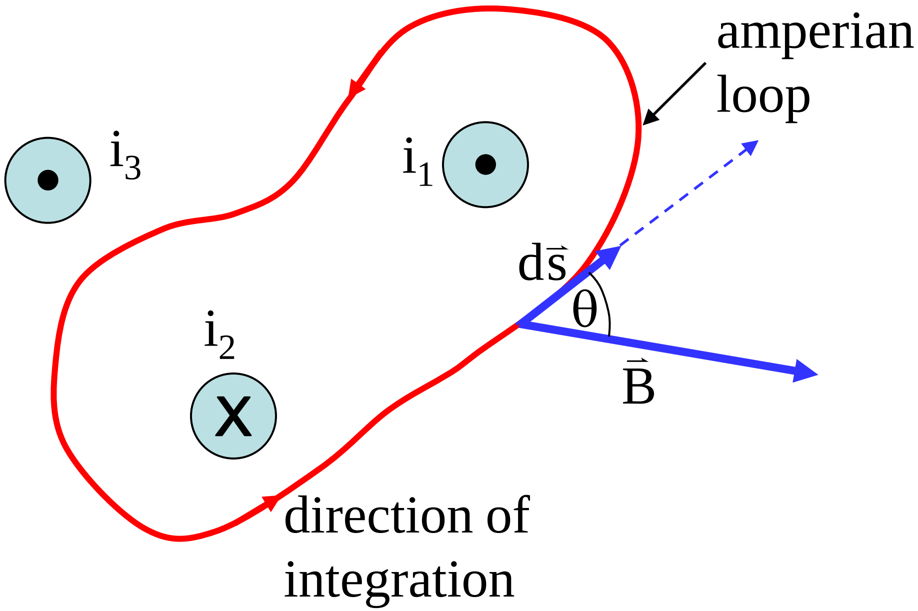
Lecture 15

- In this lecture we will look at:
  - Ampere's Law.
  - Magnetic fields associated with straight wires.
  - B fields in solenoids and toroids.
  - The magnetic dipole due to a current loop.

- After this lecture, you should be able to answer the following questions:
- State Ampere's Law in words and as a mathematical formula.
- Using Ampere's Law, determine the strength of the magnetic field in the centre of a long, current carrying solenoid.
- Explain how the direction of the magnetic field due to a current loop can be determined.

Ampere's Law

- Despite the name, this law was first deduced by Maxwell!



- Ampere's law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \qquad [15.1]$$

- Can be written:

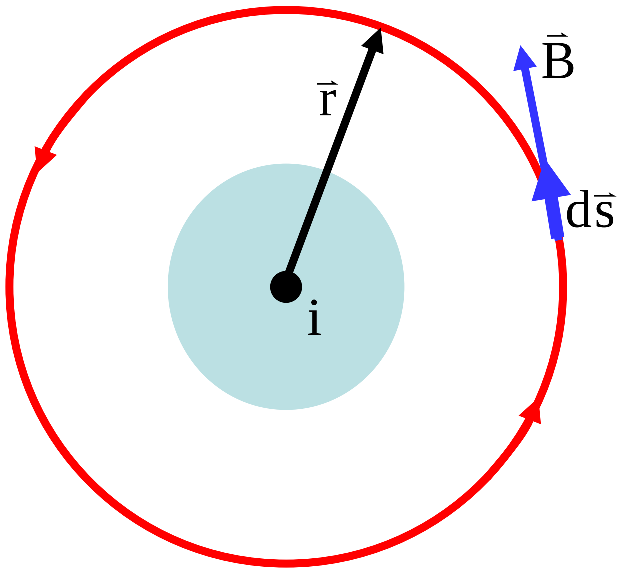
$$\oint B \cos\theta \, ds = \mu_0 i_{enc}.$$

- Determine sign of current using yet another right hand rule.
- Curl right hand round amperian loop so fingers point along direction of integration, currents in direction of thumb assigned +ive sign, in opposite direction –ive sign.
- Now have

$$\oint B \cos\theta \, ds = \mu_0 (i_1 - i_2).$$

Magnetic Field Outside/Inside Long Straight Wire

- Outside wire:

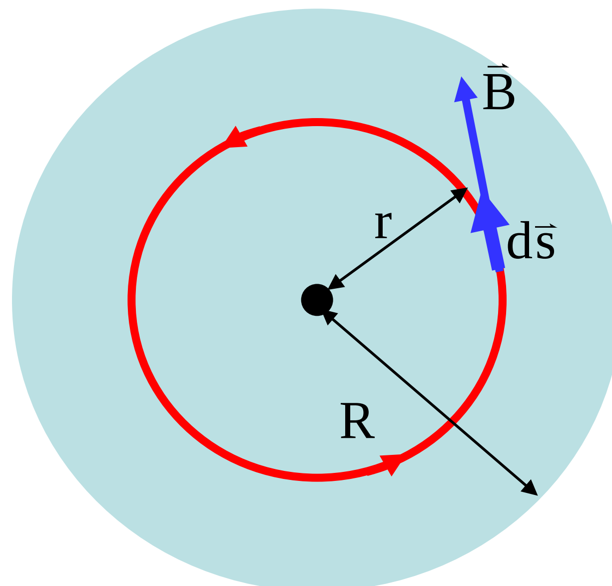


- $\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta \, ds = B \oint ds = B(2\pi r).$

- Hence: $2\pi r B = \mu_0 i$

$$\Rightarrow B = \frac{\mu_0 i}{2\pi r} \qquad [15.2]$$

- Inside wire:

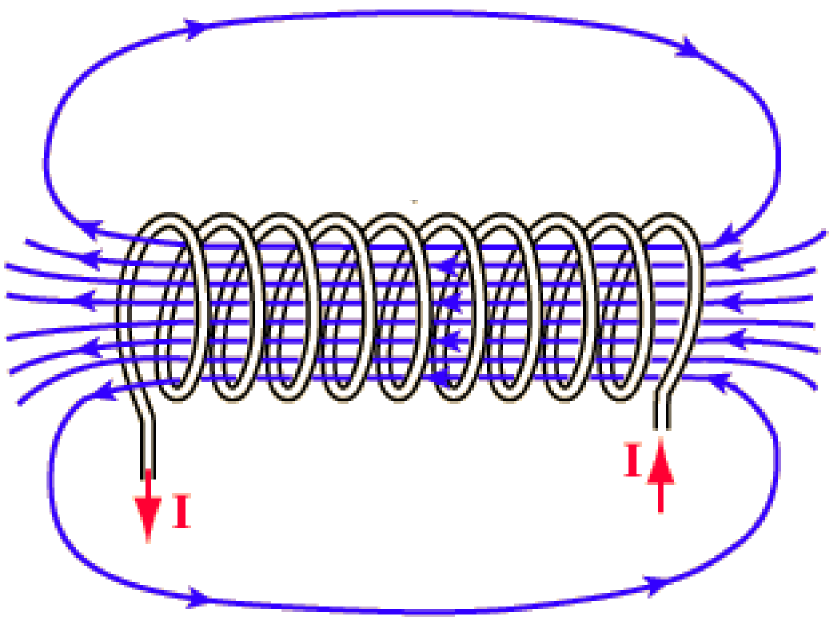


- $2\pi r B = \mu_0 i_{enc} = i \mu_0 \frac{\pi r^2}{\pi R^2}.$

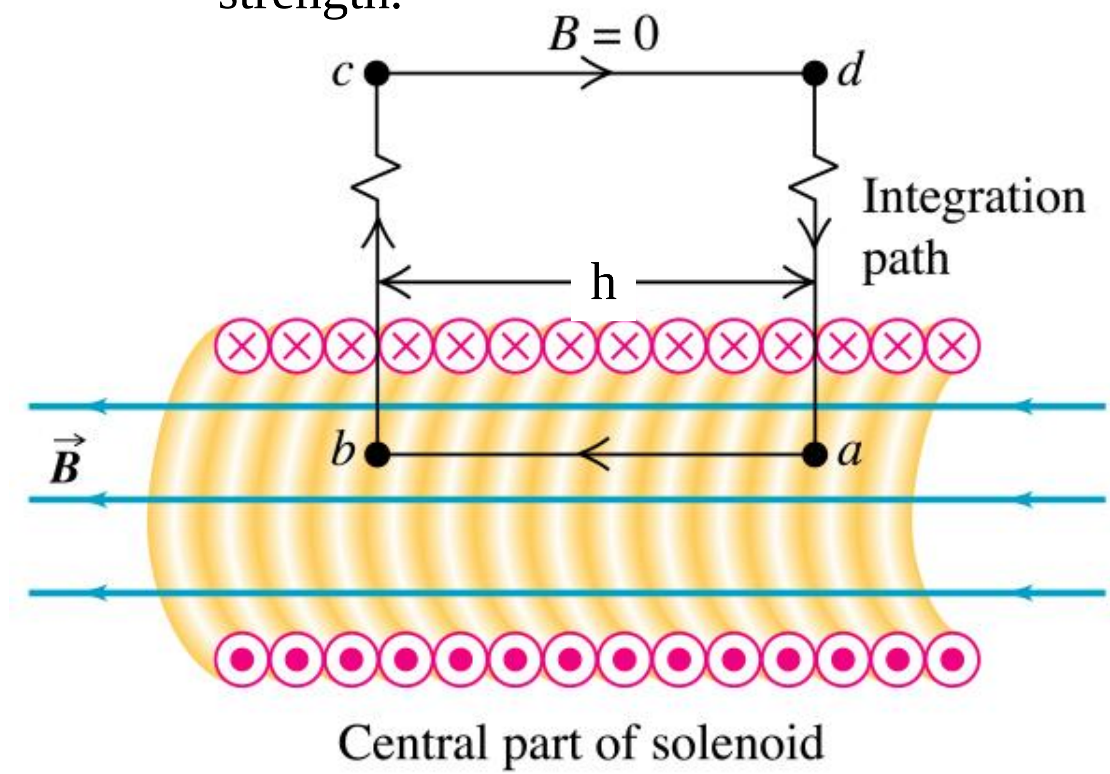
- Hence $B = \frac{\mu_0 i}{2\pi R^2} r \qquad [15.3]$

Field in a Solenoid

- Magnetic field is nearly uniform in the centre of a long solenoid.
- Field outside is relatively weak.



- Use Ampere's law to determine field strength.



- Using this amperian loop:

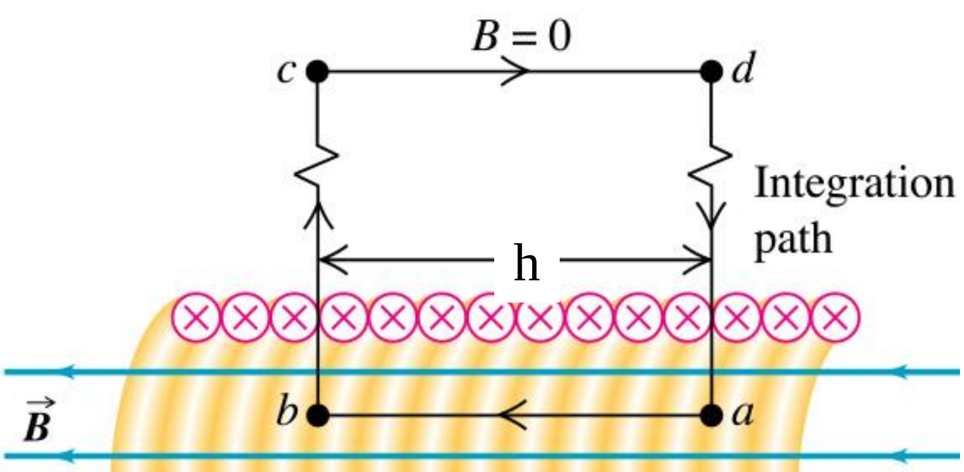
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}.$$

Field in Solenoid

- But the integral can be split up into four parts:

$$\oint \vec{B}\cdot d\vec{s} = \int_a^b \vec{B}\cdot d\vec{s} + \int_b^c \vec{B}\cdot d\vec{s}$$

$$+\int_c^d \vec{B}\cdot d\vec{s} + \int_d^a \vec{B}\cdot d\vec{s}.$$



- No of turns per unit length n.

- $\int_a^b \vec{B}\cdot d\vec{s} = Bh.$
- $\int_b^c \vec{B}\cdot d\vec{s} = \int_d^a \vec{B}\cdot d\vec{s} = 0.$
- Because the path c to d is a long way from the solenoid, the field is negligible, hence:

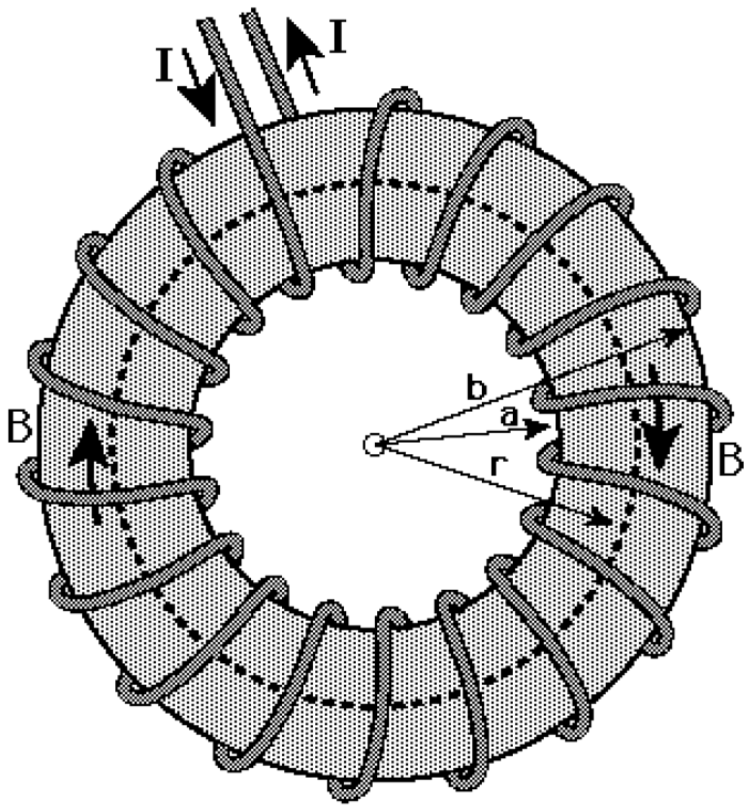
  $\int_c^d \vec{B}\cdot d\vec{s} = 0.$
- The result is thus: $\oint \vec{B}\cdot d\vec{s} = Bh.$
- The enclosed current is: $i_{enc} = i(nh).$
- Hence:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc} \Rightarrow Bh = \mu_0 inh$$

$$\text{or } B = \mu_0 in \qquad [15.4]$$

Field in Toroid

- Toroid is solenoid bent into ring:



- Total number of turns N.

- From Ampere's law (amperian loop at radius r traversed in clockwise direction) we get:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$$

$$\Rightarrow B(2\pi r) = \mu_0 iN$$

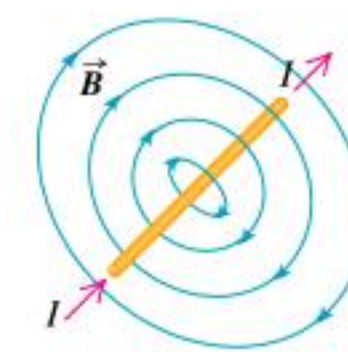
$$\text{or } B = \frac{\mu_0 iN}{2\pi r} \qquad [15.5]$$

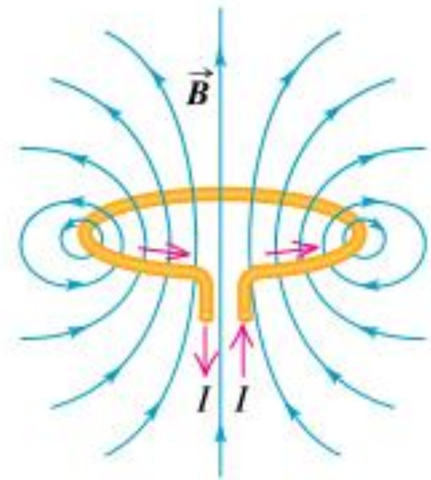
- Note that field is not uniform as in solenoid, but decreases with increasing radius.
- Field outside toroid (i.e. $r < a$ or $r > b$ or above or below toroid) is zero.

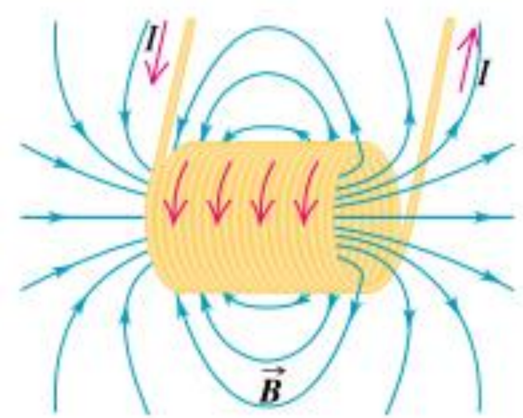
Current Loop as Magnetic Dipole

- Have seen torque on current carrying loop in B field can be expressed in terms of a magnetic dipole: $\vec{\tau} = \vec{\mu} \times \vec{B}$.
- As "loop" configuration is of importance (electrons in atoms!), now calculate field due to loop.
- Use Biot-Savart law as opposed to Ampere's law due to limited symmetry.
- (Have already calculated the field at the centre of the loop: $B = \mu_0 i/2R$.)

- Fields due to wire, current loop and solenoid:

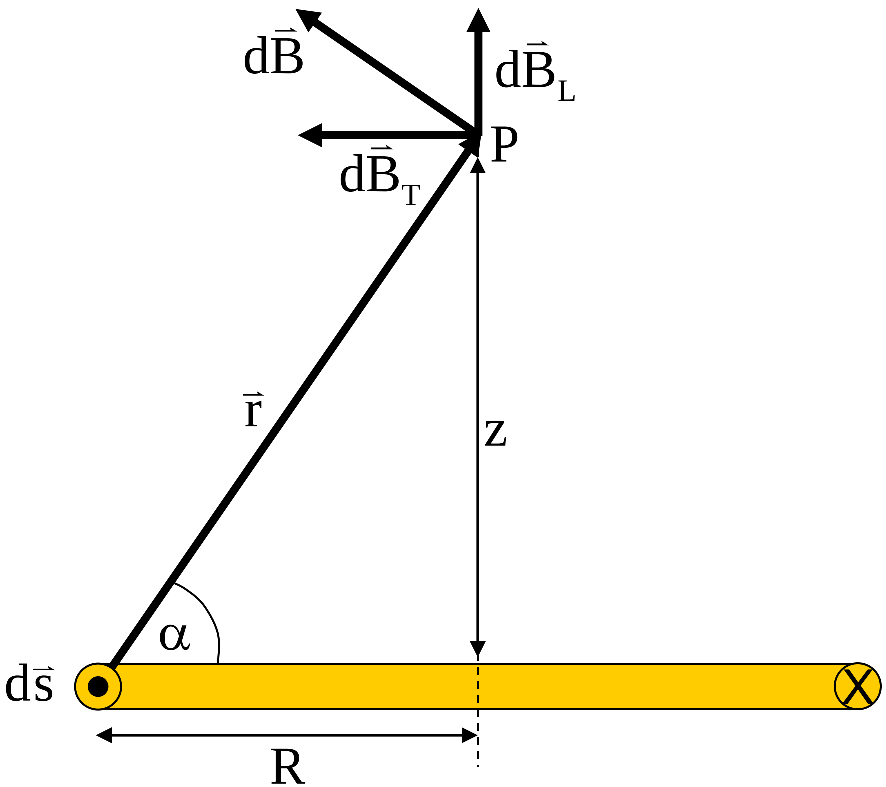






Current Loop as Magnetic Dipole

- Diagram shows rear half of current loop, radius R, loop perpendicular to transparency:



- Angle θ between $d\vec{s}$ and $\vec{r}$ is 90°.
- Biot-Savart law:

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{i\,ds}{r^2}.$$

- Transverse field components cancel.

$$dB_L = \frac{\mu_0 i\cos\alpha\,ds}{4\pi r^2}.$$

- Use $r^2 = R^2 + z^2$ and $\cos\alpha = \dfrac{R}{\sqrt{R^2+z^2}}$.

- Hence: $dB_L = \dfrac{\mu_0 iR}{4\pi(R^2+z^2)^{\frac{3}{2}}}ds$

$$\text{and } B = \int dB_L = \frac{\mu_0 iR}{4\pi(R^2+z^2)^{\frac{3}{2}}}\int ds$$

$$= \frac{\mu_0 iR\times 2\pi R}{4\pi(R^2+z^2)^{\frac{3}{2}}} = \frac{\mu_0 iR^2}{2(R^2+z^2)^{\frac{3}{2}}}.$$

Current Loop as Magnetic Dipole

- For points along the z axis far from the loop we have:

  $$B \approx \frac{\mu_0 i R^2}{2z^3}.$$

- Using $A = \pi R^2$: $B \approx \frac{\mu_0}{2\pi}\frac{iA}{z^3}.$

- If we allow the coil to have N turns this becomes:

  $$B \approx \frac{\mu_0}{2\pi}\frac{NiA}{z^3}.$$

- Writing $\mu = NiA$, we have:

  $$\vec{B} \approx \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3} \qquad [15.6]$$

- Curl RH fingers round loop in direction of i, thumb gives direction of B field in loop and of dipole.

- If set z = 0 we get agreement with previous calculation using Ampere's Law:

  $$B = \frac{\mu_0 i}{2R}.$$