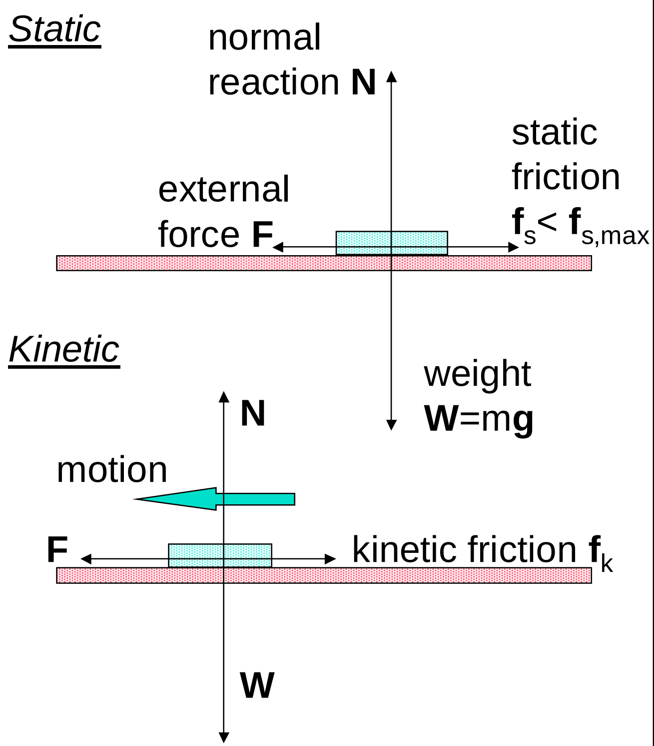
Lecture 2

- Friction
  - Static friction
  - Kinetic friction
  - Drag
  - Terminal Speed
- Circular Motion

Friction

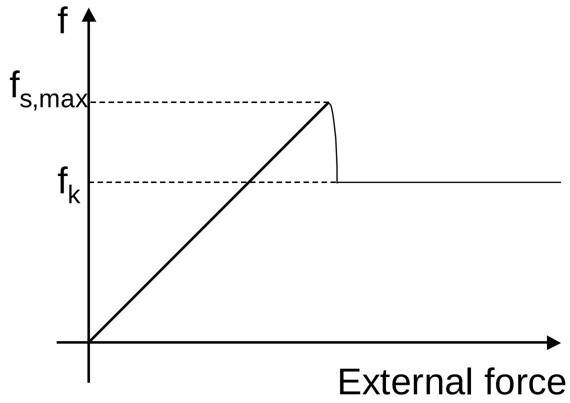
- Fundamental cause electro-magnetic force between atoms and molecules, cf. van-der-Waals forces.
- Surfaces microscopically rough, “cold welds” form at points of close contact.
- Friction is result of welds forming, breaking, forming, breaking, can produce audible squeaks (e.g. brakes) and wear (e.g. piston rings, tyres).
- Consider here friction between unlubricated solid surfaces.

Friction cont.



Friction cont.

Relationship between external and frictional forces



Friction cont.

- Static friction

  $f_s < f_{s,max} = \mu_s N$

  μ_s coeff. of static friction.
- Kinetic friction

  $f_k = \mu_k N$

  μ_k coeff. of kinetic friction.

Friction, a worked example.

A block rests on a plane. The angle this makes to the horizontal is increased to θ_c at which point the block starts to slide. What is θ_c and what is the subsequent acceleration of the block if the plane remains at this angle?

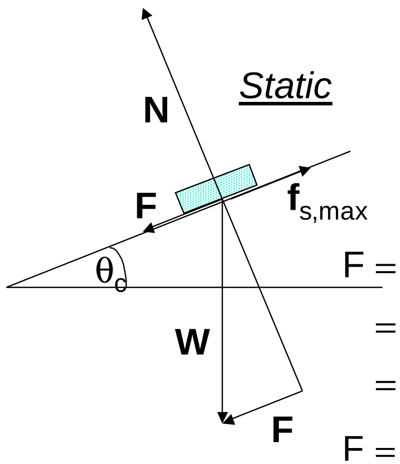
(μ_s= *0.25,* μ_k= *0.20,* g=*9.8 ms*$^{-2}$)

For you to think about:

What happens as μ_k tends to μ_s?

Friction, a worked example.

Static

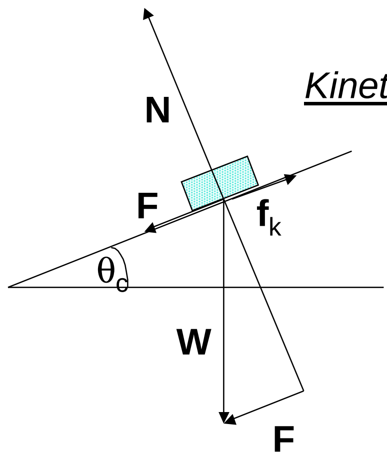


$$F = f_{s,max}$$
$$= \mu_s N$$
$$= \mu_s W \cos\theta_C$$
$$F = W \sin\theta_c$$

$$W \sin\theta_c = \mu_s W \cos\theta_C$$
$$\mu_s = \frac{\sin\theta_c}{\cos\theta_c}$$
$$= \tan\theta_c$$
$$\theta_c = \arctan\mu_s = 14^\circ$$

Friction, a worked example.

Kinetic



$$\mathbf{F} + \mathbf{f_k} = \frac{d}{dt}\mathbf{p}$$
$$W \sin\theta_c - \mu_k N = m\frac{d}{dt}v$$
$$mg\sin\theta_c - \mu_k mg\cos\theta_c = ma$$
$$g(\sin\theta_c - \mu_k \cos\theta_c) = a$$
$$a = 0.475 ms^{-2}$$

The Drag Force

- Relative motion between fluid and body leads to drag force D, which opposes motion.
- For motion of "blunt" objects in air such that air flow turbulent:

$$D = \frac{1}{2} C \rho A v_r^2$$

ρ - density of air,

A - effective cross-section,

v_r - relative speed,

C - drag coeff. (about 0.4 for car).

Drag cont.

- Terminal speed.

Objects falling in air eventually reach a terminal speed v_t at which the forces of drag and weight are equal and opposite, eg. a raindrop of radius 1.5 mm reaches v_t of 7 m s^{-1} after falling about 10 m.

$$\frac{1}{2} C \rho A v_t^2 = mg$$

$$v_t = \sqrt{\frac{2mg}{C \rho A}}$$

Drag cont.

- Stokes' Law

  A sphere falling through a liquid of viscosity η such that the flow is laminar experiences a drag force:

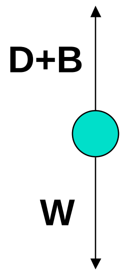
  $$D = 6\pi r \eta v_r$$

- Units of viscosity:

  N s m^{-2}, Pa s, kg m^{-1} s^{-1}.

Drag, a worked example.

- What is the terminal speed of a steel ball, radius 2 mm, falling in glycerine? (ρ_{st}=3000 kg m^{-3}, ρ_{gl}=1300 kg m^{-3},η_{gl}=0.83 Pa s)



$$D + B = 6\pi r \eta_{gl} v_t + \frac{4}{3}\pi r^3 \rho_{gl} g$$

$$W = \frac{4}{3}\pi r^3 \rho_{st} g$$

At terminal speed:

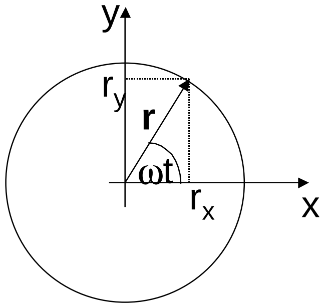
$$W = D + B$$

$$\frac{4}{3}\pi r^3 \rho_{st} g = \frac{4}{3}\pi r^3 \rho_{gl} g + 6\pi r \eta_{gl} v_t$$

$$v_t = \frac{2r^2 g(\rho_{st} - \rho_{gl})}{9\eta_{gl}} = 0.07 ms^{-1}$$

Circular Motion

- Determine velocity of particle undergoing uniform circular motion.



$$\mathbf{r} = (r\cos\omega t, r\sin\omega t)$$

$$\mathbf{v} = \frac{d}{dt}\mathbf{r}$$
$$= (-r\omega\sin\omega t, r\omega\cos\omega t)$$
$$v = \sqrt{r^2\omega^2\sin^2\omega t + r^2\omega^2\cos^2\omega t}$$
$$= r\omega$$

Circular Motion cont.

- Now acceleration

$$\mathbf{a} = \frac{d}{dt}\mathbf{v} = \frac{d^2}{dt^2}\mathbf{r}$$
$$= (-r\omega^2\cos\omega t, -r\omega^2\sin\omega t)$$
$$= -\omega^2\mathbf{r}$$
$$a = r\omega^2$$
$$= \frac{v^2}{r}$$

- We see **a** antiparallel to **r**

$$\mathbf{v}\cdot\mathbf{r} = (-r^2 + r^2)\omega\sin\omega t\cos\omega t = 0$$
$$\Rightarrow \mathbf{v}\perp\mathbf{r}$$

- What is **r**x**a**?

Circular motion cont.

- From second law get centripetal force

$$\mathbf{F} = -m\omega^2\mathbf{r}$$

$$F = -mr\omega^2$$

$$= -\frac{mv^2}{r}$$