Notes for Demonstrators

UNIVERSITY OF LIVERPOOL

DEPARTMENT OF PHYSICS

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Contents

Introduction

This manual is intended to provide you with some general information that will help you when you are working as a demonstrator for one of the physics modules, be it in a laboratory, problems or computer class. Specific information will be given in the instructions for each module by the module organiser, for example in the laboratory handbooks. As most demonstrating is carried out in the laboratories, some information is given here about lab safety and error analysis, the latter as it is one of the areas which students consistently find most difficult to understand.

Your job as a demonstrator

Your job as a demonstrator in the laboratories or elsewhere is first and foremost to ensure that everyone works safely. There are many potential hazards in a laboratory, some of which are discussed below, but accidents can even happen in problems and computer classes. In fact, the most common accidents in laboratories or elsewhere are due to people falling over things, so please make sure that aisles, corridors and doorways, particularly fire exits, are free of obstructions such as coats, bags, chairs etc!

Laboratories, problems and computer classes are all designed to try and help our students learn more about physics. Good demonstrators can make an enormous difference to the effectiveness of these activities. You can do this by being pro-active. Don’t sit at a desk in a corner reading a newspaper (or even Physics Review D!) and grunting discouragingly at anyone who has the temerity to ask you a question. Walk round, see what the students are doing and offer help if you see they are struggling. Remember that some students will be reluctant to approach you if they have a problem: you are a figure of authority for them and they will be nervous about asking questions, at least initially!

When talking to students who have asked you for help, or where you have spotted that they are having difficulties, try and get them to solve their problems by themselves. You can do this, for example, by asking them a series of questions that direct them towards the solution. We can illustrate this with an example. Suppose you are in a Mechanics Problems Class and the problem that one group of students has got stuck with is calculating the torque about the origin exerted by the force $\vec{F} = \begin{pmatrix} 0 & 4 & 0 \end{pmatrix} \mathrm{N}$ acting at the point $\vec{r} = \begin{pmatrix} 3 & 0 & 0 \end{pmatrix} \mathrm{m}$. Don’t just say:

“Well that’s obvious, the answer is $\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 0 \\ 0 & 4 & 0 \end{vmatrix} = \begin{pmatrix} 0 & 0 & 12 \end{pmatrix} \mathrm{Nm}$.”

Much better is to try and make sure they understand the problem, e.g. by asking them to explain which axis the force is directed along and where the point is at which the force acts, perhaps getting them to draw a diagram illustrating this. At this point they will probably be able to see what the torque is going to be (its magnitude and direction). You can then remind them of the formula for torque and get members of the group to work out a component each, or something like that. This will take you more time than just giving the answer, but it will be much more beneficial to the students and also more fun for you!

Your help with demonstrating is much appreciated. We hope you find it to be an enjoyable and rewarding job!

What do I do if I am going to miss a class?

If you are going to have to miss a class for which you are demonstrating, it is your responsibility to find someone who can replace you. Please organise this in good time, tell the module organiser and make sure that your replacement knows what is expected of them and can do your job effectively.

Safety in the laboratories

General comments

The following points supplement the general information about Safety and Fire Precautions in the Physics Department Handbook, which was given to you when you registered. It is your responsibility to read these regulations and those below.

You should ensure that particular care is exercised when experiments involve cryogenic liquids, lasers or radioactive materials and take note of any local rules posted near such experiments.

- You must make certain that students in the laboratories work at all times in a way that ensures their own safety and that of other persons in the laboratory.

- None of the experiments in the laboratories are dangerous provided that normal practices are followed.

- The treatment of serious injuries must take precedence over all other actions, including the containment or cleaning up of radioactive materials. The first aider in the Chadwick Laboratory is Paul Rossiter (Tel. 43800).

- If you are uncertain about any safety matter for any activity in the laboratory, you must consult the laboratory manager (Paul Rossiter, Tel 43800) and the module organiser.

- All accidents must be reported. Contact first the laboratory manager (Paul Rossiter, Tel. 43800) to ensure that any hazard is dealt with and then the Safety Officer (Keith Williams, Tel. 43431) and the module organiser. The Safety Officer has Accident Report Forms which must be completed. The person responsible for radiation protection issues is Dr. Joost Vossebeld (Tel. 43346).

Particular hazards

Ensure that students are aware of the following points when their experiments involve any of the particular hazards listed.

Liquid nitrogen

- The goggles provided should always be used when pouring liquid nitrogen.

- Care should be taken when transferring liquid nitrogen as contact with the skin can cause “cold burns”.

- Liquid nitrogen should not be confined in an enclosed vessel without some means of venting the gas.

Lasers

- Never look directly into a laser beam.

- Experiments are arranged to minimise the use of reflected beams. Care should be taken with any operation that could reflect the beam into the eyes of the person doing the experiment or those of others nearby.

Radioactive sources

- Obey the local safety rules and all specific instructions given in the descriptions of particular experiments.

- Always wash your hands after using equipment involving sources, particularly before handling food.

Vacuum equipment

- Ensure needle valves and pumping equipment are used with due care.

Lead

- In some experiments lead bricks and sheets are used. Ensure anyone who has handled lead washes their hands, particularly before they touch any food. Note that lead can rub off onto benches and other equipment.

Miscellaneous points

- Ensure benches are not cluttered with unnecessary materials, e.g. do not allow students to place coats or clothing on them.

- Ensure aisles and escape routes are free of obstructions such as bags and coats.

- Do not allow food or drink to be brought into the laboratories.

Faculty of Science regulations

The Regulations of the Faculty of Science as outlined in the Handbook apply to all the experiments carried out in the Physics Department.

Error analysis

One of the areas that students find most difficult is the analysis of errors. This section of the Handbook is meant to provide you with some material that will help you to explain error analysis to students, and includes some revision questions.

Why estimate errors?

A measurement of a physical quantity is unlikely to yield exactly the true value. An estimate of the size of the likely difference between the true and measured value is important if significant conclusions are to be drawn from the results. Since the true value is not known, the most likely size of the error on the measured value must be estimated and quoted along with the result.

Random and systematic errors

A *random* error occurs when repeated measurements vary around the central value without any pattern. For example, reading the position of a pointer on the scale of a meter has a random error associated with it. Random errors can usually be estimated from the spread in repeated measurements.

Systematic errors are usually more difficult to estimate and must be identified by consideration of the experimental method. A simple example of a systematic error is one which is the same throughout a set of readings. For example if a meter is not zeroed correctly, all the readings taken with it will be offset by the same amount. More subtly, a systematic error can vary during an experiment due to, say, drift in the calibration of an instrument. See Chapter 8 of Squires' book (available in the laboratory) for a useful discussion of systematic errors.

At the research level, rigorous error analysis can be difficult and sometimes impossible. It is however important to attempt an error analysis and to state clearly in any report of an experiment how the error has been estimated so that the reader can understand the significance of the result and compare it with results from other experiments or theoretical calculations. This is why it is important to develop an understanding of error analysis and why you will always be expected to perform an error analysis of experiments.

Estimating errors in practice

- In experiments in the undergraduate laboratories, you should identify possible sources of systematic errors and eliminate or reduce them by correct use and calibration of the instruments. An estimate of remaining systematic errors should be made by considering operation of the instruments. For example the zero of a scale can be set to within, say ± 0.2 of the smallest division.

- Random errors can be estimated from the spread in repeated readings.

- All the errors can then be combined to determine the error on the final result using the rules given later in this document.

- Estimates of error are liable to have errors themselves! As a rule-of-thumb it is usually adequate to determine the errors themselves to within about 10%. In consequence, error calculations can often be simplified by ignoring errors that are small compared with other errors. Care is needed here as the analysis may be more sensitive to errors in some quantities than others.

Errors in reading instruments

The error in reading a scale, for example on a rule or an analogue meter, can arise from a number of sources:

(a) *Parallax error*. If the line of sight is not at right angles to the scale, a gap between the object being measured (the pointer in the case of a meter) and the scale will cause an error. This can be reduced by careful alignment of the eye, a process aided in better quality meters by a mirror built into the scale so that the pointer and its image can be lined up to ensure the scale is viewed at right angles.

(b) *Zero error.* Most instruments have the provision to set the reading to zero when zero input is present. If the instrument is not correctly zeroed, actual readings will be offset by the offset of the zero. This offset can be measured and a correction applied but it is good practice to always zero the instrument so that the readings can be used without correction.

(c) *Scale reading errors.* The scale can only be read to some accuracy which depends on how finely the scale is engraved. A conservative rule-of-thumb is to assume that the scale can be read to a half of the smallest division. However a fifth of the smallest interval can often be achieved. In practice, you make a judgement based on your use of the particular instrument tempered with experience and common sense.

    A digital scale can be read to $\pm$ 1 in the least significant digit displayed provided the reading is stable. As in the case of an analogue instrument, there will be a zero error.

    A vernier scale can never be read to better than **one** unit of the smallest division. Usually the accuracy is printed on the instrument.

Calibration errors

The accuracy of the reading of every instrument, analogue or digital, will depend on the calibration. Manufacturers will usually supply details of the accuracy of the calibration of the instruments at the point of manufacture. A data sheet from the manufacturer with this information should be available close to the equipment in the laboratory. Unless told otherwise in the instructions, you can assume the calibration is correct. However, possible calibration errors should not be ignored if you have to do any trouble-shooting on the data.

Calibration errors may take the form of an overall multiplicative constant. Instruments will often have some internal adjustment to set this. More commonly, calibration errors will manifest themselves as small deviations around the marked scale due, for example, to the quality of construction of the instrument. These can be accounted for by calibrating the scale against a (usually expensive) standard instrument. If this is advisable, the module organisers will tell you.

Often uncertainties on digital multi-meters are given, for example, as ± (2 + 3). The first number is the uncertainty in the calibration of the instrument, in percent. The second number is associated with the granularity of a digital scale and therefore the uncertainty in displaying a measured value. The 3 in this example means that one component of the uncertainty is 3 times the smallest digit displayed on the scale. E.g.: A multi-meter reads U = 7.34 V. The error according to the manual is ± (2 + 3) i.e. 2% and 3 times the smallest digit (in this case the second digit after the decimal point). Since both errors are independent, they are added in quadrature. The total error is:

$$\Delta U = [(\underbrace{7.34\ \text{V} \times \mathbf{2}/100}_{\text{Calibration}})^2 + (\underbrace{\mathbf{3} \times 0.01\ \text{V}}_{\text{Scale}})^2]^{1/2} = 0.15\ \text{V}.$$

The result must therefore be quoted as: **U = (7.34 ± 0.15) V**

For values much greater than the smallest division, the scale error is usually negligible, whereas for small values both errors contribute.

The mean and its standard error

The best estimate of the true value of a quantity that has been repeatedly measured is the mean of the measurements. If the individual readings are x_i then the mean is given by $\bar{x} = \sum x_i/N$, where N is the total number of readings.

The spread of the readings gives information on the error. The Standard Error on a single reading (usually σ_{n-1} on a calculator) is a measure of this spread. It is given by

$$\sigma_{N-1} = \pm\sqrt{\sum\left(x_i - \bar{x}\right)^2 / (N-1)}.$$

The Standard Error on a single reading remains essentially the same, irrespective of the number of measurements taken. However it is obvious that the uncertainty in the mean value must get less as more individual readings contribute to this mean value. This concept is embodied in the quantity Standard Error in the Mean, $\Delta x = \sigma_{N-1}/\sqrt{N}$, which is used as the error in a measurement of this type. Thus the result above would be quoted as $\bar{x} \pm \Delta x$.

Consistency

If you have two independent measurements of the same quantity, these should be consistent i.e. the same within the errors. A reasonable criterion for the consistency of two independent measurements of the same quantity $x_1 \pm \Delta x_1$ and $x_2 \pm \Delta x_2$ is that

$$|x_1 - x_2| < 3\sqrt{\Delta x_1^2 + \Delta x_2^2}.$$

Systematic errors

So far we have been considering Random Errors, which cause the measured values to spread around the true value and only affect the precision of the determination. If the instrument used has adequate sensitivity the random error can be reduced by taking more readings. However there may also be Systematic Errors which shift all the readings in one direction away from the true value, as for example when a manufacturer quotes the accuracy of an instrument as being ± 1%, in this situation any measurement made with a particular instrument would be subject to a systematic error of 1% and this will affect the accuracy of the result. This could not be reduced by taking multiple readings with the same instrument. Alternatively the experimenter may introduce a systematic error into a measurement by poor technique.

There is no general rule for dealing with systematic errors; you should try and be aware of possible systematic errors and eliminate or reduce them if possible, and then make an estimate of their contribution.

Combination of errors

The final result of an experiment is often obtained by combining various measured values, or it may be some function of those values. There are certain procedures for obtaining the error in the final result from the errors in the individual measured quantities, and these are given in the Data Analysis Summary Sheets below. The theoretical justifications for these procedures are also briefly described. Before stating these, it is useful to note other alternative ways of specifying the error δx in a value x such as the fractional error δx/x and the percentage error [(100 δx) x]%, as these values are often used in the derivation of combined errors. So if z is a function of two quantities A and B, usually written z = f(A,B), then a small error δz in z is related to small errors δA and δB in A and B. If A and B are independent of each other and their errors δA and δB are random, then the error in z, δz, is obtained by adding the individual contributions to δz in quadrature (squaring), so that: $\delta z^2 = (\partial z/\partial A)^2 \delta A^2 + (\partial z/\partial B)^2 \delta B^2$. All the necessary formulae for combining errors can be derived from this equation and are given in the Data Analysis Summary Sheets below. For example, the refractive index n, of the glass of a prism can be found from measurements made with a spectrometer and use of the formula

$$n = \frac{\sin\frac{1}{2}(D+A)}{\sin A/2}.$$

The error on the refractive index arising from the errors in the spectrometer measurements can be found using the expressions $\delta n^2 = (\partial n/\partial D)^2 \delta D^2 + (\partial n/\partial A)^2 \delta A^2$, $(\partial n/\partial D) = (\partial n/\partial D)_{A\ const}$ and $(\partial n/\partial A) = (\partial n/\partial A)_{D\ const}$.

Data analysis summary sheets

Errors

The result of every physical measurement should be accompanied by an error (or uncertainty) i.e. $x \pm \Delta x$ where Δx is typically given by:

(i) the standard error in the mean of a set of N measurements of which x would then be the mean (see below).

(ii) half the smallest reading we can take on the instrument used in the measurement (or 1 digit for a digital instrument).

(iii) an estimate of the possible systematic error (e.g. from the manufacturers specification of the accuracy of the instrument).

Strictly, the contributions to Δx from (i), (ii) and (iii) should be combined, but usually we take whichever is the largest.

The distribution of measurements

A set of measurements of the same quantity free of systematic errors will typically show a distribution centred about the true value. The best estimate of the true value of the quantity will be given by the mean of these readings:

Mean $$\bar{x} = \frac{\Sigma x_i}{N},$$

where N is the number of readings and x_1, $x_2 ... x_i ... x_N$ are the N values of x.

The error on a single reading is related to the spread of the distribution. The appropriate estimate of the spread of the distribution for N readings is given by:

Standard error on a single reading $$\sigma_{n-1} = \sqrt{\frac{\Sigma\left(x_i - \bar{x}\right)^2}{N-1}}$$

Note: This can be obtained using the button labelled σ_{n-1} on a calculator. This should be used rather than the button labelled σ_n, which is a different measure of the spread called the standard deviation.

Multiple readings reduce the error on the mean by the factor $\frac{1}{\sqrt{N}}$.:

Standard error on the mean: $$\Delta x = \sqrt{\frac{\Sigma\left(x_i - \bar{x}\right)^2}{N(N-1)}} = \frac{\sigma_{n-1}}{\sqrt{N}}.$$

Quoting results

Whenever a physical quantity is reported, it must be quoted in a standard form. This form contains either 5 or 6 elements.

1. A sentence describing the result.
2. The name of the measured quantity.
3. The measured value, rounded according to the error.
4. The uncertainty, given to at most 2 significant digits.
5. If applicable, a global order of magnitude.
6. The unit.

E.g.: The volume of a box was measured to be 294.58344 cm^3. The error analysis shows that the uncertainty of the result is 0.5664 cm^3. The result should be rounded to 1 significant figure and presented as:

The volume of the box is $\underline{V = (294.6 \pm 0.6)\ cm^3}$

If the result is a very large or a very small number, it is convenient to explicitly give the order of magnitude for both the measured value and the uncertainty. If we wanted to show the volume of the same box in m^3, we could do it as

$$V = (0.0002946 \pm 0.0000006)\ m^3.$$

This is difficult to read, so it is much more convenient to quote it as

$$\underline{V = (294.6 \pm 0.6) \times 10^{-6}\ m^3}.$$

One often finds great uncertainty whether to quote one or two significant figures. As a general rule of thumb any error beginning with 3 or greater should be rounded and only be quoted to one significant figure. The only exception are values 14, 15, 16 and possibly 24, 25 and 26. These can be left in their present form, rounded to two significant figures.

Examples: $L = (7.3789 \pm 0.0453)$ m is shown as $\underline{L = (7.38 \pm 0.05)\ m}$

$T = (25687 \pm 1424)$ s is shown as $\underline{T = (25700 \pm 1400)\ s}$

$\rho = (0.00005489 \pm 3.79 \times 10^{-6})\ kg/m^3$ as $\underline{\rho = (5.5 \pm 0.4) \times 10^{-5}\ kg/m^3}$

$v = (344.347 \pm 0.152)$ m/s is shown as $v = (344.35 \pm 0.15)$ m/s

Consistency

Two independent measurements of the same quantity $x_1 \pm \Delta x_1$ and $x_2 \pm \Delta x_2$ are consistent if

$$|x_1 - x_2| < 3\sqrt{\Delta x_1^2 + \Delta x_2^2}.$$

Rules for combining errors

The rules below show how an error in a measured quantity propagates into an error in a derived quantity. ΔA refers to the error in A and is assumed small compared to A and ΔB refers to the error in B and is assumed small compared to B.

1. Z = A + B or Z = A – B.

   $\Delta Z = \sqrt{(\Delta A)^2 + (\Delta B)^2}$. $\Rightarrow$ Add the errors in quadrature.

2. Z = k A.

   $\Delta Z = k\,\Delta A$. $\Rightarrow$ Multiply the actual error by k.

   or

   $\frac{\Delta Z}{Z} = \frac{\Delta A}{A}$. $\Rightarrow$ The fractional errors of A and Z are equal.

3. Z = k A × B or Z = $\frac{kA}{B}$.

   $\frac{\Delta Z}{Z} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$. $\Rightarrow$ Add the fractional errors in quadrature.

4. Z = kA^n, where n is a constant.

   $\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$. $\Rightarrow$ Multiply the fractional error by n, independent of k.

5. Z = f(A) where f is a function.

   $\Delta Z = \frac{dZ}{dA}\Delta A$. $\Rightarrow$ Applies where $\frac{dZ}{dA}$ can be expressed analytically.

   For example, if Z = $\log_e A$, then $\frac{dZ}{dA} = \frac{1}{A}$ and $\Delta Z = \frac{\Delta A}{A}$.

Also remember: $d(\sin\theta) = \cos\theta\, d\theta$ etc. and that the error $d\theta$ in θ is in radians.

For more complicated functions, the above rules must be applied in sequence.

Care must be taken if a quantity appears more than once in the calculation to avoid the effects of correlated errors.

Mathematical basis of the rules for combining errors

If z = f(x,y), then a small error δz in z is related to small errors δx in x and δy in y by:

$$\delta z^2 = \left(\frac{\partial f}{\partial x}\right)^2 \delta x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \delta y^2.$$

Examples

i) z = A + B. $\Rightarrow$ Example of rule 1.

$$\frac{\partial f}{\partial A} = 1 = \frac{\partial f}{\partial B}. \quad \Rightarrow \quad \delta z^2 = \delta A^2 + \delta B^2.$$

ii) $z = k\,A^n$. $\Rightarrow$ Example of rule 4.

$$\frac{\partial z}{\partial A} = knA^{n-1}.$$

$$\delta z = k\,n\,A^{n-1}\,\delta A \quad \Rightarrow \quad \frac{\delta z}{z} = n\frac{\delta A}{A}.$$

iii) $z = k\sin\theta$. $\Rightarrow$ Example of the use of rule 5.

$$\frac{\partial z}{\partial \theta} = k\cos\theta \quad \Rightarrow \quad \delta z = k\cos\theta\,\delta\theta \quad (\theta \text{ in radians}).$$

iv) $z = cA + k\,B$. $\Rightarrow$ Combination of rules 1 and 2.

$$\Rightarrow \quad \delta z^2 = c^2\,\delta A^2 + k^2\,\delta B^2.$$

v) $z = k\,AB$. $\Rightarrow$ Example of rule 3.

$$\frac{\partial z}{\partial A} = kB \text{ and } \frac{\partial z}{\partial B} = kA \qquad \Uparrow$$

$$\delta z^2 = k^2y^2\delta A^2 + k^2x^2\delta B^2 \;\Rightarrow\; \left(\frac{\delta z}{z}\right)^2 = \left(\frac{\delta A}{A}\right)^2 + \left(\frac{\delta B}{B}\right)^2.$$

Revision exercises

1. A quantity is measured to be 10 ± 1. What is the value of the uncertainty of:
   a) 6x, b) x^3, c) $1/x$ and d) e^x?

2. A certain length is measured 9 times independently and the values obtained are: 8.61, 8.72, 8.65, 8.67, 8.62, 8.80, 8.58, 8.66, 8.65, all in cm.

   Write down a) the best estimate of the length, b) the standard error in a single reading, c) the error in the mean and d) the final result of the measurement.

   A different method of measuring the same length gives the result 8.90 ± 0.10 cm. Is this consistent with the above measurement?

3. The current in an AC circuit is monitored using both an ammeter (reading RMS values), and an oscilloscope which measures the peak-to-peak voltage across a (200 ± 1) Ω resistor. The values obtained are (7.3 ± 0.1) mA and (4.0 ± 0.2) V. Are these consistent?

4. If i = (45 ± 2)° and r = (35 ± 3)°, what is μ = sin i/sin r and what is the uncertainty in μ?

5. The length of a cylinder is measured to be 219.325 ± 0.002 mm. The diameter of the cylinder is approximately 200 mm. What is the largest error in the diameter measurement which will result in an error in the volume measurement of 10^{-5} m^3? What instrument should you use?

6. The total impedance of an LCR circuit is given by

$$Z = \left[R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2 \right]^{1/2}.$$

   The following quantities are measured:
   R = (1001.6 ± 0.5) Ω.
   L = (127.2 ± 0.2) mH.
   f = (562.57 ± 0.03) Hz.
   C = (0.565 ± 0.016) μF.
   What is Z, its error and in what units is it expressed?