Lecture 8

- In this lecture we will look at:
  - The energy stored in a capacitor.
  - The energy density of an electric field.
  - Dielectrics.
  - Electric fields in the presence of a dielectric.
  - Dielectrics and Gauss' Law.
  - Dielectric strength.

- After this lecture, you should be able to answer the following questions:
- What is a dielectric and what is the difference between a polar and a non-polar dielectric?
- How does the electric field in a parallel plate capacitor whose plates are separated by a distance d and are held at a potential difference V change when a dielectric with relative permittivity ε_r is inserted between the plates?
- How does the capacitance of the above capacitor alter?

Energy Stored in a Capacitor

- In order to charge a capacitor, work must be done (e.g. by a battery).
- Hence charged capacitor has stored potential energy.
- Consider capacitor C with charge q'.
- Potential is then $V = q'/C$.
- If a small additional amount of charge dq' is transferred, this requires that an amount of work dW be done, where:

$$dW = V\,dq' = \frac{q'}{C}dq'.$$

- The total amount of work done in charging the capacitor is thus:

$$W = \int dW = \frac{1}{C}\int_0^q q'dq' = \frac{q^2}{2C}.$$

- Hence the potential energy stored by the capacitor is:

$$U = \frac{q^2}{2C} \qquad [8.1]$$

- Using q = CV, we can rewrite this:

$$U = \tfrac{1}{2}CV^2 \qquad [8.2]$$

- Where is this potential energy stored?
- In the forces between the +ive and –ive charges, i.e. in the electric field.

Energy Density of an Electric Field

- Recall that, for a parallel plate capacitor, $C = \dfrac{\varepsilon_0 A}{d}$.
- The energy stored in the capacitor is U, giving an energy density u of: $u = \dfrac{U}{Ad} = \dfrac{\frac{1}{2}CV^2}{Ad}$.
- Using the expression for C above:

$$u = \frac{CV^2}{2Ad} = \frac{\left(\varepsilon_0 A/d\right)V^2}{2Ad} = \tfrac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2.$$

- But V/d is the electric field strength E, so we get:

$$u = \tfrac{1}{2}\varepsilon_0 E^2 \qquad [8.3]$$

Dielectrics

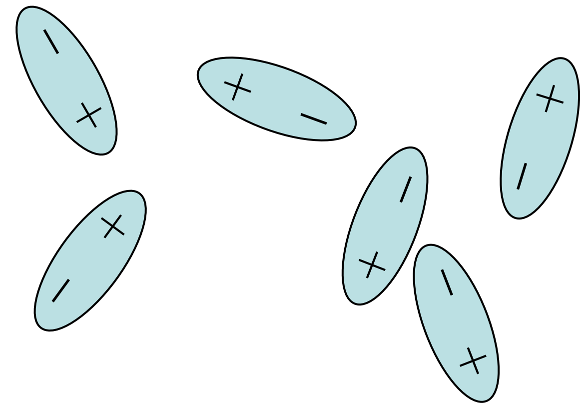
- So far we have considered the space between capacitor plates to be filled with "free space" or vacuum (~ air in this context!).
- What happens if we insert a dielectric (i.e. an insulator) between the plates?
- Find the capacitance increases by a factor κ, the dielectric constant:

$$C' = \kappa C \qquad [8.4]$$

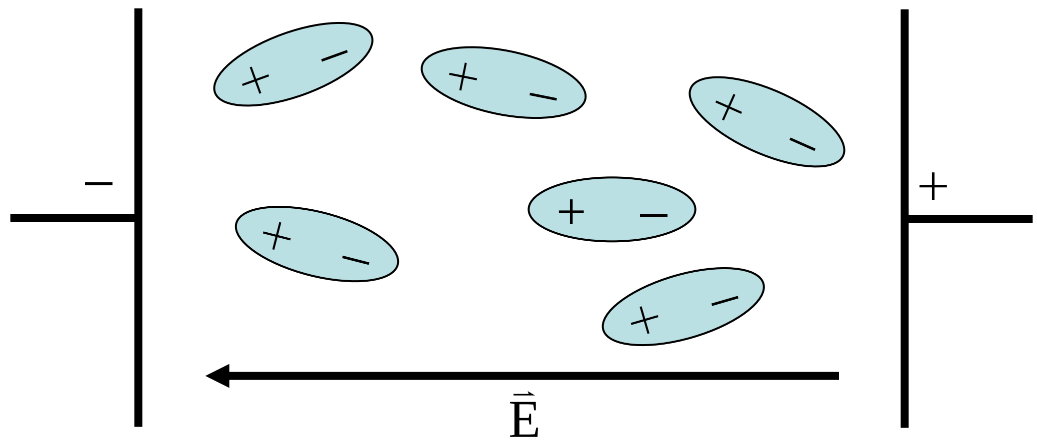
- Why?

Dielectrics, Atoms and Molecules

- There are two types of dielectric.
  - Polar dielectrics.
  - Non-polar dielectrics.
- Polar dielectrics contain atoms or molecules that have a permanent electric dipole moment (e.g. water).
- When these are placed in a capacitor, the E field causes (partial) alignment of the atoms/molecules.
- Polar dielectric in absence of E field:



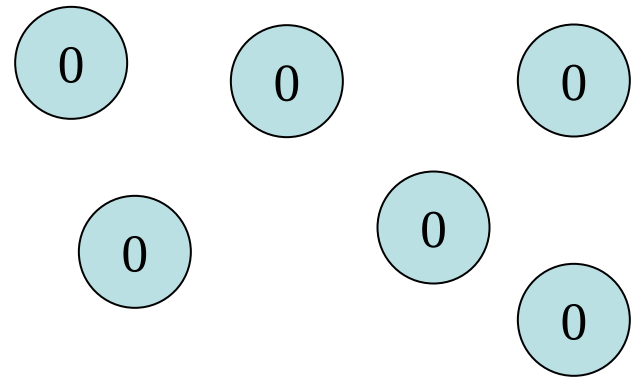
- Polar dielectric in E field:



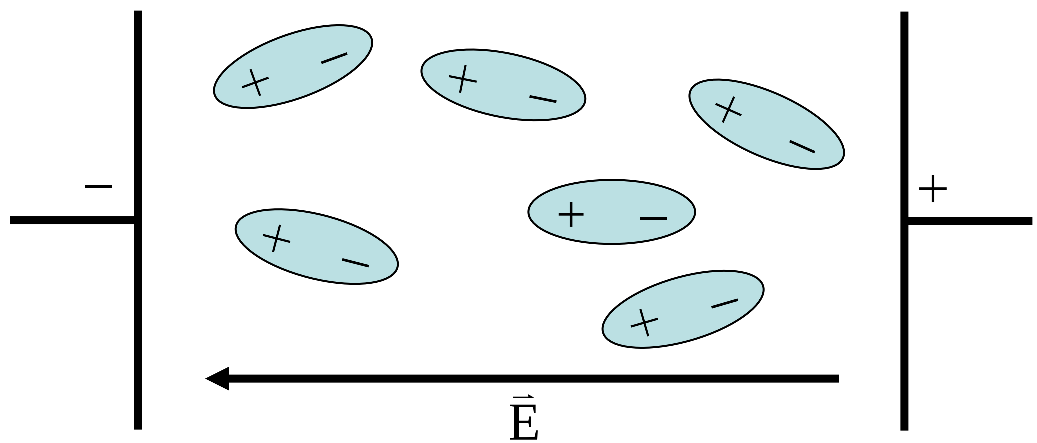
Dielectrics, Atoms and Molecules

- The atoms or molecules of non-polar dielectrics do not have an intrinsic electric dipole moment.
- When such dielectrics are placed in an E field, the field “stretches” slightly the atoms or molecules, separating the mean positions of the +ive and –ive charges.
- The atoms or molecules acquire an induced electric dipole moment.
- Example of a non-polar dielectric: paper.

- Polar dielectric in absence of E field:

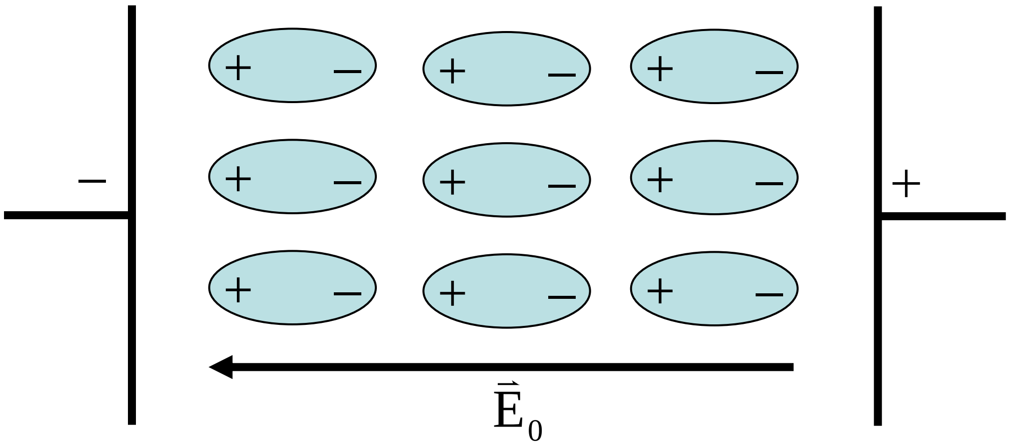


- Polar dielectric in E field:

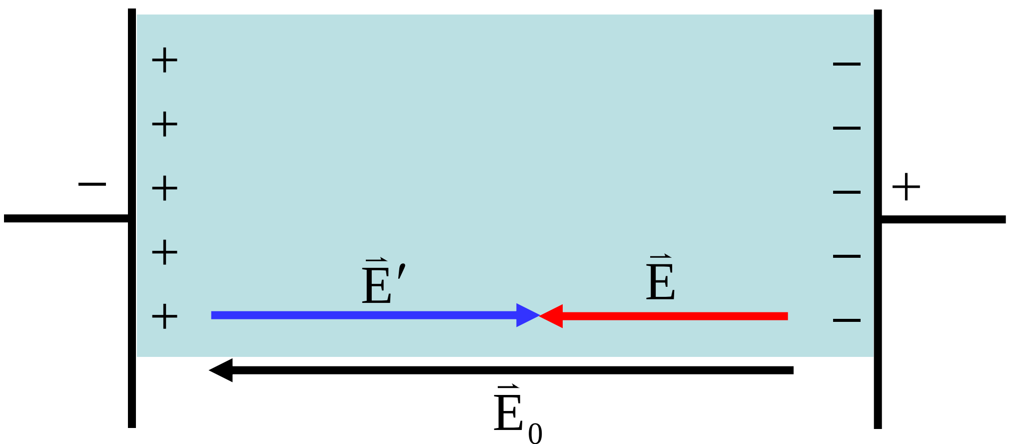


Electric Fields in the Presence of a Dielectric

- Apply electric field $\vec{E}_0$ to dielectric.



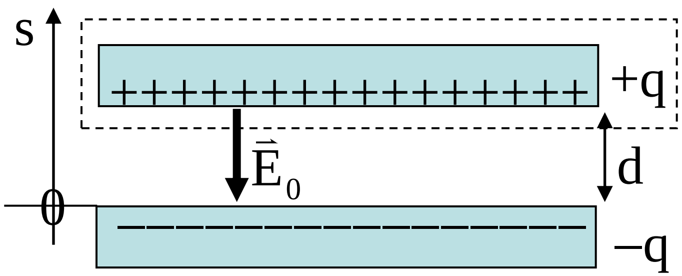
- Induce surface charge in dielectric, resulting field $\vec{E}'$ opposite to $\vec{E}_0$.



- Net field inside capacitor is $\vec{E} = \vec{E}_0 + \vec{E}'$ [8.5]
- Effect of dielectric is to weaken the field in the capacitor.
- This means the voltage across the capacitor will drop, as $V = Ed$.
- (Note, this is not true if a battery is connected across the capacitor, as this will then supply more charge to push the potential difference back up again!)
- See that electrostatic forces will tend to pull dielectrics into capacitors.

Dielectrics and Gauss' Law

- Remember capacitance without dielectric:

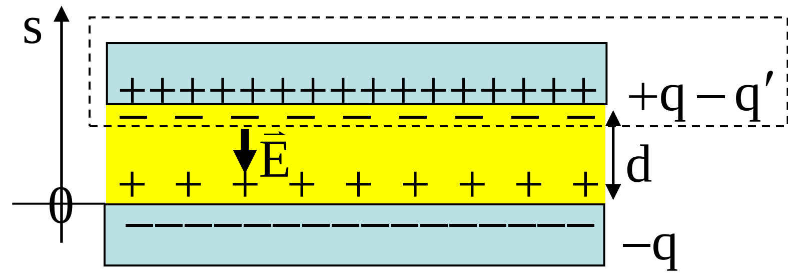


- Magnitude of E field due to charge +q on upper plate from Gauss' law:

$$\oint \vec{E}_0 \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E_0 A = \frac{q}{\varepsilon_0} \text{ or } E_0 = \frac{q}{\varepsilon_0 A}.$$

- Now add dielectric:



- Gaussian surface now encloses free charge (+q, on plate) plus the surface charge (–q′, on surface of dielectric).

$$\oint \vec{E} \cdot d\vec{A} = \frac{q - q'}{\varepsilon_0}$$

$$EA = \frac{q - q'}{\varepsilon_0}$$

$$\text{or } E = \frac{q - q'}{\varepsilon_0 A} \qquad [8.6]$$

Dielectrics and Gauss' Law: Dielectric Strength

- The effect of the dielectric is to weaken the original field by the factor κ, so:

$$E = \frac{E_0}{\kappa} = \frac{q}{\kappa\varepsilon_0 A}.$$

- Comparing this with $E = \dfrac{q - q'}{\varepsilon_0 A}$,

  we see $q - q' = \dfrac{q}{\kappa}$ [8.7]

- This equation allows us to write Gauss' Law in a form that is appropriate in the presence of a dielectric:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q - q'}{\varepsilon_0} = \frac{q}{\kappa\varepsilon_0}$$

$$\Rightarrow \oint \kappa\vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}. \qquad [8.8]$$

- The flux integral now involves $\kappa\vec{E}$, not just $\vec{E}$.
- The charge enclosed by the Gaussian surface is now the free charge only.
- The quantity κ may not be constant, so we leave it inside the integral.
- $\kappa = \varepsilon_r$ is called the (static) relative permittivity.
- Sometimes use electric displacement $\vec{D} = \kappa\varepsilon_0\vec{E}$, then Gauss' Law is written $\oint \vec{D} \cdot d\vec{A} = q$ [8.9]
- In very high E fields, the dielectric may break down; the field at which this occurs is called the dielectric strength.