference energy) takes much larger values than the transverse variables.



To achieve reasonable accuracy, we need much higher order maps when including acceleration. But even a ninth-order map is not sufficient: we have to change the reference energy at some point(s) during the acceleration.

Spanning the Space of Lattice Configurations

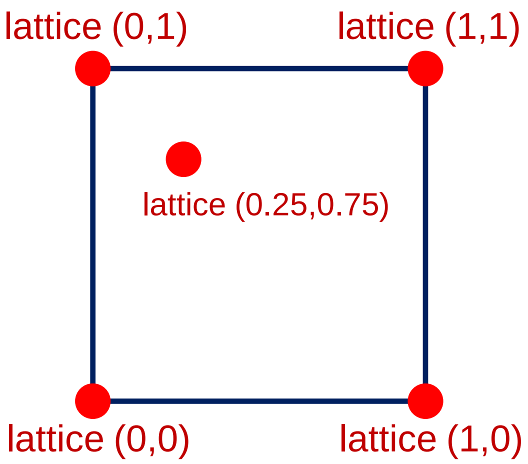
Each lattice configuration in EMMA is specified by four variables: the strengths and positions of the two magnets in each cell.

Characterising the properties of any given lattice either numerically (using e.g. Zgoubi) or using transfer maps (e.g. COSY) takes around half an hour.

Optimising the lattice for desired properties requires (in general) characterising many different lattices.

Optimisation based on numerical tracking is not really practicable.

But using transfer maps, we can "estimate" the transfer map for any desired configuration by interpolation between known reference configurations, and then obtain the dynamical properties directly from the map.



Optimisation based on interpolation of transfer maps can be very fast...

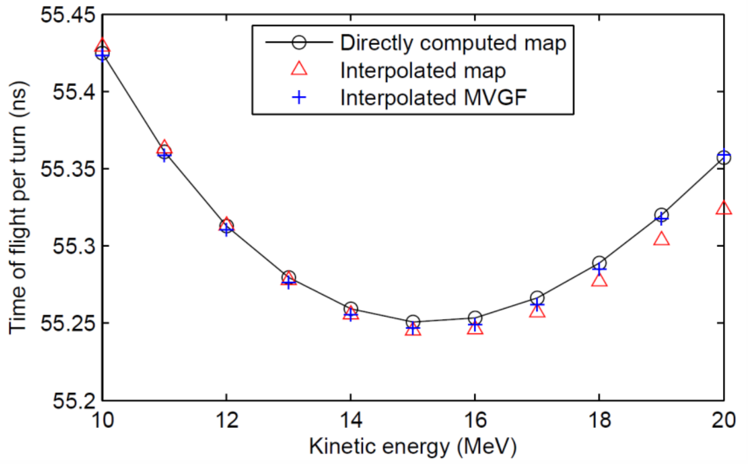
... *but is it accurate?*

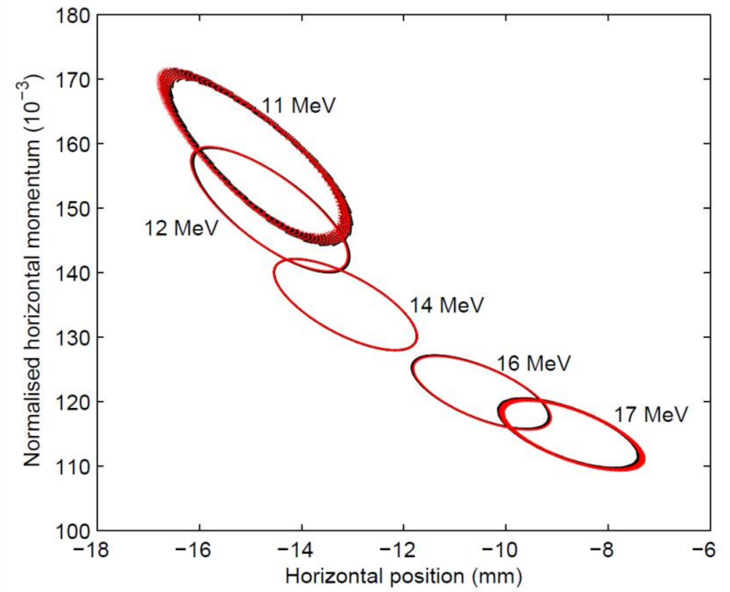
Obtaining Transfer Maps by Interpolation

Comparison between maps obtained by interpolation and by direct computation suggest that interpolation may be reasonably accurate.

However, without applying a constraint to force the interpolated map to be symplectic, the effects of the symplectic error become very obvious.

This can affect the computation of important quantities, such as the betatron tunes.





Mixed Variable Generating Functions

A mixed-variable generating function expresses a transfer map as a single function, rather than a set of functions relating the new values of the dynamical variables to the old values.

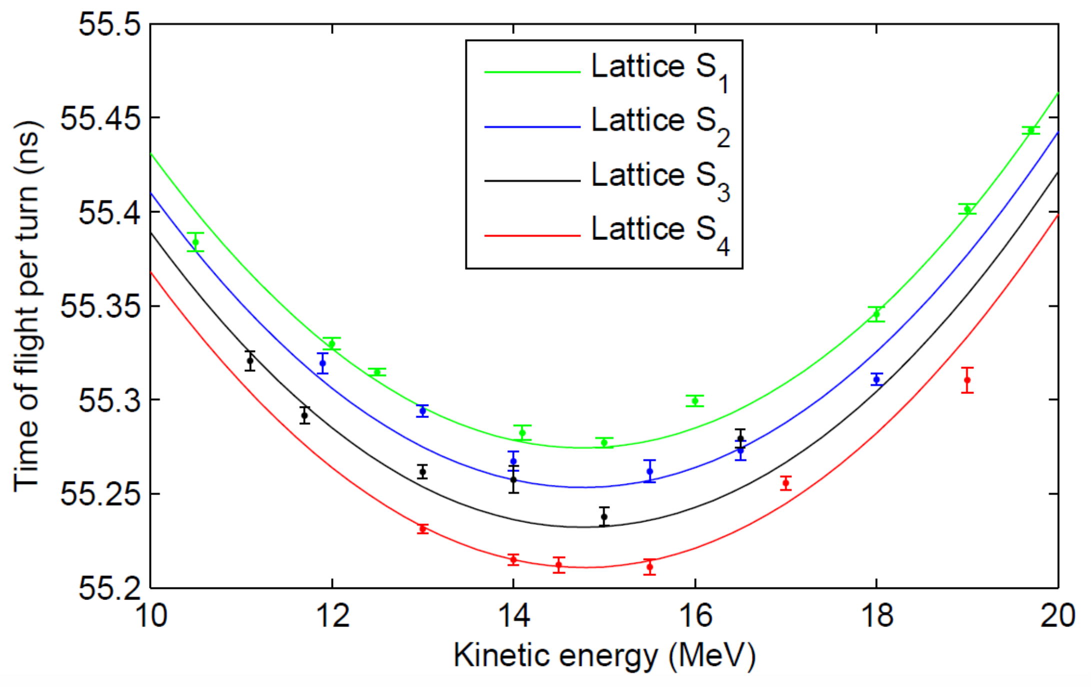
$$
\begin{aligned}
x_2 &= \frac{\partial F(x_1, p_{x2})}{\partial p_{x2}} \\
p_{x1} &= \frac{\partial F(x_1, p_{x2})}{\partial x_1},
\end{aligned}
$$

A map expressed in the form of a mixed-variable generating function is guaranteed to be symplectic.

We can improve the interpolation by converting the power-series maps to mixed-variable generating function maps, and then interpolating the mixed-variable generating functions.

A Predictive Model for EMMA

1. The model lattice is fitted (using an optimisation routine) to a measured time-of-flight curve.
2. The fitted model does not agree terribly well in absolute terms to the lattice set up in the machine. However…
3. …the model correctly predicts the effects of *changes* to the lattice configuration.



Measured time-of-flight (data points with error bars) compared with predictions from the model (curves).

The lattices S_1 etc. are obtained by making specified changes from a fitted initial lattice configuration.

Conclusions

We have made significant progress towards the development of a predictive model for EMMA, based on transfer maps representing the dynamics in accurate models of the magnetic fields.

Some issues still remain: in particular, why does the fitted model not agree better in absolute terms with the known experimental configuration?

References:

- Y. Giboudot, "Study of beam dynamics in nsFFAG EMMA with dynamical map", PhD Thesis, Brunel University (July 2011).
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