Lecture 6

- Collisions
  - Impulse and Linear Momentum
  - Elastic Collisions
  - Inelastic Collisions

Collisions

- Definition

  The objects participating in a collision exert a relatively strong force on each other for a relatively short time.
- Examples involving contact:
  - snooker balls;
  - cricket bat and ball.
- Examples without contact:
  - “sling-shot” collision of satellite and planet;
  - Rutherford’s alpha particles colliding with gold nucleii.

Impulse

- The impulse of a force **F**(t) between times t_i and t_f is defined by:

$$\mathbf{J} = \int_{t_i}^{t_f} \mathbf{F}(t)\, dt$$

- Constant force, impulse is product of force and the time it acts.
- Varying force, impulse is the product of the average force and the time it acts.
- The SI unit of impulse is Kg m s^{-1}
- Note that impulse is not work.

Impulse and Momentum

- From Newton's second law we see that the impulse acting on a body determines the change in momentum:

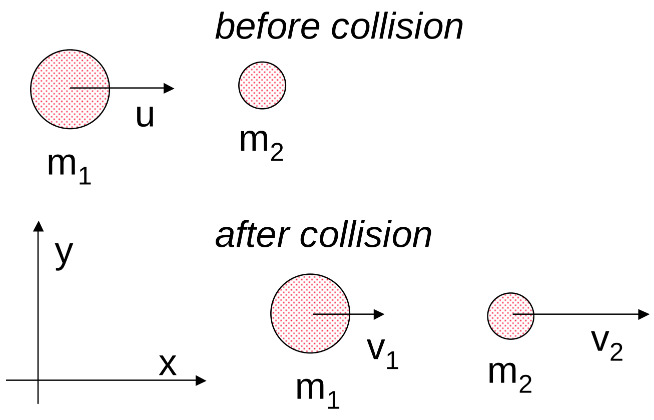
$$\mathbf{F} = \frac{d}{dt}\mathbf{p}$$

$$\int_{t_i}^{t_f} \mathbf{F} dt = \int_{\mathbf{p}_i}^{\mathbf{p}_f} d\mathbf{p}$$

$$\mathbf{J} = \mathbf{p}_f - \mathbf{p}_i$$

Elastic Collisions

- Elastic collisions are collisions in which the total kinetic energy remains constant.
- Example, collision between two objects in one dimension, in frame in which "target" initially at rest.



Elastic collisions cont.

- Conservation of momentum

$$m_1 u = m_1 v_1 + m_2 v_2$$

- Constant total kinetic energy

$$\frac{m_1 u^2}{2} = \frac{m_1 v_1^{\,2}}{2} + \frac{m_2 v_2^{\,2}}{2}$$

- Two unknowns, two equations, so can solve, find:

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u$$

$$v_2 = \frac{2m_1}{m_1 + m_2} u$$

Elastic collisions cont.

- Motion of c. of m., before collision:

$$u_{cm} = \frac{m_1 u}{m_1 + m_2}$$

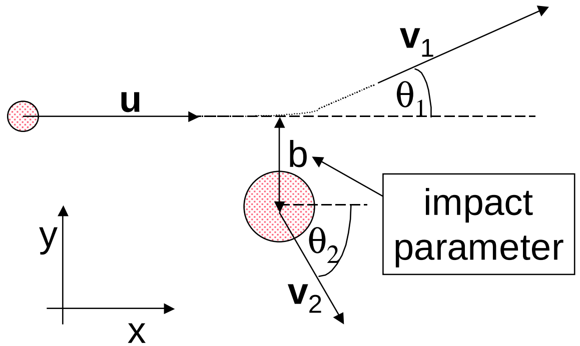
- After collision, using conservation of momentum:

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{m_1 u}{m_1 + m_2} = u_{cm}$$

Elastic collisions cont.

- Example, elastic collisions in 2D



- Momentum conservation:

$$m_1 \mathbf{u} = m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2$$

Elastic collisions cont.

- Conservation of kinetic energy

$$\frac{m_1 u^2}{2} = \frac{m_1 v_1^{\ 2}}{2} + \frac{m_2 v_2^{\ 2}}{2}$$

- Four unknowns, three equations:
  - Can obtain relationships between variables but cannot solve without further measurement.
  - Typically scattering angle or energy of one of outgoing particles (e.g. Rutherford scattering experiment).
- <u>Exercise</u>, show that motion of c. of m. unaffected by collision.

Elastic collisions cont.

- Example, snooker balls, all masses same:

$$\mathbf{u} = \mathbf{v_1} + \mathbf{v_2}$$

$$\mathbf{u}^2 = \mathbf{v_1}^2 + 2\mathbf{v_1} \cdot \mathbf{v_2} + \mathbf{v_2}^2$$

- Also have

$$u^2 = v_1^{\ 2} + v_2^{\ 2}$$

- See that cue ball and target ball move away from one another at right angles (no friction, spin!) as:

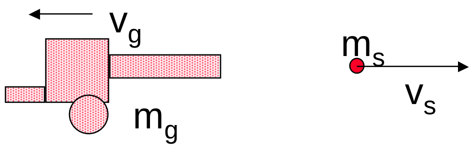
$$\mathbf{v_1} \cdot \mathbf{v}_2 = 0$$

$$\cos(\theta_1 + \theta_2) = 0$$

$$\theta_1 + \theta_2 = \frac{\pi}{2}$$

Inelastic Collisions

- In inelastic collisions the K.E. is not conserved.
- K.E. may increase, e.g. explosion, nuclear decay (internal energy converted to K.E.).
- K.E. may decrease, car crash, nuclear collision in which nucleii are excited (K.E. converted to internal energy).
- Example in one dimension, gun firing shell. Energy released in explosion B.



Inelastic collisions cont.

- Initial momentum zero, so from conservation of momentum:

$$0 = m_g v_g + m_s v_s$$

$$v_s = -\frac{m_g v_g}{m_s}$$

- Energy (not just K.E.!) conservation:

$$B = \frac{m_g {v_g}^2}{2} + \frac{m_s {v_s}^2}{2}$$

$$= \frac{m_s {v_s}^2}{2m_g}(m_s + m_g)$$

$$v_s = \sqrt{\frac{2m_g B}{m_s(m_s + m_g)}}$$

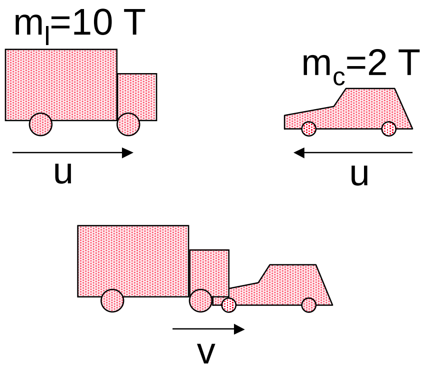
$$\approx \sqrt{\frac{2B}{m_s}}$$

Inelastic collisions cont.

- Similarly

$$v_g \approx \sqrt{\frac{2m_s B}{m_g^2}}$$

- Further example, crash in which vehicles, both doing 30 mph, become entangled.



Inelastic collisions cont.

- Conservation of momentum

$$m_l u - m_c u = (m_l + m_c)v$$

$$v = \frac{(m_l - m_c)}{(m_l + m_c)}u$$

- Compare kinetic energies before, K_i, and after, K_f, collision.

$$K_i = \left(\frac{m_l}{2} + \frac{m_c}{2}\right)u^2$$

$$K_f = \frac{m_l + m_c}{2}v^2 = \frac{1}{2}\frac{(m_l - m_c)^2}{m_l + m_c}u^2$$

$$K_f - K_i = -\frac{2m_l m_c}{m_l + m_c}u^2 = -0.6\text{MJ}$$

- The lost K.E. goes into deforming vehicles and their passengers!