Answers to lecture problems – lectures 21 to 22

Lecture 21

Slide 1

What is the order of the equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$?

Is this equation linear? Is it homogeneous?

Order is 2.

Equation is linear and homogeneous.

Lecture 22

Slide 1

Prove that the function $G = r^{-l-1}$ is a solution of the equation $\frac{1}{G}\frac{d}{dr}\left(r^2 \frac{dG}{dr}\right) = l(l+1).$

$$\frac{dG}{dr} = (-l-1)r^{-l-2}$$

$$\frac{d}{dr} r^2 \frac{dG}{dr} = \frac{d}{dr} r^2 (-l-1) r^{-l-2} = \frac{d}{dr} (-l-1) r^{-l} = (-l-1)(-l) r^{-l-1}$$

Hence:

$$\frac{1}{G}\frac{d}{dr}\left(r^2 \frac{dG}{dr}\right) = \frac{1}{r^{-l-1}} (-l-1)(-l) r^{-l-1} = l(l+1) \text{ QED.}$$