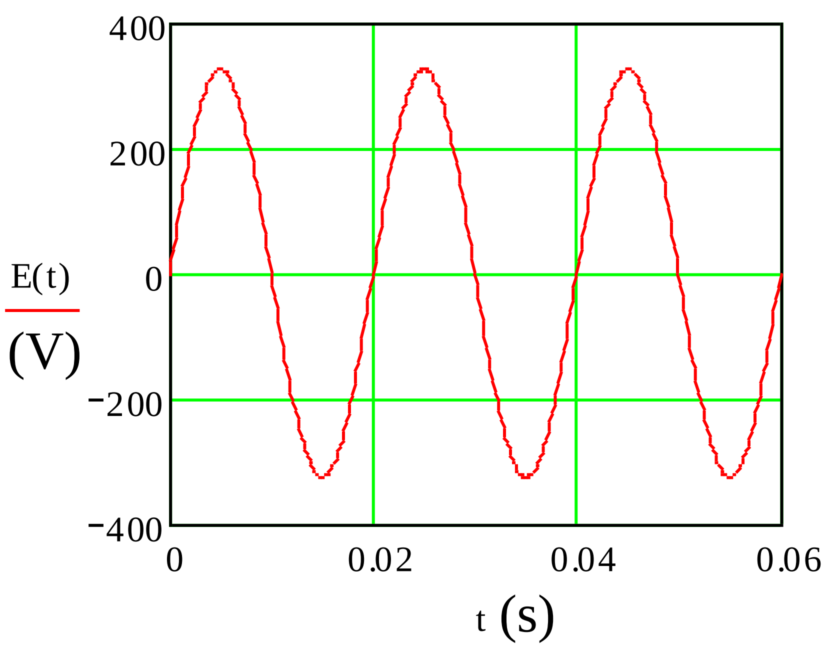
Lecture 19

- In this lecture we will look at:
  - Alternating current.
  - Resistive loads and phasors.
  - Capacitive loads and phasors.
  - Inductive loads and phasors.

- After this lecture, you should be able to answer the following questions:
- Write down the equations which define the capacitive and inductive reactances. In what units are these measured?
- Is the reactance of a capacitor largest for high or low frequencies?
- Is the reactance of an inductor largest for high or low frequencies?
- Describe the phase relationships between an AC voltage applied (separately) across a resistor, a capacitor and an inductance and the resulting current in each case.

Alternating Current

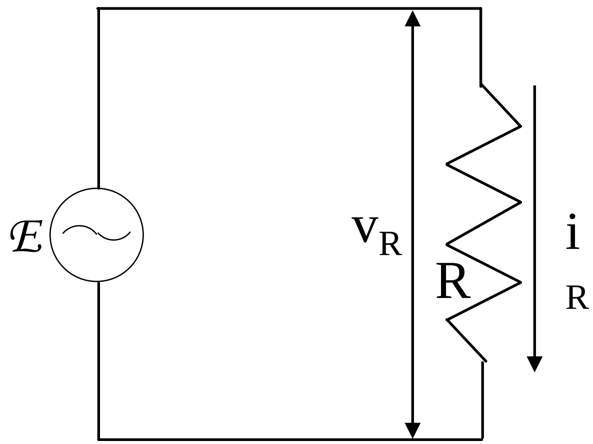
- Recall electricity generator, coil rotated in magnetic field.
- Freq. f_d, angular freq. $\omega_d = 2\pi f_d$.
- Get induced emf:
  $\mathcal{E}(t) = \mathcal{E}_m \sin \omega_d t$ [19.1]
- E.g for mains, $\mathcal{E}_m = 325$ V, $f_d = 50$ Hz



- Need to understand behaviour of electrical devices when driven by emf varying sinusoidally with time.
- This emf will induce current in electrical components.
- Label amplitude of current I.
- May be out of phase with emf, so write $i = I\sin(\omega_d t - \phi)$, allowing for phase shift of ϕ w.r.t. driving voltage.
- Consider separately effects of resistive, capacitive and inductive loads.

Resistive Load

- Circuit consists of alternating emf and resistance:



- Using Kirchoff's loop rule, we have: $\mathcal{E} - v_R = 0.$
- Inserting the expression for the emf we have $v_R = \mathcal{E}_m \sin \omega_d t$ or, using V_R to represent the amplitude of the potential across R, $v_R = V_R \sin \omega_d t.$
- The current through the resistor is given by i = v/R, therefore:

$$i_R = \frac{V_R}{R} \sin \omega_d t \qquad [19.2]$$

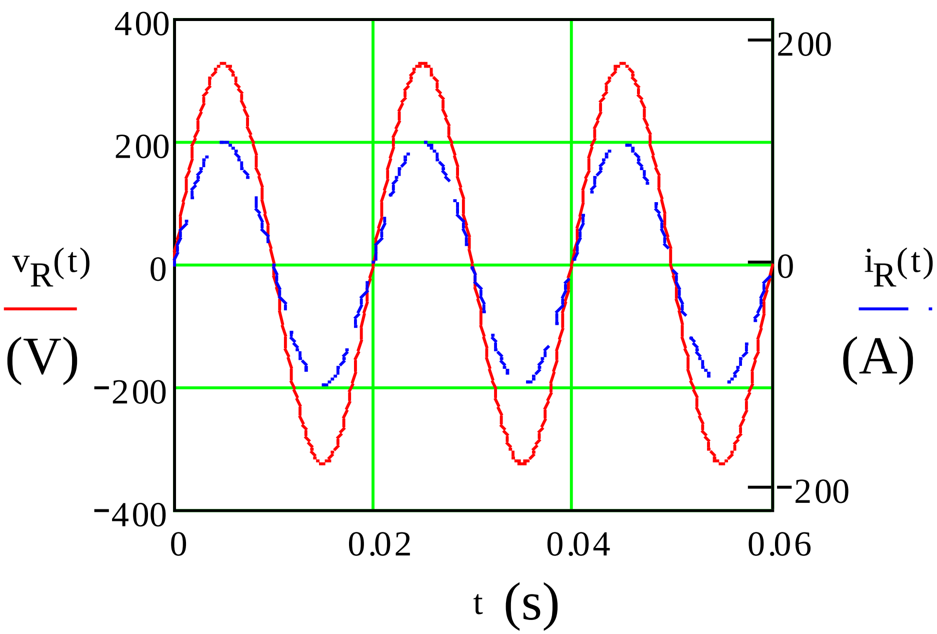
- We can write this as $i_R = I_R \sin \omega_d t.$
- The amplitudes of the current and voltage are related by:

$$V_R = I_R R \qquad [19.3]$$

- The phase shift $\phi = 0$: the voltage across the resistor and the current through it are in phase.
- These relationships apply for all resistors in AC circuits.

Resistive Loads and Phasors

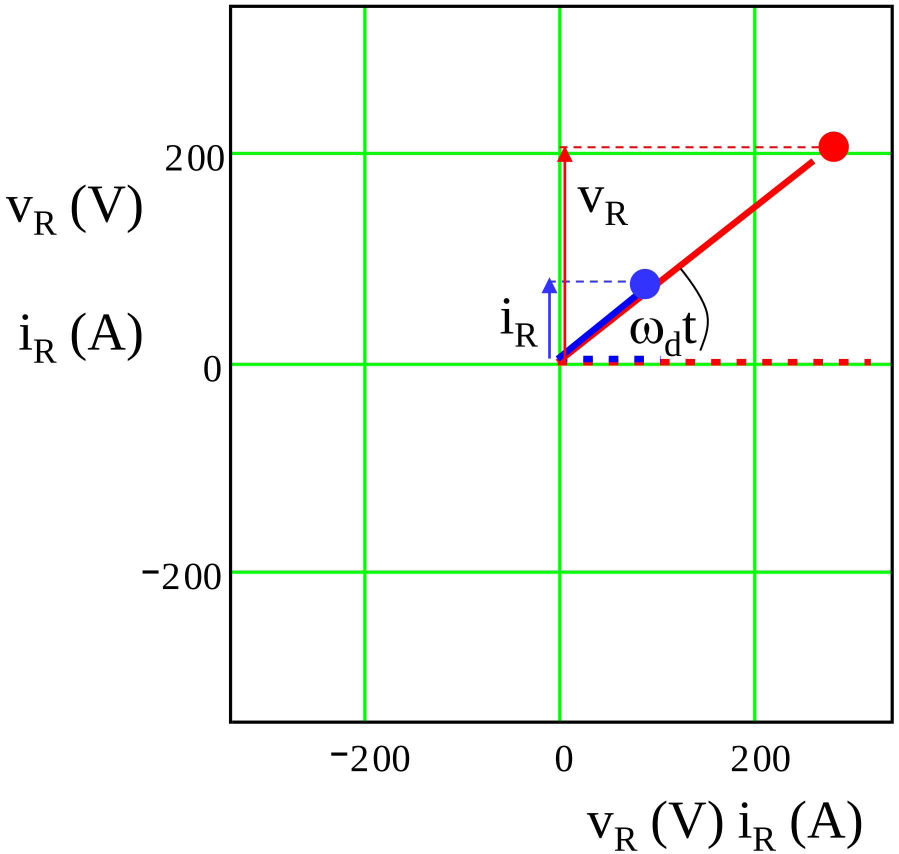
- Plotting the voltage across the resistor and the current through it (using $\mathcal{E}_m = 325$ V and R = 3 Ω):



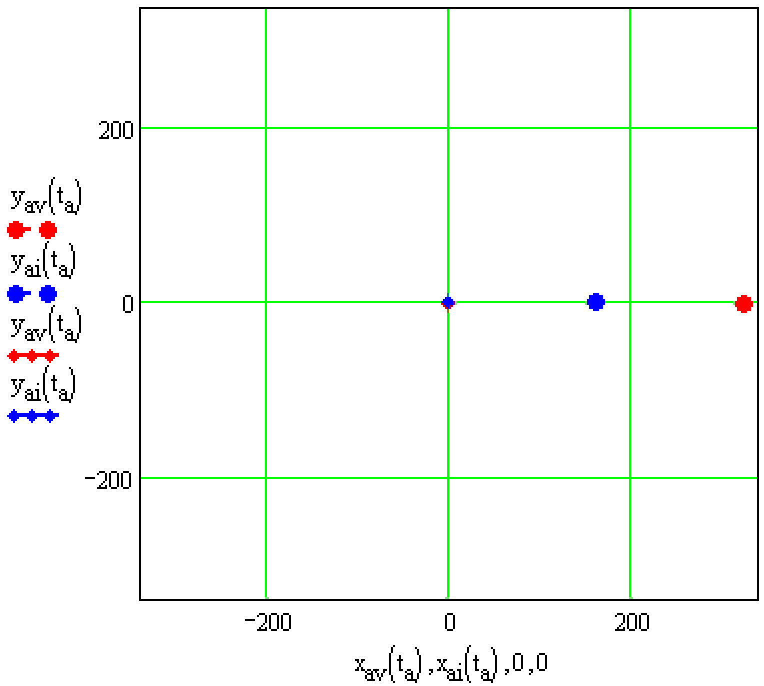
- See the voltage and the current oscillate together, i.e. are in phase.
- The voltage and current can also be represented as phasors.
- Phasors are vectors whose length represents the magnitude of the voltage or current.
- The projection of the phasor on the vertical axis represents the voltage or current at a particular time.
- The phasors rotate in a positive direction (i.e. anticlockwise) around the origin with angular velocity ω_d.

Resistive Loads and Phasors

- Static picture phasors for voltage across, and current through, resistor (parameters as before) at time t:

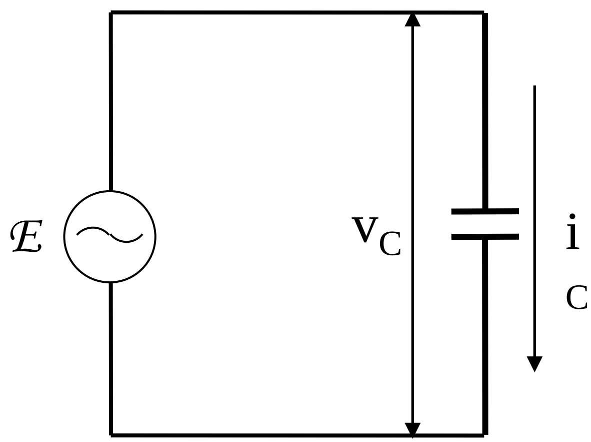


- Allowing the phasors to rotate, we see how they describe the time variation of the current and voltage:



Capacitive Load

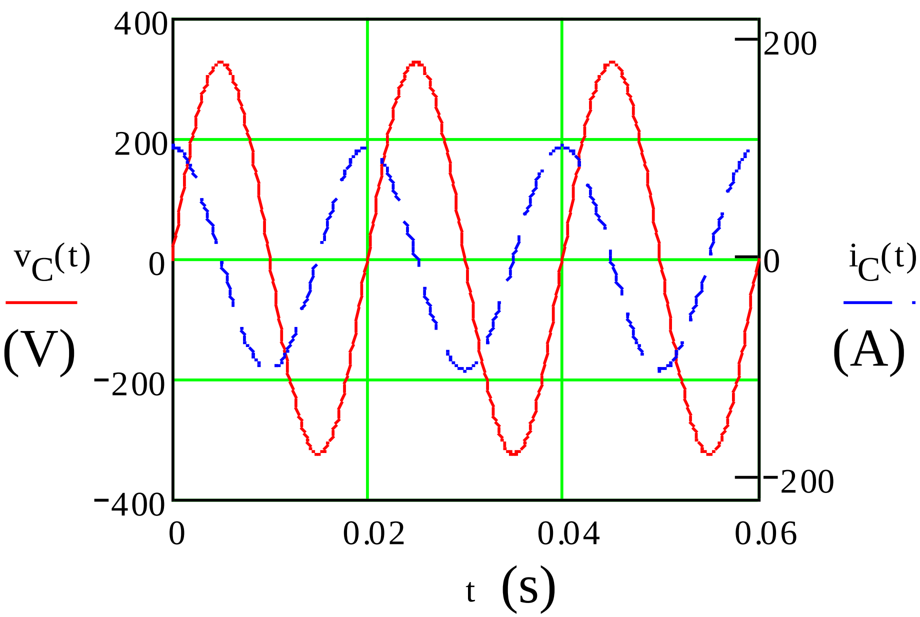
- Circuit consists of alternating emf and capacitance:



- Using Kirchoff's loop rule, we have:
  $\mathcal{E} - v_C = 0.$
- Inserting the expression for the emf we have:
  $v_C = \mathcal{E}_m \sin\omega_d t$ or
  $v_C = V_C \sin\omega_d t$ [19.4]
- From the definition of capacitance:
  $q_C = Cv_C = CV_C \sin\omega_d t.$
- From this we can find the current:
  $$i_C = \frac{dq_C}{dt} = \omega_d C V_C \cos\omega_d t.$$
- Introduce the capacitive reactance:
  $$X_C = \frac{1}{\omega_d C} \quad \text{(unit } \Omega\text{)} \qquad [19.5]$$
- Using the substitution $\cos\theta = \sin\left(\theta + \frac{\pi}{2}\right)$ the expression for i_C becomes:
  $$i_C = \frac{V_C}{X_C}\sin\left(\omega_d t + \frac{\pi}{2}\right).$$
- Writing this in our standard form,
  $$i_C = I_C \sin(\omega_d t - \phi)$$
  with $\phi = -\pi/2$ and $V_C = I_C X_C$ [19.6]

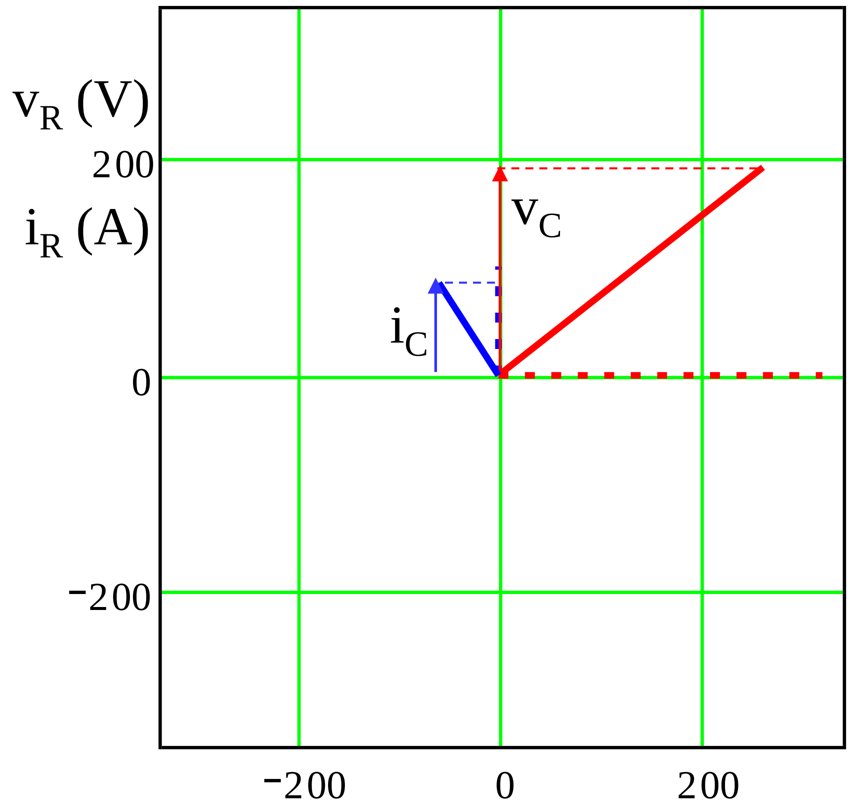
Phasors for Capacitive Loads

- Plotting the voltage across the inductor and the current through it (using $\mathcal{E}_m = 325$ V and C = 1 mF):



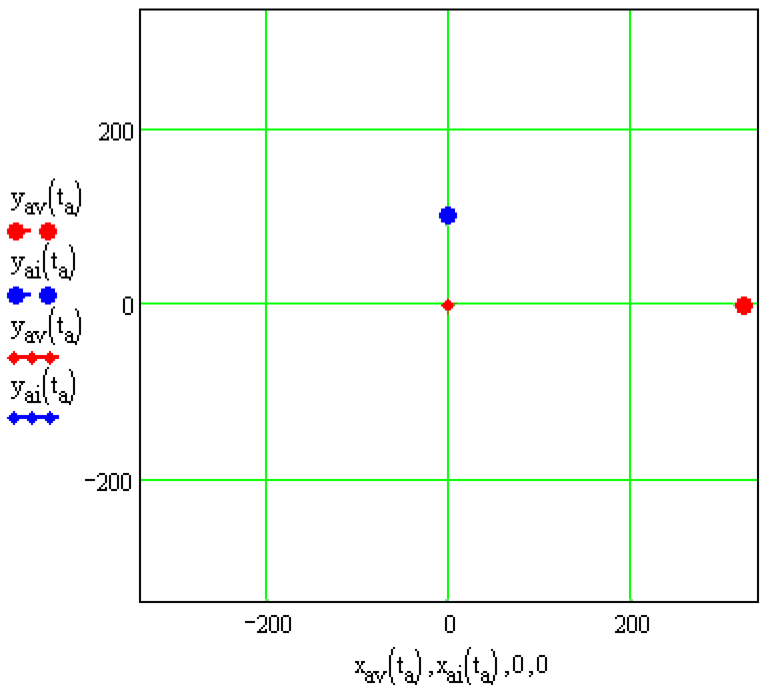
- We see the current leads the voltage by $\pi/2$, (i.e. current reaches peak before voltage).

- In terms of phasors, static picture:



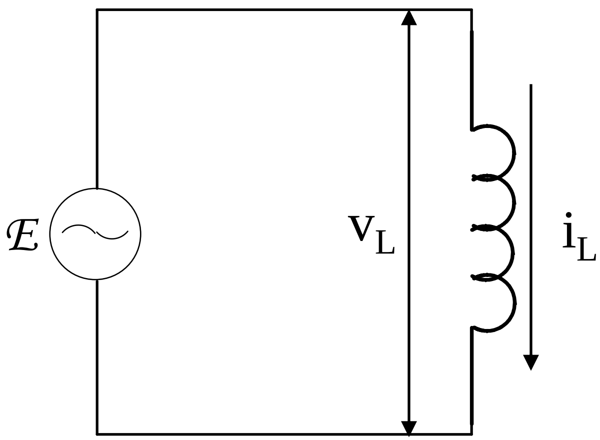
Phasors for Capacitive Loads

- Animation of phasors for capacitive loads:



Inductive Load

- Circuit consists of alternating emf and inductance:

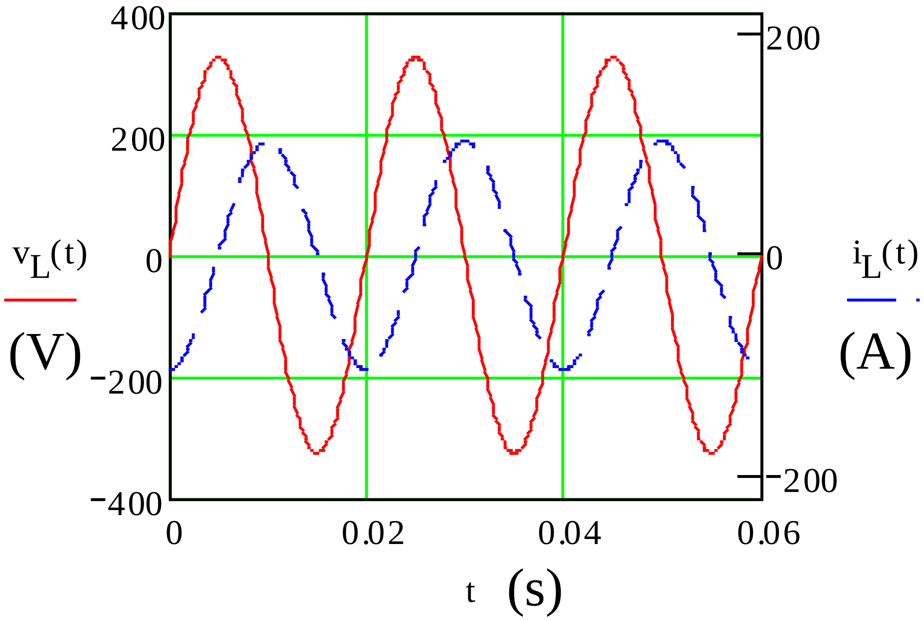


- Using Kirchoff's loop rule, we have:
  $\mathcal{E} - v_L = 0.$
- Inserting the expression for the emf we have:
  $v_L = \mathcal{E}_m \sin\omega_d t$ or
  $v_L = V_L \sin\omega_d t$ [19.7]

- From the definition of inductance:
  $v_L = L\dfrac{di_L}{dt}.$
- We thus have: $\dfrac{di_L}{dt} = \dfrac{V_L}{L}\sin\omega_d t.$
- $i_L = \int \dfrac{di_L}{dt}dt = \dfrac{V_L}{L}\int \sin\omega_d t\, dt = -\dfrac{V_L}{\omega_d L}\cos\omega_d t.$
- Introduce the inductive reactance:
  $X_L = \omega_d L$ (units Ω) [18.8]
- Using the substitution $-\cos\theta = \sin\left(\theta - \frac{\pi}{2}\right)$ the expression for i_L becomes:
  $i_L = \dfrac{V_L}{X_L}\sin\left(\omega_d t - \dfrac{\pi}{2}\right).$
- Writing this in our standard form,
  $i_L = I_L \sin(\omega_d t - \phi)$
  with $\phi = \pi/2$ and $V_L = I_L X_L$ [19.9]

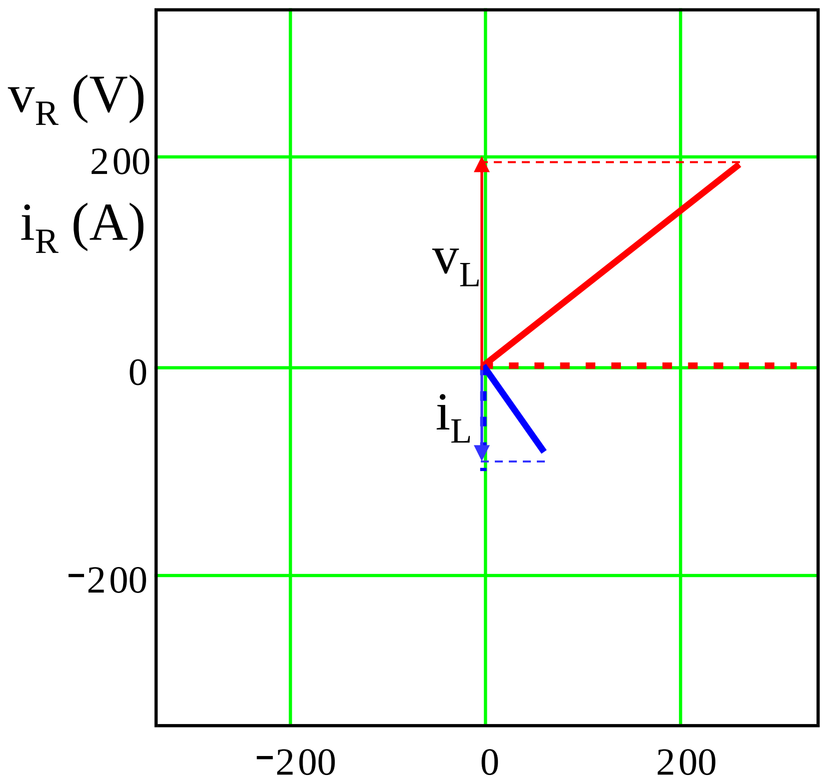
Inductive Loads and Phasors

- Plotting the voltage across the resistor and the current through it (using $\mathcal{E}$ = 325 V and L = 10 mH):



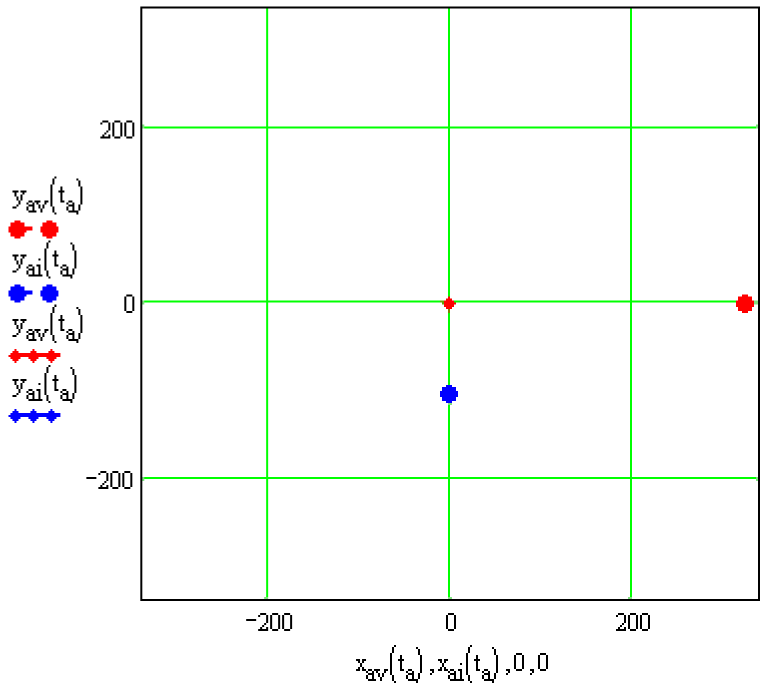
- We see the current lags the voltage by π/2, (i.e. current reaches peak after voltage).

- In terms of phasors, static picture:



Phasors for Inductive Loads

- Animation of phasors for inductive loads:



- Mnemonics for remembering phase relationships etc:
  - C i v i L

    C, i leads v   i lags v, L
  - ELI positively is the ICE man. (i.e. $\mathcal{E}$ leads I for L, which has +ive phase, and I leads $\mathcal{E}$ for C).
- $X_C = \omega_d C$ or $1/\omega_d C$? Remember that C is in Cellar, i.e. in the denominator.