Answers to lecture problems – lectures 19

Lecture 19

Slide 1

$$
\begin{aligned}
V_{out} &= \int_{-\infty}^{t} \cos\omega t\, e^{-(t-\tau)}\, d\tau \\
&= \int_{-\infty}^{t} e^{-(t-\tau)}\, d\left(\frac{\sin\omega t}{\omega}\right) \\
&= \frac{\sin\omega t}{\omega} e^{-(t-\tau)}\Big|_{-\infty}^{0} - \frac{1}{\omega}\int_{-\infty}^{t} \sin\omega t\, e^{-(t-\tau)}\, d\tau \\
&= \frac{\sin\omega t}{\omega} + \frac{1}{\omega}\int_{-\infty}^{t} e^{-(t-\tau)}\, d\left(\frac{\cos\omega t}{\omega}\right) \\
&= \frac{\sin\omega t}{\omega} + \frac{1}{\omega}\frac{\cos\omega t}{\omega} e^{-(t-\tau)}\Big|_{-\infty}^{0} - \frac{1}{\omega^2}\int_{-\infty}^{t} \cos\omega t\, e^{-(t-\tau)}\, d\tau \\
&= \frac{\sin\omega t}{\omega} + \frac{\cos\omega t}{\omega^2} - \frac{1}{\omega^2} V_{out}
\end{aligned}
$$

$$
\left(1+\frac{1}{\omega^2}\right) V_{out} = \frac{\omega\sin\omega t + \cos\omega t}{\omega^2}
$$

$$
V_{out} = \frac{\omega\sin\omega t + \cos\omega t}{1+\omega^2}
$$