Answers to lecture problems – lectures 7…9

Lecture 7

Slide 1

$$\phi = \int_C \vec{E}.d\vec{r}$$

$$= \int_0^1 E_x \frac{dx(t)}{dt} dt + \int_0^1 E_y \frac{dy(t)}{dt} dt + \int_0^1 E_z \frac{dz(t)}{dt} dt$$

$$= \int_0^1 E_x x\, dt + \int_0^1 E_y y\, dt + \int_0^1 E_z z\, dt$$

$$= \int_0^1 \left(yt\, zt + 2xt\, yt\right) x\, dt + \int_0^1 \left((xt)^2 + xt\, zt + (zt)^2\right) y\, dt + \int_0^1 \left(xt\, yt + 2yt\, zt\right) z\, dt$$

$$= \int_0^1 xyzt^2 + 2x^2yt^2\, dt + \int_0^1 x^2yt^2 + xyzt^2 + yz^2t^2\, dt + \int_0^1 xyzt^2 + 2yz^2t^2\, dt$$

$$= \left[\frac{xyzt^3}{3} + \frac{2x^2yt^3}{3} + \frac{x^2yt^3}{3} + \frac{xyzt^3}{3} + \frac{yz^2t^3}{3} + \frac{xyzt^3}{3} + \frac{2yz^2t^3}{3}\right]_0^1$$

$$= xyz + x^2y + yz^2 + \phi_0$$

Lecture 8

Slide 1

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6xy^2 + 2xz^3 & 6x^2y - 6y^2z & 3x^2z^2 - 2y^3 \end{vmatrix}$$

$$= \begin{pmatrix} -6y^2 + 6y^2 & 6xz^2 - 6xz^2 & -6y^2 + 6y^2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$$

Yes.

Mass.

Lecture 9

Slide 1

Laplace Equation is $\nabla^2\phi(x, y, z) = 0$.

Slide 7

$$A = \left| \int_0^{2\pi} \int_0^{\pi} R^2 \sin\theta\, d\theta\, d\phi \right|$$

$$= 2\pi R^2 \int_0^{\pi} \sin\theta\, d\theta$$

$$= 2\pi R^2 \left[-\cos\theta\right]_0^{\pi}$$

$$= 2\pi R^2 \left(1 - (-1)\right)$$

$$= 4\pi R^2$$