Lecture 5

- This lecture, we will look at:
  - Electric potential.
  - Calculating the electric potential from the electric field.
  - Electric potential due to a point charge.
  - Illustrating the electric potential and an example from Particle Physics: the drift chamber.
  - The potential due to a dipole.
- After this lecture, you should be able to answer the following questions:
- In what units is electric potential measured and how is electric potential related to potential energy?
- What are the formulae for the potential due to a point charge and a dipole?

Electric Potential

- A charge q_0 in an electric field experiences a force $\vec{F} = q_0\vec{E}$.
- If it moves under the influence of the force, or if it is moved against the influence of the force, work is done.
- Associate the electric potential energy, U, with the charge in the electric field.
- The change in electric PE between the initial configuration, i, and the final configuration, f, is given by:

$$\Delta U = U_f - U_i = -W.$$

- Here, W is the work done by the electric field on the charge (–W is the work done on the field in moving the charge).
- Define configuration with charge at infinite distance from source of field to have $U_\infty = 0$.
- Then, U = –W.
- Define electric potential, the electric potential energy per unit charge:

$$V = \frac{U}{q_0} \qquad [5.1]$$

- Units $J\,C^{-1}$ = Volts (V).
- The electric potential difference between points i and f is the difference in potential energy per unit charge between the two points:

$$\Delta V = V_f - V_i = \frac{U_f}{q_0} - \frac{U_i}{q_0} = \frac{\Delta U}{q_0}.$$

Calculating the Electric Potential from the Field

- Force on charge: $\vec{F} = q_0 \vec{E}$.
- Work done in moving charge a short distance $d\vec{s}$ is $dW = \vec{F} \cdot d\vec{s}$.
- Difference in potential energy between configurations i and f:

$$U_f - U_i = -\int_i^f dW = -\int_i^f \vec{F} \cdot d\vec{s} = -\int_i^f q_0 \vec{E} \cdot d\vec{s}.$$

- Hence difference in electric potential:

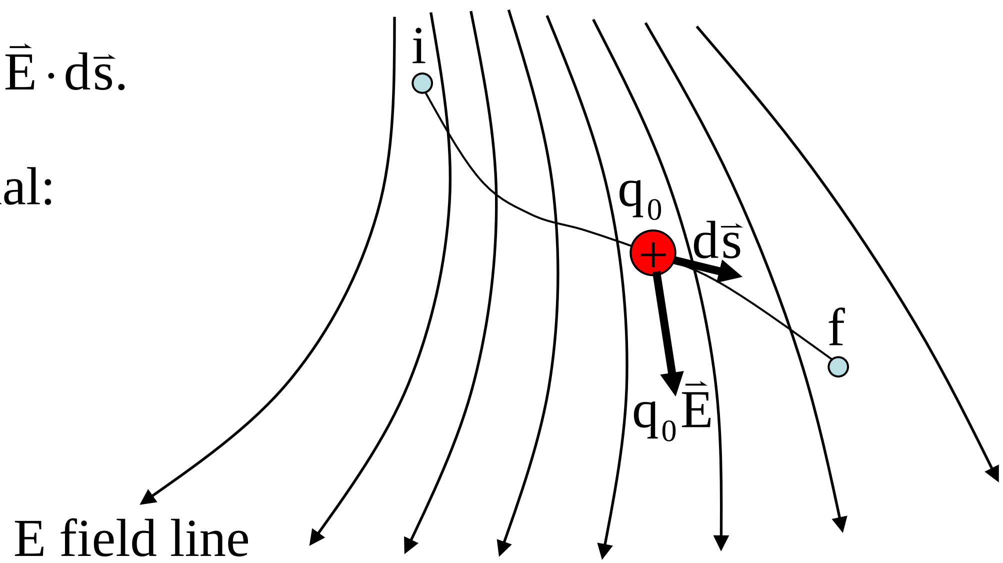
$$V_f - V_i = \frac{U_f}{q_0} - \frac{U_i}{q_0}$$

$$= \frac{-1}{q_0} \int_i^f q_0 \vec{E} \cdot d\vec{s}$$

$$= -\int_i^f \vec{E} \cdot d\vec{s}.$$

- Defining $V_i = 0$ at infinite distance:

$$V_f = -\int_\infty^f \vec{E} \cdot d\vec{s} \qquad [5.2]$$



Electric Potential due to Point Charge

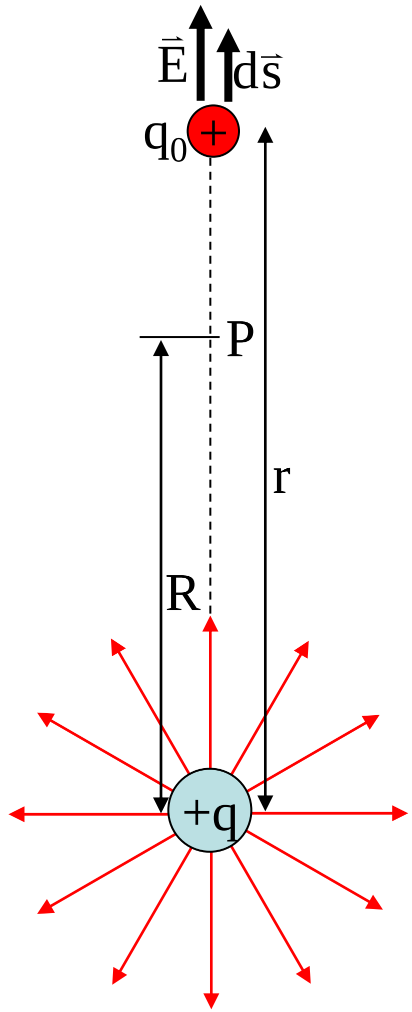
- Determine potential due to point charge.
- $\vec{E}\cdot d\vec{s} = E\,dr\cos\theta = E\,dr$
- Hence:

$$V_f = -\int_{\infty}^{R} E\,dr$$

$$= -\int_{r=\infty}^{r=R} \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,dr$$

$$= \frac{-q}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right]_{\infty}^{R}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{R}.$$



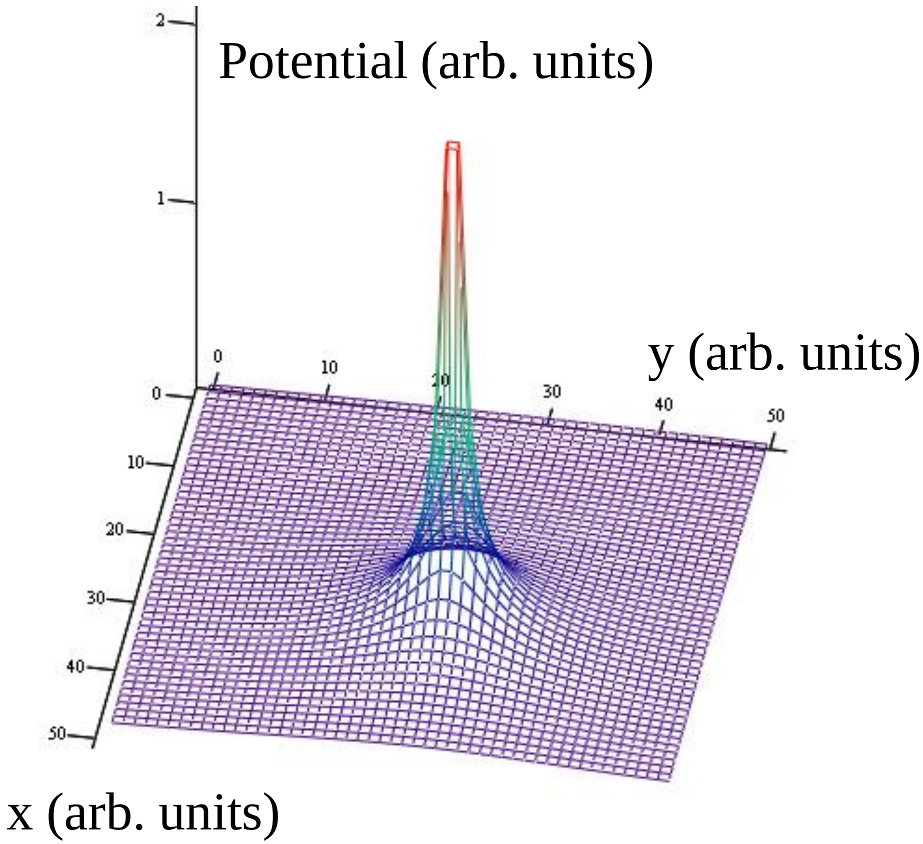
- Again, we have used $V_\infty = 0$.
- Therefore:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad [5.3]$$

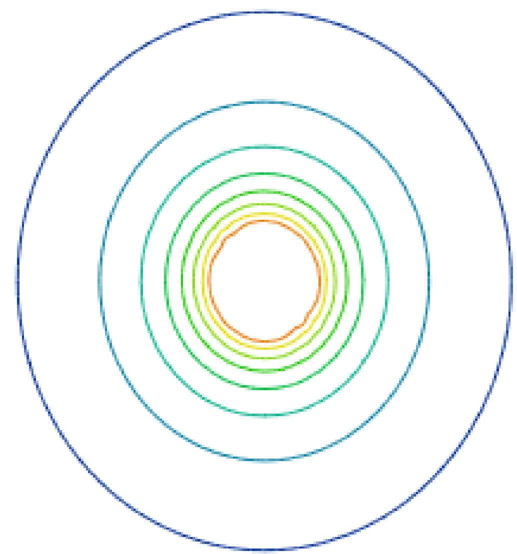
for any point at a distance r from the charge q.

Displaying the Electric Potential

- Potential due to a point charge:

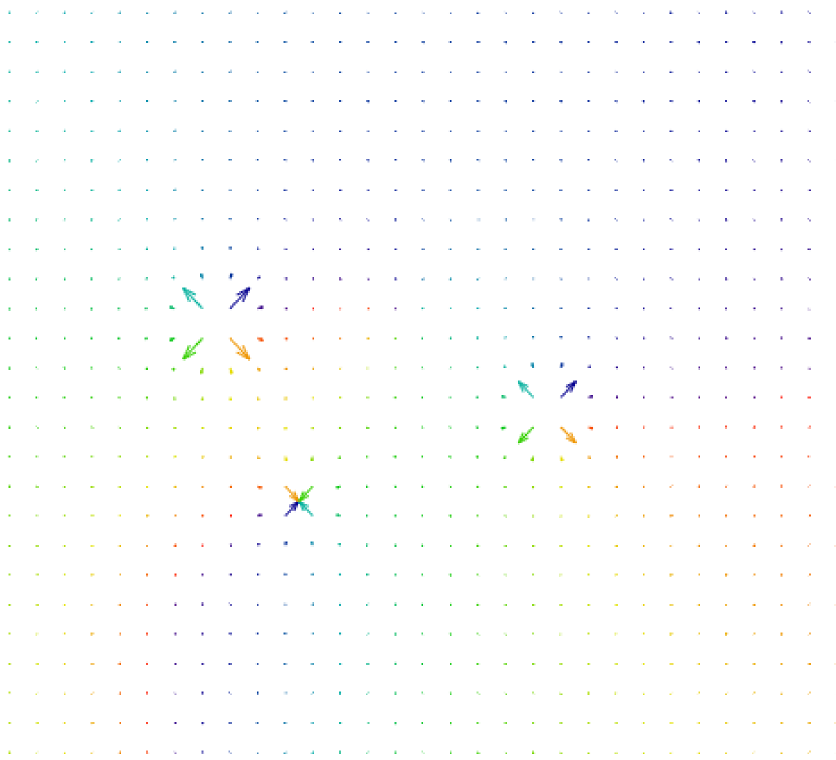


- Electric potential indicated using equipotentials (c.f. contour lines on Ordnance Survey map!).

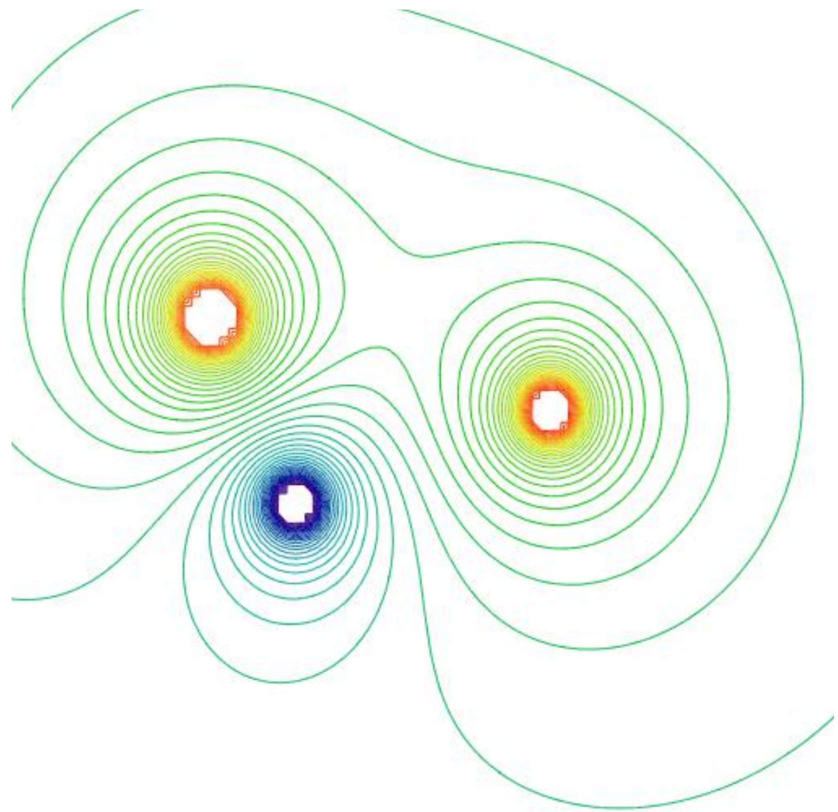


Electric Field and Associated Electric Potential

- Electric field due to 3 point charges:



- Associated electric potential:

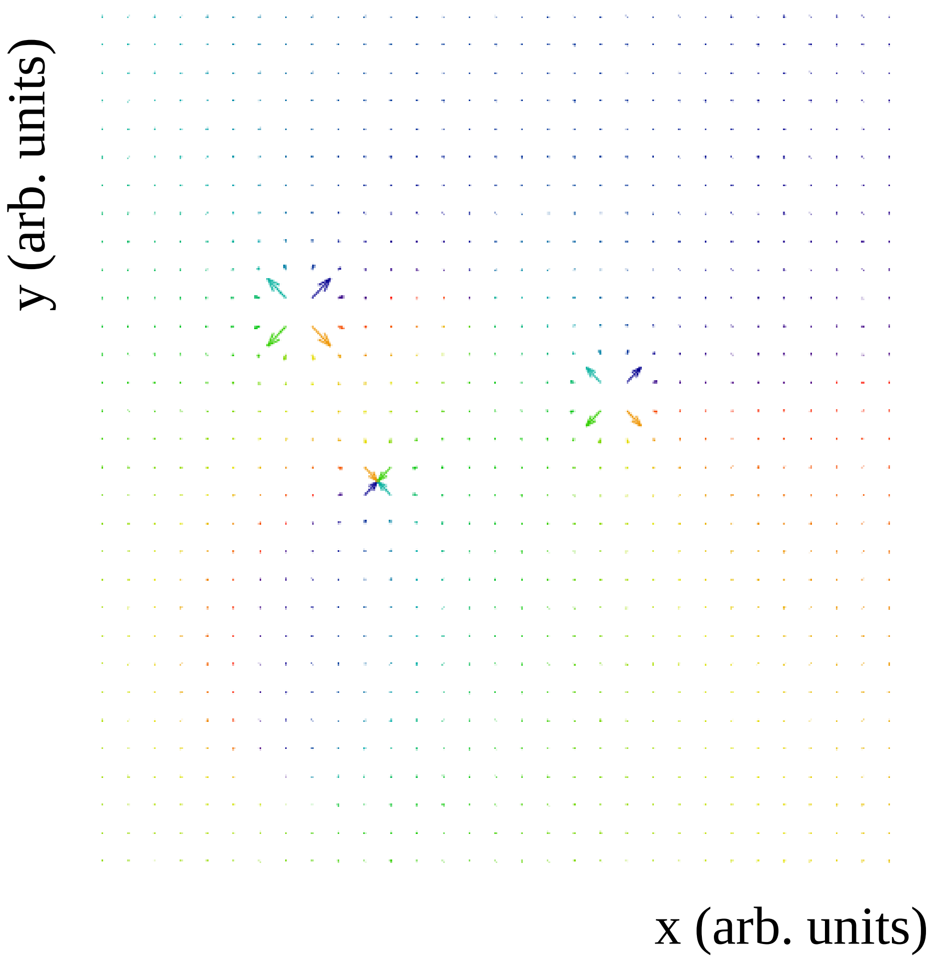


- Principle of superposition for potentials:

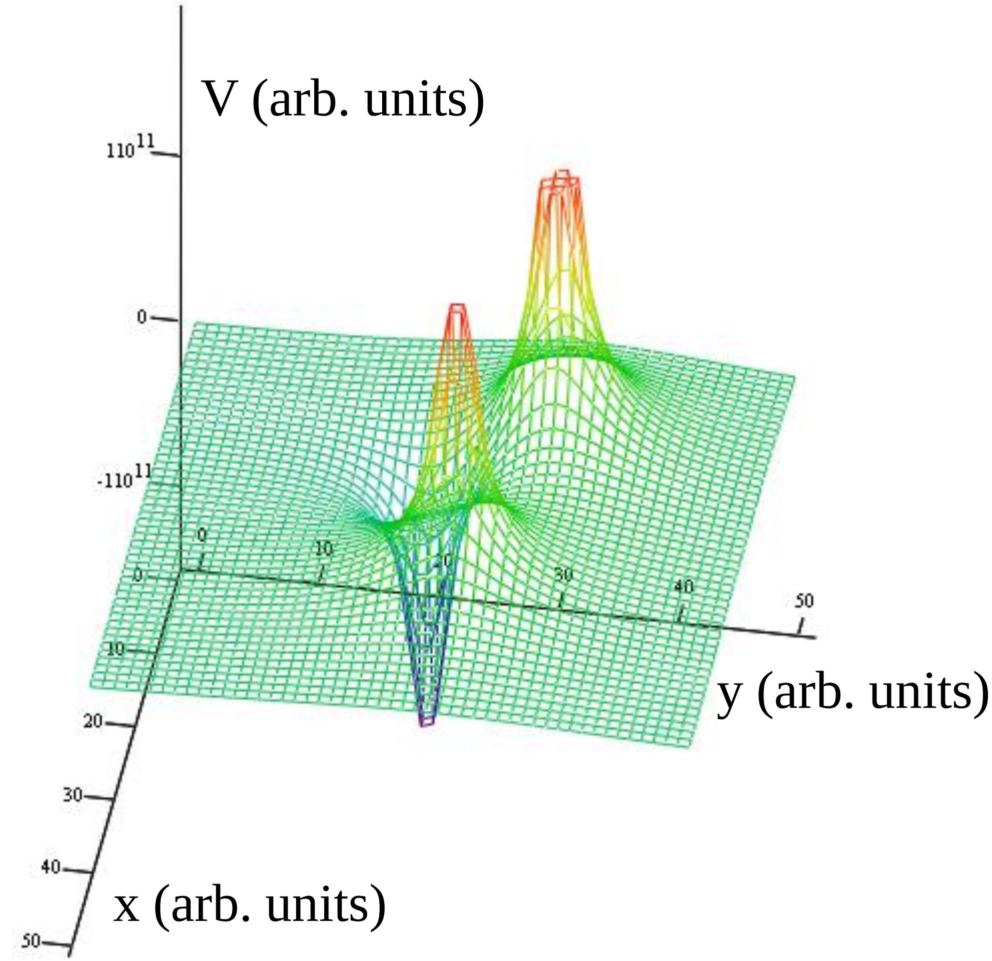
$$V = V_1 + V_2 + \ldots = \sum_{i}^{N} V_i \quad [4.4]$$

Electric Field and Associated Electric Potential

- Electric field due to 3 point charges:

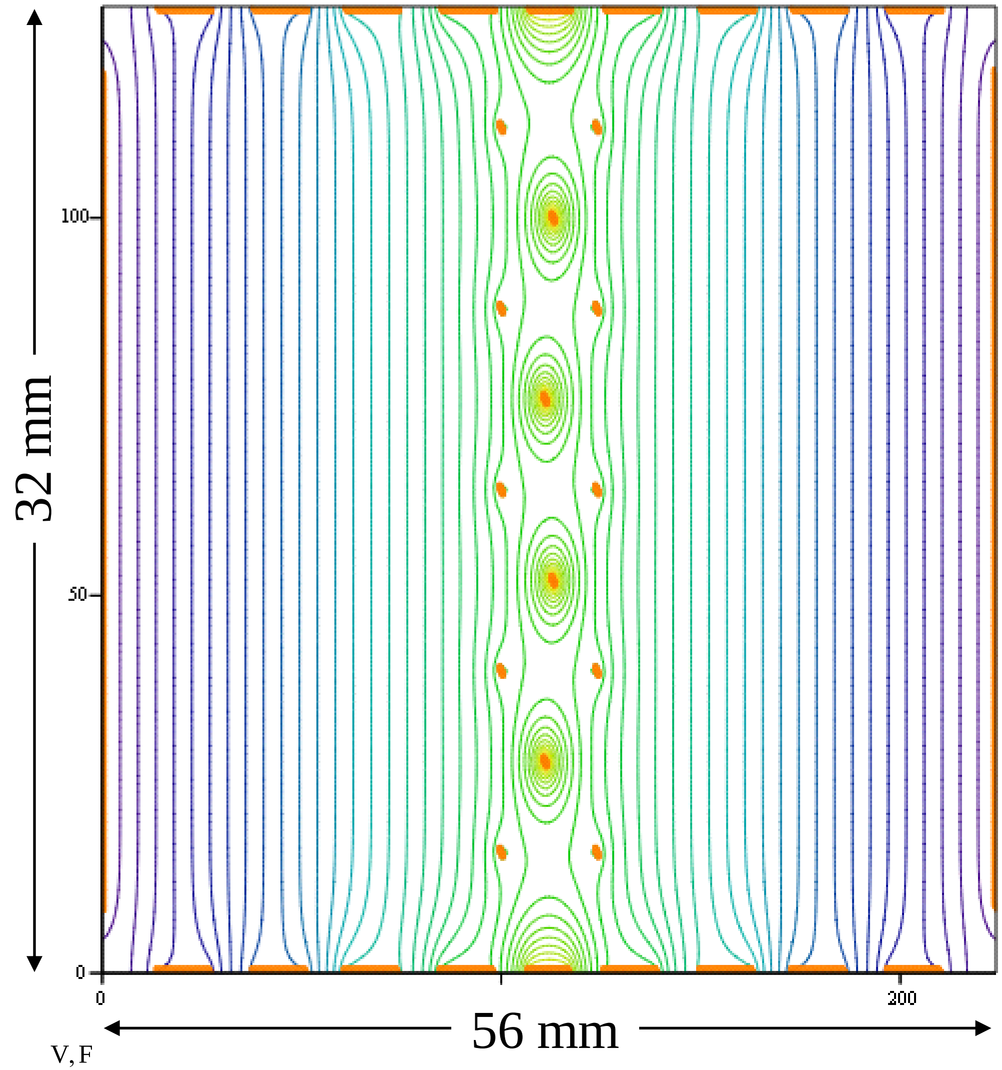


- Associated electric potential:



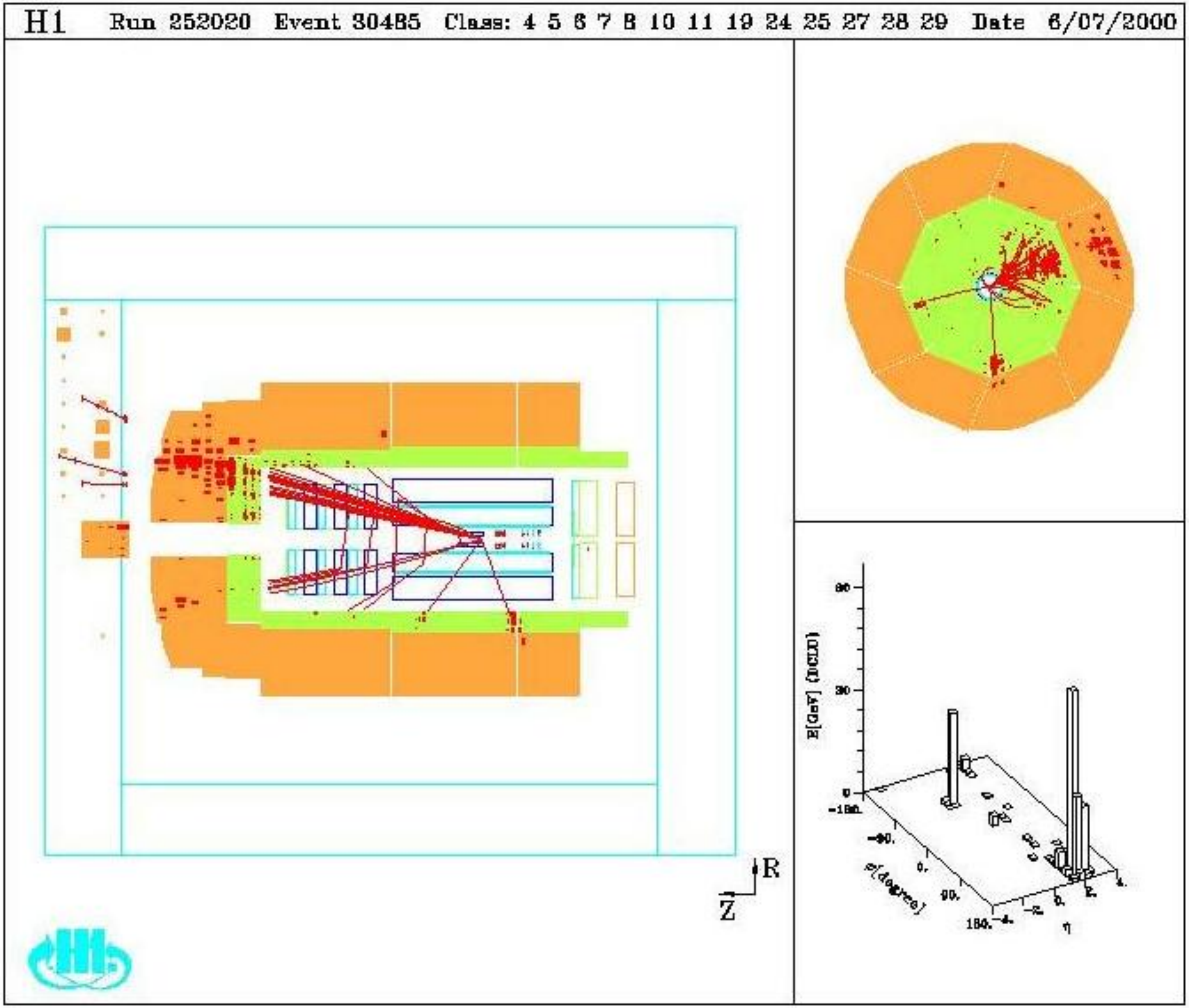
Electric Potential in a Drift Chamber

- Electric potential in drift chamber illustrated using equipotentials.
- Electric field always normal to equipotentials.
- Electrons produced in drift volume by high energy charged particle passing through gas in chamber.
- Electrons drift along electric field lines to anode wires (central potential wells) where they produce electrical signals.
- Drift electric field ~ 1 MV/m.
- Using information on time taken for electrons to reach wires, reconstruct path of high energy charged particle.

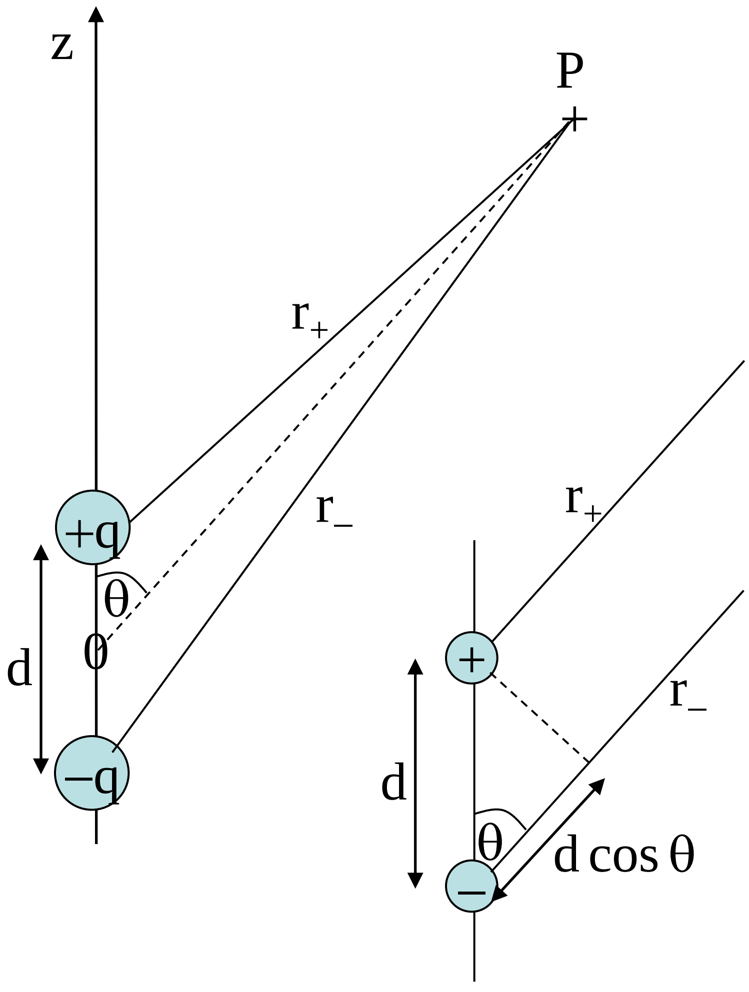


H1 Drift Chambers

- Event display of H1 detector at the HERA electron-proton collider showing the paths of charged particles and the energy deposited in the detector's calorimeters.



Potential due to Dipole



- Adding potentials due to positive and negative charges:

$$V = V_+ + V_- = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_+} - \frac{q}{r_-}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{r_- - r_+}{r_+ r_-}\right).$$

- For $r >> d$: $r_+ r_- \approx r^2$,

$$r_- - r_+ \approx d\cos\theta.$$

- Hence

$$V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2} \qquad [5.5]$$