

Answers for Tutorial 4

The marks to be awarded for each question are indicated in square brackets.

Problem 1 [5]

One mA is a charge of 1 mC per second, which corresponds to $\frac{0.001}{1.6 \times 10^{-19}} = 6.25 \times 10^{15}$ electrons. In a minute this is $60 \times 1.6 \times 10^{15} = 3.76 \times 10^{17}$ electrons. [1]

$$\text{Current density } J = \frac{i}{A} = \frac{i}{\pi r^2} = \frac{0.001}{\pi \times 0.001^2} = 318 \text{ A m}^{-2}. \quad [1]$$

$$J = nev_d \Rightarrow v_d = \frac{J}{ne} = \frac{318}{8.47 \times 10^{28} \times 1.6 \times 10^{-19}} = 2.35 \times 10^{-8} \text{ m s}^{-1}. \quad [1]$$

$$\text{Time for electron to drift 5 m is } t = \frac{s}{v_d} = \frac{5}{2.36 \times 10^{-8}} = 211 \times 10^6 \text{ s}. \quad [1]$$

The light comes on much more quickly than the time taken for the electrons to travel from the battery to the bulb as an electric field is established in the wire at a speed close to c , (much above v_d !). As soon as the field is established, the electrons in the wire start to drift under the influence of the field, so electrons will be drifting through the filament of the bulb within of the order of ten nanoseconds of turning on the switch. [1]

Problem 2 [5]

$$R = \frac{\rho L}{A} \Rightarrow \rho = \frac{AR}{L} = \frac{\pi r^2 R}{L} = \frac{\pi \times (0.5 \times 10^{-3})^2 \times 0.01}{0.5} = 1.57 \times 10^{-8} \text{ } \Omega \text{ m}. \quad [3]$$

$$\sigma = \frac{1}{\rho} = \frac{1}{1.57 \times 10^{-8}} = 63.7 \times 10^6 \text{ S m}^{-1}. \quad [2]$$

Problem 3 [10]

Current through resistor and solar cell i , emf of solar cell \mathcal{E} , potential across resistor R is V .

$$\mathcal{E} = i(r + R) \Rightarrow i = \frac{\mathcal{E}}{r + R} \Rightarrow V = iR = \frac{\mathcal{E}R}{r + R} \text{ or } r + R = \frac{\mathcal{E}R}{V}. \quad [2]$$

$$\text{With } R = 500 \text{ } \Omega \text{ we have: } r + 500 = \frac{500\mathcal{E}}{0.1} \quad (1) \quad [1]$$

With $R = 1000 \Omega$ we have: $r + 1000 = \frac{1000\mathcal{E}}{0.15}$ (2) [1]

Subtract (1) from (2): $500 = \left(\frac{1000}{0.15} - \frac{500}{0.1} \right) \mathcal{E} = 1667\mathcal{E} \Rightarrow \mathcal{E} = \frac{500}{1667} = 0.3 \text{ V}$ [1]

Substituting for \mathcal{E} in (1): $r = 5000\mathcal{E} - 500 = 5000 \times 0.3 - 500 = 1000 \Omega$. [1]

The power falling on the solar cell is $P_\gamma = \frac{5}{100 \times 100} \times 20 = 0.01 \text{ W}$. [1]

The power dissipated in the 1000Ω resistor is $P_R = \frac{V^2}{R} = \frac{0.15^2}{1000} = 22.5 \mu \text{ W}$ [2]

Hence efficiency is $\frac{22.5 \times 10^{-6}}{0.01} = 0.225\%$. [1]

Problem 4 [10]

Using Kirchoff's junction rule at junction a: current through 8Ω resistor is $i_1 + i_2$. [1]

Using Kirchoff's loop rule in the LH loop:

$$3 - 6 + 4i_2 - 2i_1 = 0 \text{ or } 3 = -2i_1 + 4i_2 \quad (1) \quad [2]$$

And in the RH loop:

$$6 - 8(i_1 + i_2) - 4i_2 = 0 \text{ or } 6 = 8i_1 + 12i_2 \quad (2) \quad [2]$$

Adding four times (1) to (2):

$$18 = 28i_2 \text{ so } i_2 = 9/14 \text{ A} = 0.643 \text{ A}. \quad [1]$$

Subtracting three times (1) from (2):

$$-3 = 14i_1 \text{ so } i_1 = -3/14 \text{ A} = -0.214 \text{ A}. \quad [2]$$

(Note, minus sign indicates that direction of current is opposite to that of the arrow in the circuit diagram!)

$$\text{Current through } 8 \Omega \text{ resistor is } i_1 + i_2 = 9/14 - 3/14 = 6/14 \text{ A} = 0.429 \text{ A}. \quad [2]$$

The maximum total mark for this Tutorial is 30.