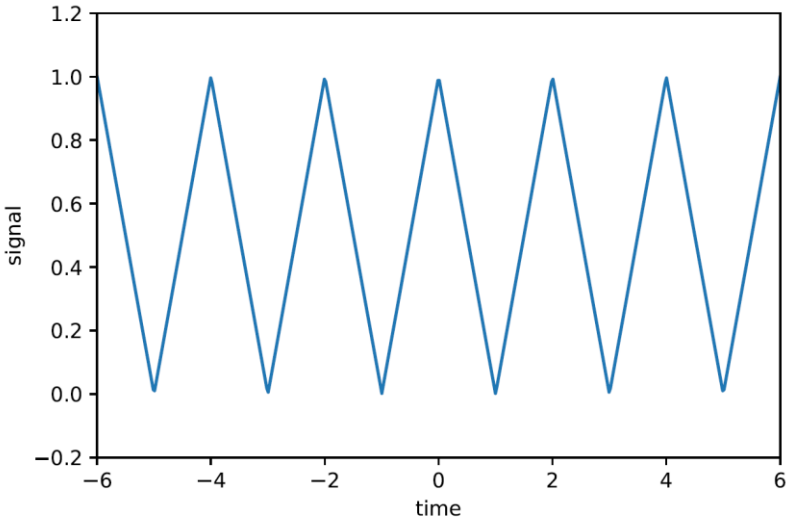
Answers to lecture problems – lectures 16…18

Lecture 16

Slide 1

Deduce what you can about Fourier coefficients for following function:



Average over one period is ½ so $a_0 = \frac{1}{2}$ and function even so $b_n = 0$.

Compare with true values.

Above "guesses" are correct, the coefficients of the cosine terms are:

$a_n = \frac{2}{2}\int_{-1}^{0}(t+1)\cos n\pi t\,dt + \frac{2}{2}\int_{0}^{1}(1-t)\cos n\pi t\,dt$. Use fact that integrating even function over symmetric range.

$= 2\int_0^1 \cos n\pi t\,dt - 2\int_0^1 t\cos n\pi t\,dt$. Integrate second term by parts.

$$= \frac{2}{n\pi}\sin n\pi t\Big|_0^1 - 2\int_0^1 t\,d\left(\frac{\sin n\pi t}{n\pi}\right)$$

$$= -\frac{2}{n\pi}t\sin n\pi t\Big|_0^1 + \frac{2}{n\pi}\int_0^1 \sin n\pi t\,dt$$

$$= -\frac{2}{n^2\pi^2}\cos n\pi t\Big|_0^1$$

$$= -\frac{2}{n^2\pi^2}(\cos n\pi - 1) = \frac{2}{n^2\pi^2}(1-\cos n\pi)$$

$$a_1 = \frac{2}{\pi^2}\times 2 = \frac{4}{\pi^2},\ a_2 = \frac{1}{2\pi^2}\times 0 = 0$$

$$a_3 = \frac{2}{9\pi^2}\times 2 = \frac{4}{9\pi^2},\ a_4 = \frac{1}{8\pi^2}\times 0 = 0$$

$$a_5 = \frac{2}{25\pi^2}\times 2 = \frac{4}{25\pi^2}\ldots$$

Lecture 17

Slide 1

Show that the Fourier Transform of the function $f(x) = -1$ if $-1 < x < 1$, $f(x) = 0$ otherwise, is

$$\tilde{f}(\omega) = -\frac{2}{\omega}\sin\omega.$$

$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(x)\exp[-i\omega x]\,dx$$

$$= -\int_{-1}^{1}\exp[-i\omega x]\,dx$$

$$= -\frac{1}{-i\omega}\exp[-i\omega x]\Big|_{-1}^{1}$$

$$= \frac{2}{2i\omega}\left(e^{-i\omega} - e^{i\omega}\right)$$

$$= -\frac{2}{\omega}\left(\frac{e^{i\omega} - e^{-i\omega}}{2i}\right)$$

$$= -\frac{2}{\omega}\sin\omega.$$

Lecture 18

Slide 1

Calculate the Fourier transform of the function given by $f(x) = 1$ if $-2 < x < 0$, $f(x) = 0$ otherwise.

$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(x)\exp[-i\omega x]\,dx$$

$$= \int_{-2}^{0}\exp[-i\omega x]\,dx$$

$$= -\frac{1}{i\omega}\exp[-i\omega x]\Big|_{-2}^{0}$$

$$= -\frac{2}{2i\omega}\left(1 - e^{2i\omega}\right)$$

$$= \frac{2e^{i\omega}}{\omega}\left(\frac{e^{i\omega} - e^{-i\omega}}{2i}\right)$$

$$= \frac{2e^{i\omega}}{\omega}\sin\omega.$$